Learning to act in noisy contexts using deep proxy learning

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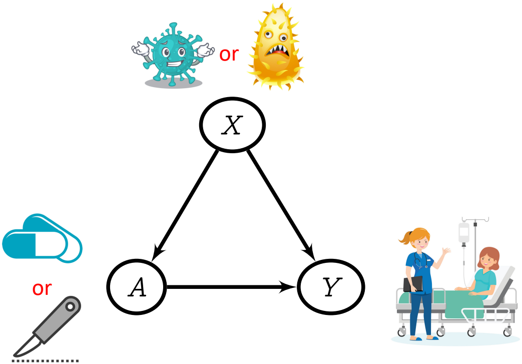
Observation vs intervention

Conditioning from observation: $\mathbb{E}[Y|A=a] = \sum_x \mathbb{E}[Y|a,x]p(x|a)$

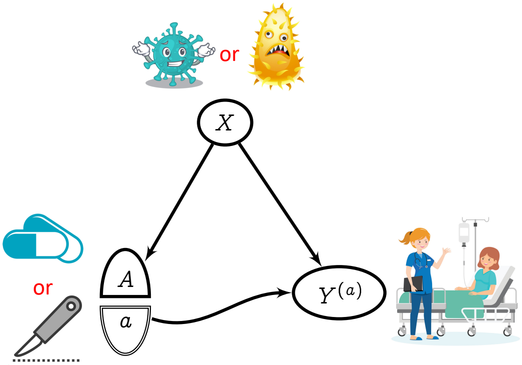


From our *observations* of historical hospital data:

- $P(Y = \text{cured}|A = \text{pills}) = 0.85$
- $P(Y = \text{cured}|A = \text{surgery}) = 0.72$

Observation vs intervention

Average causal effect (intervention): $\mathbb{E}[Y^{(a)}] = \sum_x \mathbb{E}[Y|a,x]p(x)$

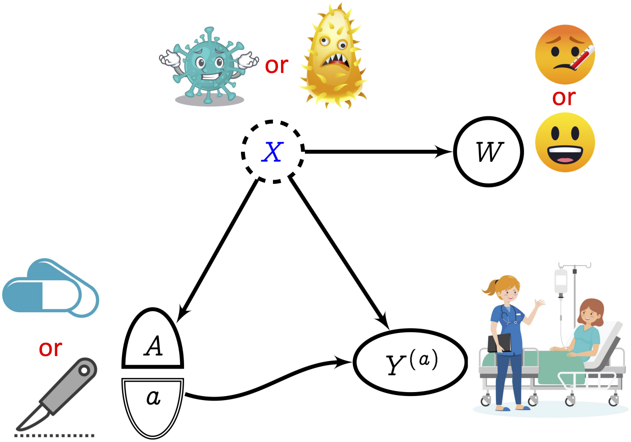


From our *intervention* (making all patients take a treatment):

- $P(Y^{(\text{pills})} = \text{cured}) = 0.64$
- $P(Y^{(\text{surgery})} = \text{cured}) = 0.75$

Richardson, Robins (2013), Single World Intervention Graphs (SWIGs): A Unification of the Counterfactual and Graphical Approaches to Causality

We record symptom W, not disease X



- $P(W = \text{fever} | X = \text{mild}) = 0.2$
- $P(W = \text{fever} | X = \text{severe}) = 0.8$

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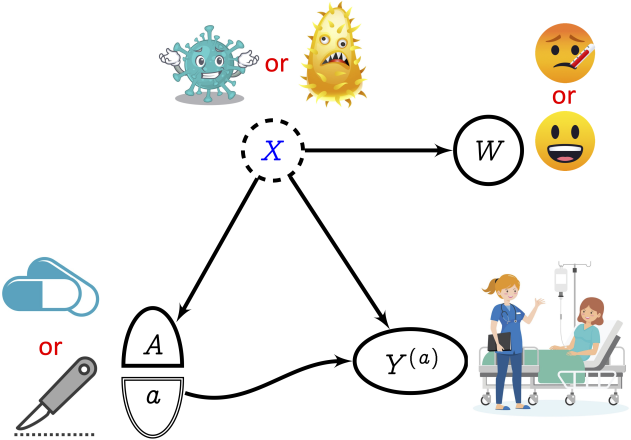

