Non-negative matrix factorization implemented by (anti-)Hebbian neural network

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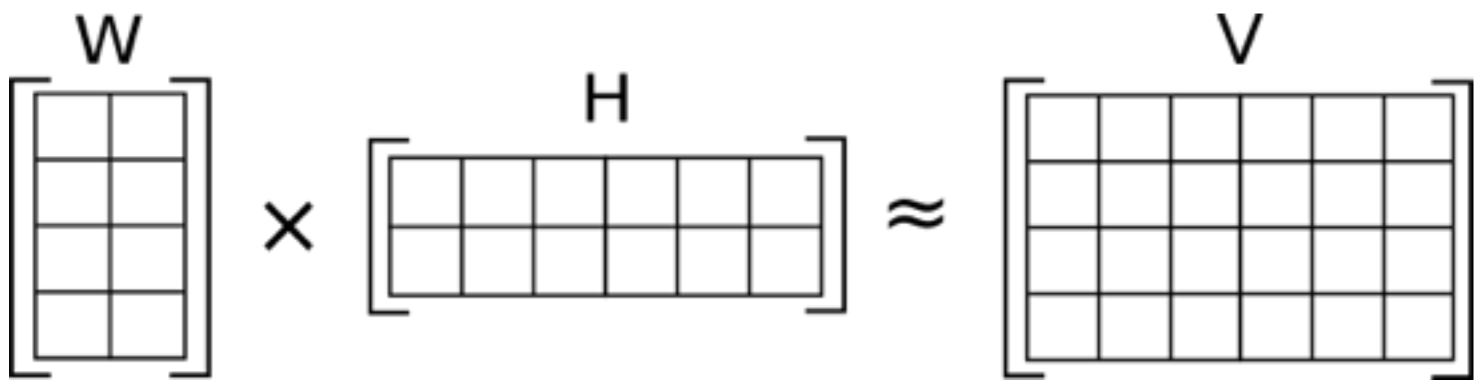
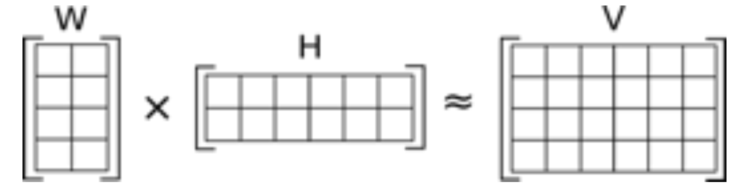
Tea talk, 19 November 2015

Non-negative matrix factorization

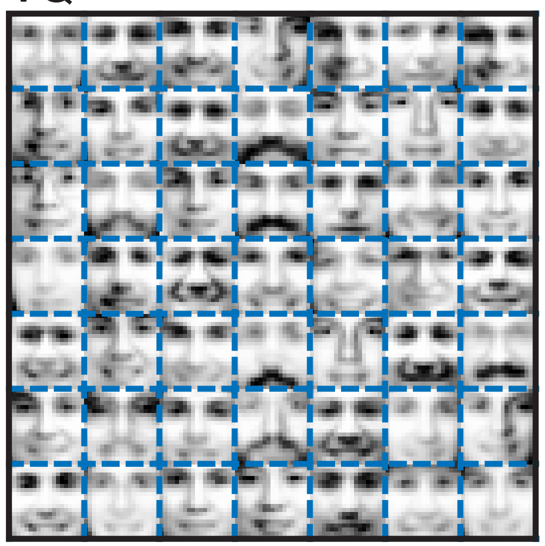


$$V_{i\mu} \approx (WH)_{i\mu} = \sum_{a=1}^{r} W_{ia} H_{a\mu}$$

Lee & Seung, Nature 1999

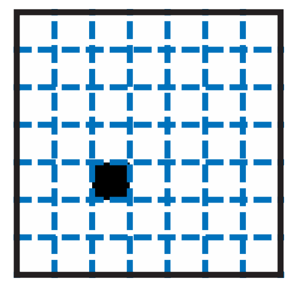


VQ



Columns of H
constrained to be unary

×



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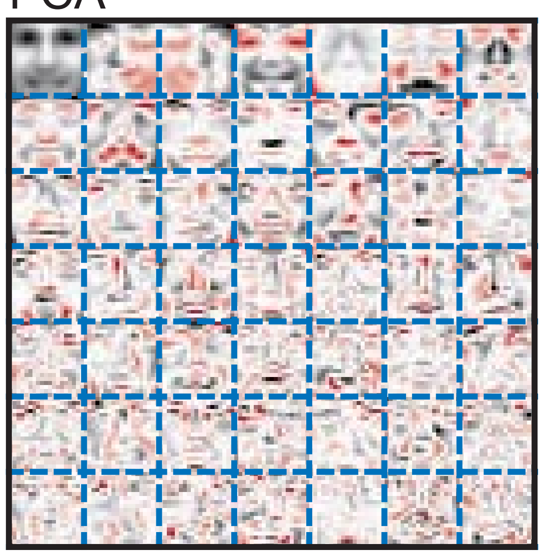
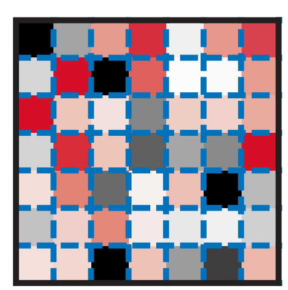
PCA