- $P(W = \text{fever} | X = \text{mild}) = 0.2$
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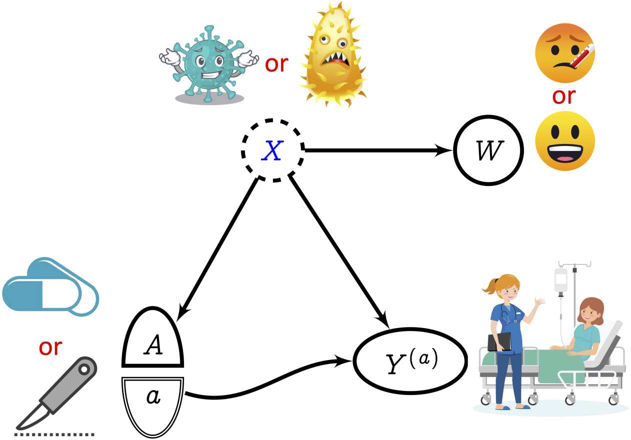
Could we just write: $P(Y^{(a)}) \overset{?}{=} \sum_{w \in \{0,1\}} \mathbb{E}[Y | a, w] p(w)$

We record symptom W, not disease X



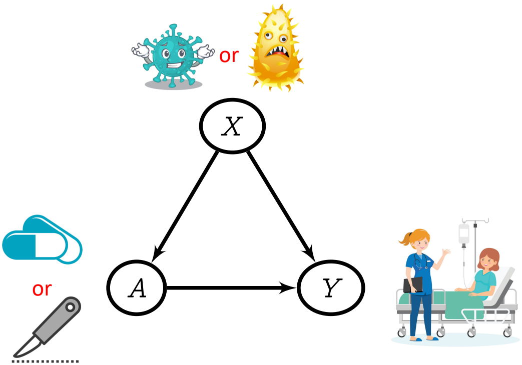
Wrong recommendation made:

- $\sum_{w \in \{0,1\}} \mathbb{E}[\text{cured}|\text{pills}, w]p(w) = 0.8 \quad (\neq 0.64)$
- $\sum_{w \in \{0,1\}} \mathbb{E}[\text{cured}|\text{surgery}, w]p(w) = 0.73 \quad (\neq 0.75)$

Correct answer impossible without observing X

Pearl (2010), On Measurement Bias in Causal Inference

Some core assumptions



Assume:

- Stable Unit Treatment Value Assumption (aka "no interference"),
- Conditional exchangeability $Y^{(a)} \perp\!\!\!\perp A | X$.
- Overlap.

Outline

Causal effect estimation, with hidden covariates X:

- Use proxy variables (negative controls)

Applications: effect of actions under

- privacy constraints (email, ads, DMA)
- data gathering constraints (edge computing)
- fundamental limitations (preferences, state of mind)

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What's new and why?

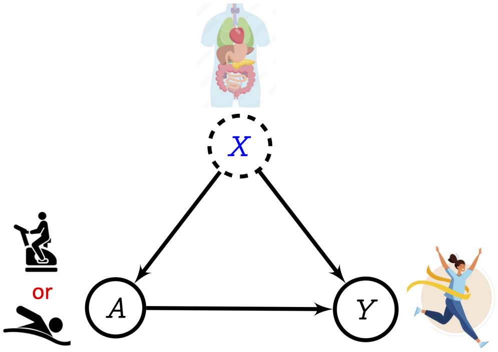
- Treatment A, proxy variables, etc can be multivariate, complicated...
- ...by using adaptive neural net feature representations
- Don't ~~meet your heroes~~ model your hidden variables!

What are proxies, and when are they useful?