Columns of W constrained
to be orthonormal

×



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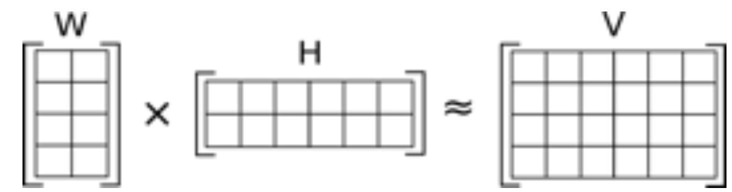
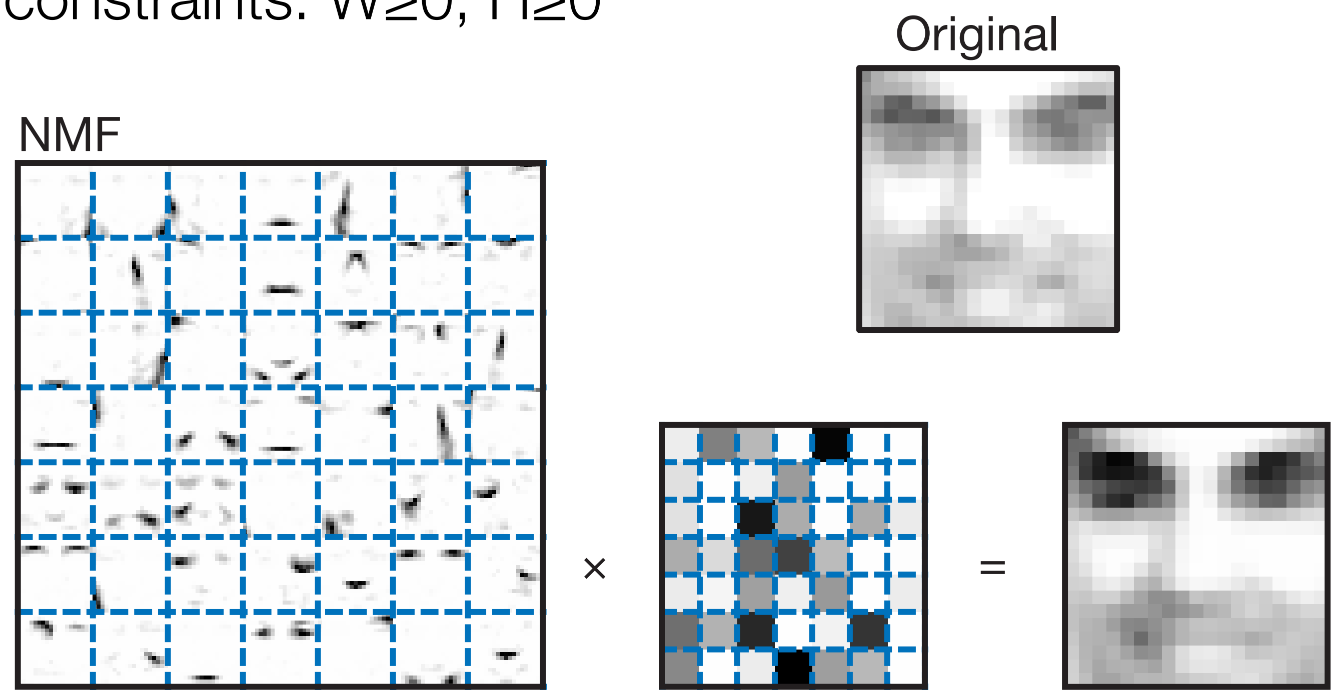
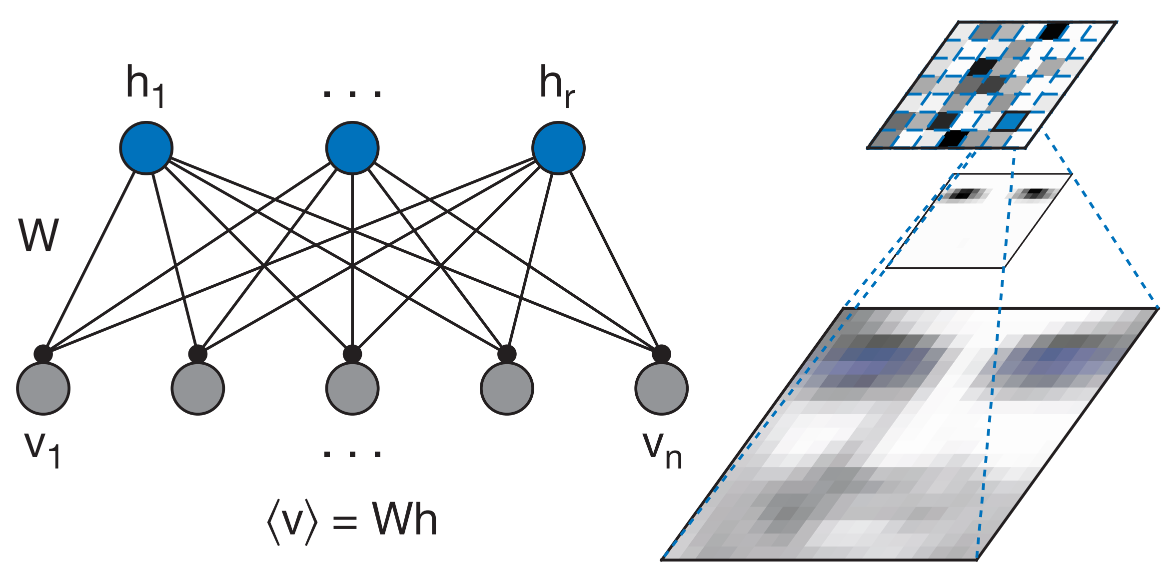