Unobserved X with (possibly) complex nonlinear effects on A, Y

In this example:

- X: true physical status
- A: exercise regimes
- Y: fitness goal



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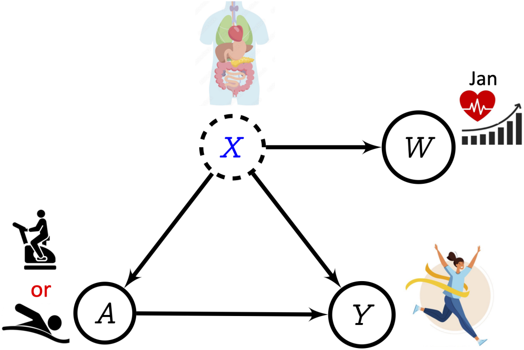

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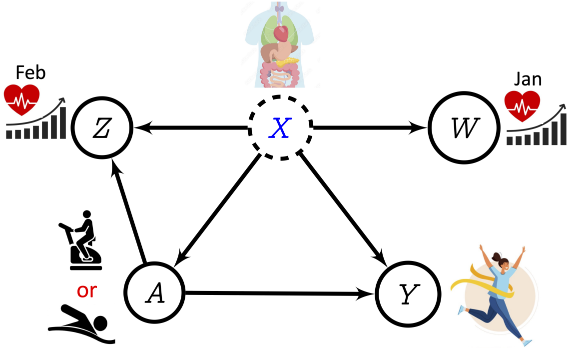

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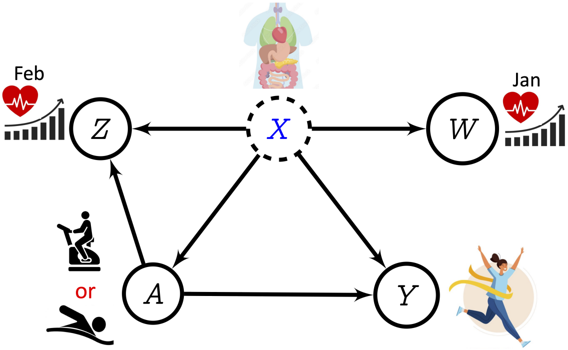

$\Longrightarrow$ Can recover $\mathbb{E}(Y^{(a)})$ from observational data

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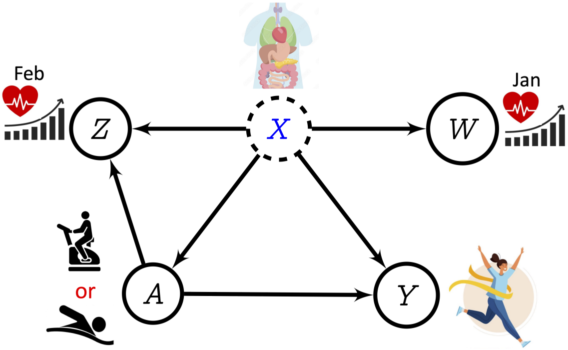


$\Longrightarrow$ Can recover $\mathbb{E}(Y^{(a)})$ from observational data

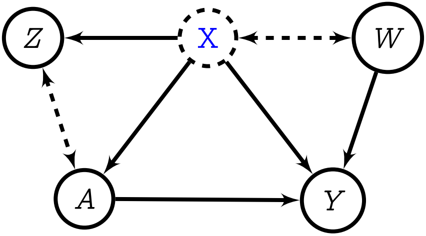
$\Longrightarrow$ More usefully: evaluate novel policy.

Proxy variables: general setting

Unobserved X with (possibly) complex nonlinear effects on A, Y

The definitions are:

- X: unobserved confounder.
- A: treatment
- Y: outcome
- Z: treatment proxy
- W outcome proxy



Miao, Geng, Tchetgen Tchetgen (2018): Identifying causal effects with proxy variables of an unmeasured confounder.

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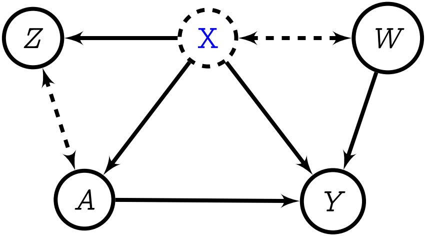

Structural assumptions:

$$W \perp\!\!\!\perp (Z, A) | X$$

$$Y \perp\!\!\!\perp Z | (A, X)$$

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Why proxy variables? A simple proof

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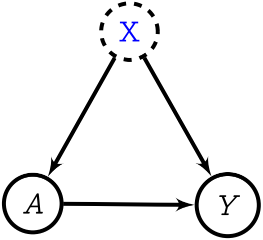

If X were observed,

$$\underbrace{P(Y^{(a)})}_{d_y \times 1} := \sum_{i=1}^{d_x} P(Y|x_i, a)P(x_i)$$

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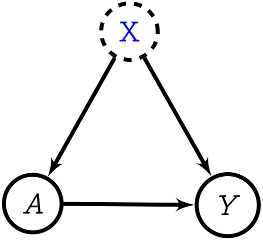

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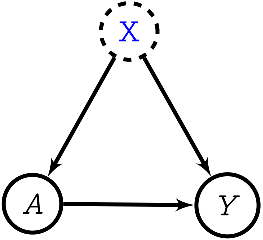

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Goal: "get rid of the blue" X

...add the outcome proxy W

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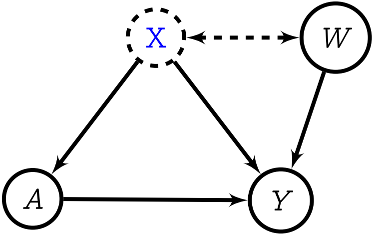

For each a, if we could solve:

$$\underbrace{P(Y|X, a)}_{d_y \times d_x} = \underbrace{H_{w,a}}_{d_y \times d_w} \underbrace{P(W|X)}_{d_w \times d_x}$$

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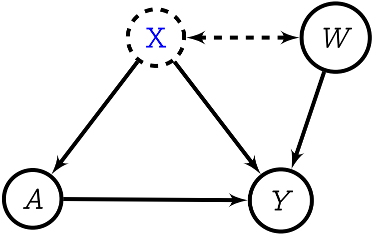

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.....then

$$P(Y^{(a)}) = P(Y|X,a)P(X)$$

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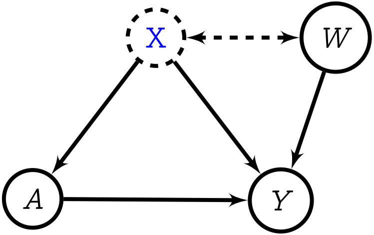

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$$= H_{w,a} P(W|X)P(X)$$

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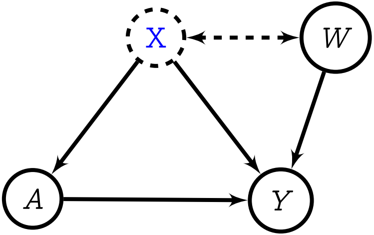

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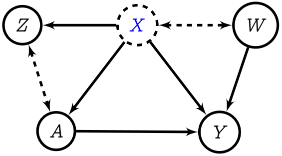
.....then

$$\begin{aligned} P(Y^{(a)}) &= P(Y|X,a)P(X) \\ &= H_{w,a}P(W|X)P(X) \\ &= H_{w,a}P(W) \end{aligned}$$

...now project onto $p(X|Z,a)$

From last slide,

$$P(Y|X,a) = H_{w,a}P(W|X)$$



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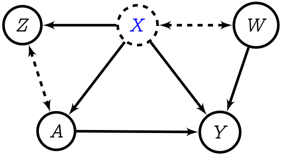

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$$P(Y|X,a)\underbrace{p(X|Z,a)}_{d_x \times d_z} = H_{w,a} P(W|X)\underbrace{p(X|Z,a)}_{d_x \times d_z}$$

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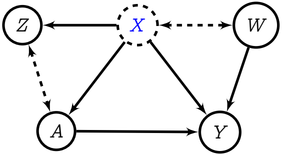

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$$P(Y|X,a)\underbrace{p(X|Z,a)}_{d_x \times d_z} = H_{w,a} P(W|X) \underbrace{p(X|Z,a)}_{d_x \times d_z}$$

Because $W \perp\!\!\!\perp (Z,A)|X$,

$$P(W|X)p(X|Z,a) = P(W|Z,a)$$

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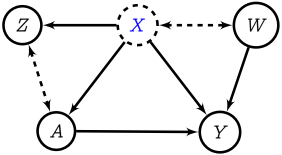

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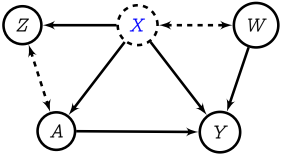

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Because $Y \perp\!\!\!\perp Z|(A,X)$,

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Solve for $H_{w,a}$:

$$P(Y|Z,a) = H_{w,a} P(W|Z,a)$$

Everything observed!

Proxy/Negative Control Methods in the Real World

Unobserved confounders: proxy methods

Kernel features (ICML 2021):

arXiv.org > cs > arXiv:2105.04544

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[Submitted on 10 May 2021 (v1), last revised 9 Oct 2021 (this version, v4)]

Proximal Causal Learning with Kernels: Two-Stage Estimation and Moment Restriction

Afsaneh Mastouri, Yuchen Zhu, Limor Gultchin, Anna Korba, Ricardo Silva, Matt J. Kusner, Arthur Gretton, Krikamol Muandet

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Deep Proxy Causal Learning and its Application to Confounded Bandit Policy Evaluation

Liyuan Xu, Heishiro Kanagawa, Arthur Gretton

Code for NN and kernel proxy methods:

https://github.com/liyuan9988/DeepFeatureProxyVariable/

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Road map: NN proxy learning

We'll proceed as follows:

- Proxy relation for continuous variables
- Loss function for deep proxy learning
- Define primary (ridge) regression with this loss
- Define secondary (ridge) regression as input to primary

Proxy relation, general domains

If X were observed, we would write (dose-response curve)

$$\mathbb{E}(Y^{(a)}) = \int_x \mathbb{E}(Y|a, x)p(x)dx.$$

....but we do not observe X.

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Main theorem: Assume we solved for link function:

$$\mathbb{E}(Y|a, z) = \mathbb{E}_{W|a,z} h_y(W, a)$$

- "Primary" $\mathbb{E}(Y|a, z)$, "secondary" $\mathbb{E}_{W|a,z}$ linked by h_y
- All variables observed, X not seen *or modeled.*

Fredholm equation of first kind. Link existence requires $\diamondsuit$, identification of ATE requires $\triangle$ (and further technical assumptions) [XKG: Asspumption 2, Prop. 1,Corr. 1; Deaner]

$$\mathbb{E}[f(X)|A = a, Z = z] = 0,\ \forall (z, a) \iff f(X) = 0,\ \mathbb{P}_X \text{ a.s.} \quad \triangle$$

$$\mathbb{E}[f(X)|A = a, W = w] = 0,\ \forall (w, a) \iff f(X) = 0,\ \mathbb{P}_X \text{ a.s.} \quad \diamondsuit$$

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Dose-response curve via $p(w)$:

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Dose-response curve via $p(w)$:

$$\mathbb{E}(Y^{(a)}) = \int_w h_y(a,w)p(w)dw$$

Challenge: need a loss function for h_y

Primary loss function for $h_y(w, a)$

Goal:

$$\mathbb{E}(Y|a, z) = \mathbb{E}_{W|a,z} h_y(W, a)$$

Primary loss function:

$$\hat{h}_y = \arg\min_{h_y} \mathbb{E}_{Y,A,Z} \left(Y - \mathbb{E}_{W|A,Z} h_y(W, A) \right)^2$$

Why?

Deaner (2021).

Mastouri, Zhu, Gultchin, Korba, Silva, Kusner, G., Muandet (2021).

Xu, Kanagawa, G. (2021).

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Why?

$f^*(a, z) = \mathbb{E}(Y|a, z)$ solves

$$\underset{f}{\operatorname{argmin}}\, \mathbb{E}_{Y,A,Z} (Y - f(A, Z))^2$$

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$f^*(a, z) = \mathbb{E}(Y|a, z)$ solves

$$\operatorname*{argmin}_{f} \mathbb{E}_{Y,A,Z} (Y - f(A, Z))^2$$

...and by the proxy model above,

$$\mathbb{E}(Y|a, z) = \mathbb{E}_{W|a,z} h_y(W, a)$$

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Mastouri, Zhu, Gultchin, Korba, Silva, Kusner, G., Muandet (2021).
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NN for link $h_y(a, w)$

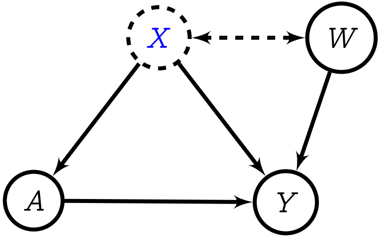
The link function is a function of two arguments

$$h_y(a, w) = \gamma^\top [\varphi_\theta(w) \otimes \varphi_\xi(a)] = \gamma^\top \begin{bmatrix} \varphi_{\theta,1}(w)\varphi_{\xi,1}(a) \\ \varphi_{\theta,1}(w)\varphi_{\xi,2}(a) \\ \vdots \\ \varphi_{\theta,2}(w)\varphi_{\xi,1}(a) \\ \vdots \end{bmatrix}$$

Assume we have:

- output proxy NN features $\varphi_\theta(w)$
- treatment NN features $\varphi_\xi(a)$
- linear final layer γ

(argument of feature map indicates feature space)



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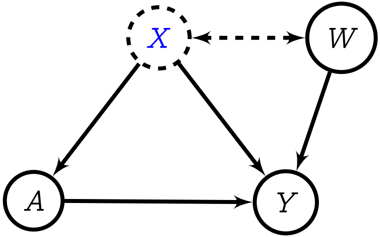
- linear final layer γ

(argument of feature map indicates feature space)



Questions:

- Why feature map $\varphi_\theta(w) \otimes \varphi_\xi(a)$?
- Why final linear layer γ?

Both are necessary (next slide)!

NN ridge regression for $h_y(w, a)$

Goal:

$$\mathbb{E}(Y|a, z) = \mathbb{E}_{W|a,z} h_y(W, a)$$

Primary regression:

$$\hat{h}_y = \arg\min_{h_y} \mathbb{E}_{Y,A,Z} \left(Y - \mathbb{E}_{W|A,Z} h_y(W, A) \right)^2 + \lambda_2 \|\gamma\|^2$$

Deaner (2021).
Mastouri, Zhu, Gultchin, Korba, Silva, Kusner, G., Muandet (2021).
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How to get conditional expectation $\mathbb{E}_{W|a,z} h_y(W, a)$?

Density estimation for $p(W|a, z)$? Sample from $p(W|a, z)$?

Deaner (2021).
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Recall link function

$$h_y(W, a) = \qquad \left[\gamma^\top \left(\varphi_\theta(W) \otimes \varphi_\xi(a) \right) \right]$$

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(this is why linear γ and feature map $\varphi_\theta(w) \otimes \varphi_\xi(a)$)

Deaner (2021).
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Ridge regression (again!)

$$\mathbb{E}_{W|a,z} \varphi_\theta(W) = \hat{F}_{\theta,\zeta} \varphi_\zeta(a, z)$$

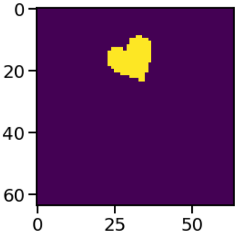
Deaner (2021).
Mastouri, Zhu, Gultchin, Korba, Silva, Kusner, G., Muandet (2021).
Xu, Kanagawa, G. (2021).

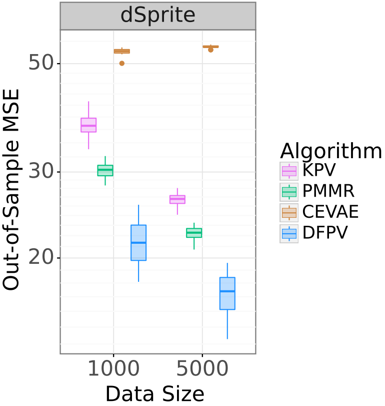
Experiments

Synthetic experiment, adaptive neural net features

dSprite example:

- $X = \{\texttt{scale}, \texttt{rotation}, \texttt{posX}, \texttt{posY}\}$
- Treatment A is the image generated (with Gaussian noise)
- Outcome Y is quadratic function of A with multiplicative confounding by posY.
- $Z = \{\texttt{scale}, \texttt{rotation}, \texttt{posX}\}$,
  W = noisy image sharing posY
- Comparison with CEVAE (Louzios et al. 2017)



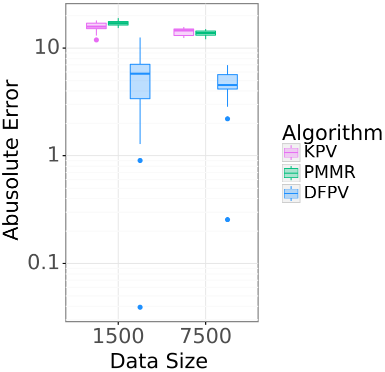


Louizos, Shalit, Mooij, Sontag, Zemel, Welling, Causal Effect Inference with Deep Latent-Variable Models (2017)

Confounded offline policy evaluation

Synthetic dataset, demand prediction for flight purchase.

- Treatment A is ticket price.
- Policy $A \sim \pi(Z)$ depends on fuel price.

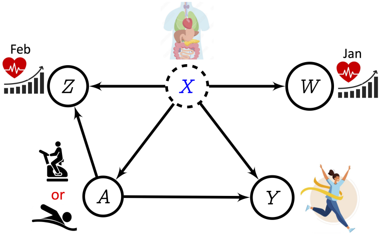


Conclusion

Causal effect estimation with unobserved X, (possibly) complex nonlinear effects on A, Y

We need to observe:

- Treatment proxy Z (interacts with A, but not directly with Y)
- Outcome proxy W (no direct interaction with A, can affect Y)



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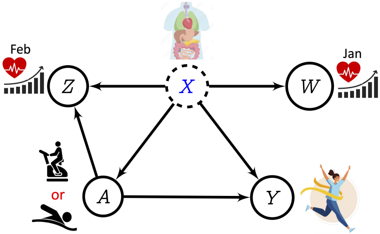

Key messages:

- Don't ~~meet your heroes~~ model/sample latents X
- Don't model all of W, only relevant features for Y
- "Ridge regression is all you need"

Code available:

https://github.com/liyuan9988/DeepFeatureProxyVariable/

Research support

Work supported by:

The Gatsby Charitable Foundation

Google Deepmind

Questions?


THE KERNEL
BREWERY
LONDON
BALTIC PORTER
7.3% ABV
THE KERNEL
EXPORT STOUT
LONDON

Failures of completeness assumptions (1)

Recall (one of the) completeness assumptions:

$$\mathbb{E}[f(X)|A = a, Z = z] = 0, \forall(a, z) \iff f(X) = 0, \ \mathbb{P}_X \text{ a.s.} \quad (\triangle)$$

For conciseness, assume conditioning on some a.

Failure 1: $Z \perp\!\!\!\perp X$ (no information about X in proxy)

$$g(X|) = \tilde{g}(X) - \mathbb{E}_X \tilde{g}(X)$$
$$\mathbb{E}(g(X)|Z, a) = \mathbb{E}g(X) = 0.$$

Failures of identifiability assumptions (2)

Failure 2: "exploitable invariance" of $p(X|z)$

$$X \sim \mathcal{N}(0,1),$$
$$Z = |X| + \mathcal{N}(0,1),$$

where $p(X|z) \propto p(z|X)p(X)$ symmetric in X. Consider square integrable *antisymmetric* function $g(X) = -g(-X) \neq 0$. Then

$$\mathbb{E}[g(X)|Z=z] = \int_{-\infty}^{\infty} g(X)p(X|z)dX$$
$$= \int_{-\infty}^{0} g(X)p(X|z)dX + \int_{0}^{\infty} g(X)p(X|z)dX$$
$$= 0.$$

If distribution of $X|Z$ retains the same "symmetry class" over a set of Z with nonzero measure, then the assumption is violated by $g(X)$ with zero mean on this class.