constraints: W≥0, H≥0



Find W, H by minimizing:

$$F = \sum_{i=1}^{n} \sum_{\mu=1}^{m} [V_{i\mu} \log(WH)_{i\mu} - (WH)_{i\mu}]$$



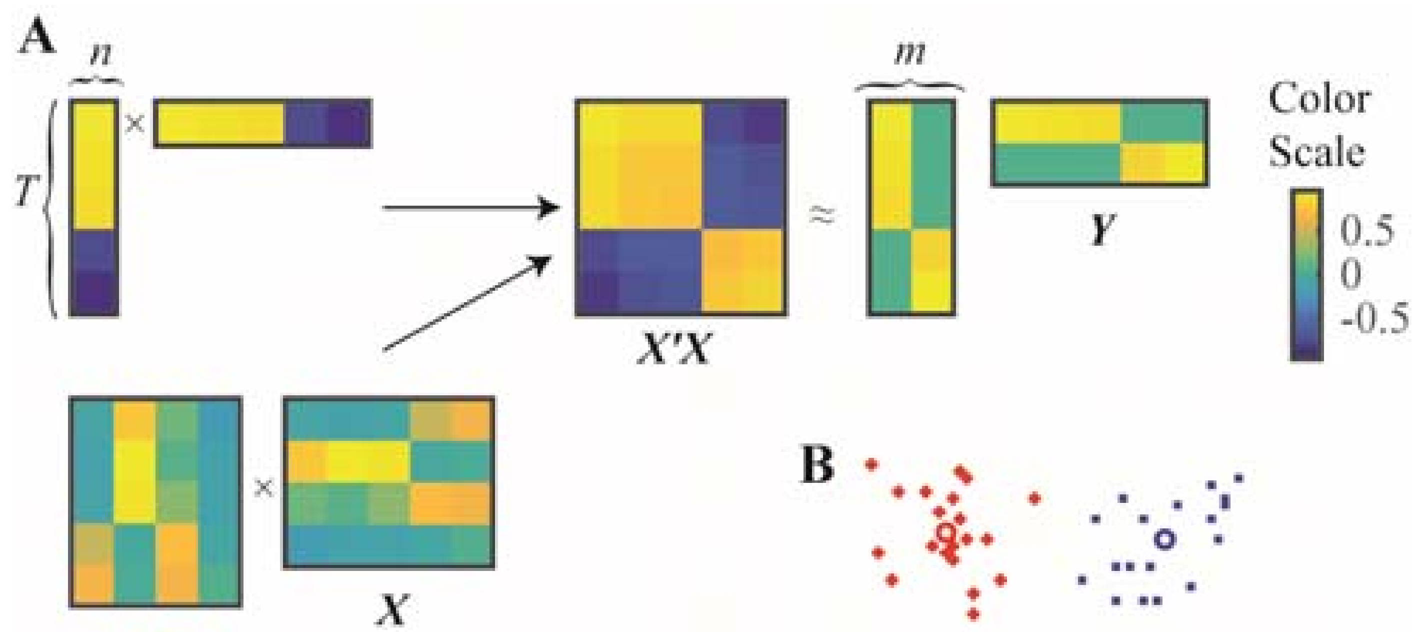
A Hebbian/Anti-Hebbian Network Derived from Online Non-Negative Matrix Factorization Can Cluster and Discover Sparse Features

C. Pehlevan, D.B. Chklovskii, 2015

- Connection between K-means and SNMF
- online SNMF performed by a single layer neuronal network
- Why K-means (therefore SNMF) discovers sparse features

  - k-means discovers ICA filters when independent components are sparse

SNMF vs k-means clustering



k-means

$$\min_{\{k\}} \sum_{k=1}^{K} \sum_{t \in C_k} \left\| \boldsymbol{x}_t - \frac{1}{n_k} \sum_{s \in C_k} \boldsymbol{x}_s \right\|_2^2$$

$$\boldsymbol{Y}^* = \arg\min_{\boldsymbol{Y} \in \Pi} \frac{1}{2} \left\| \boldsymbol{X}'\boldsymbol{X} - \boldsymbol{Y}'\boldsymbol{Y} \right\|_F^2$$

k-means: Y is binary

SNMF: Y≥0

Online SNMF performed by a single layer neuronal network

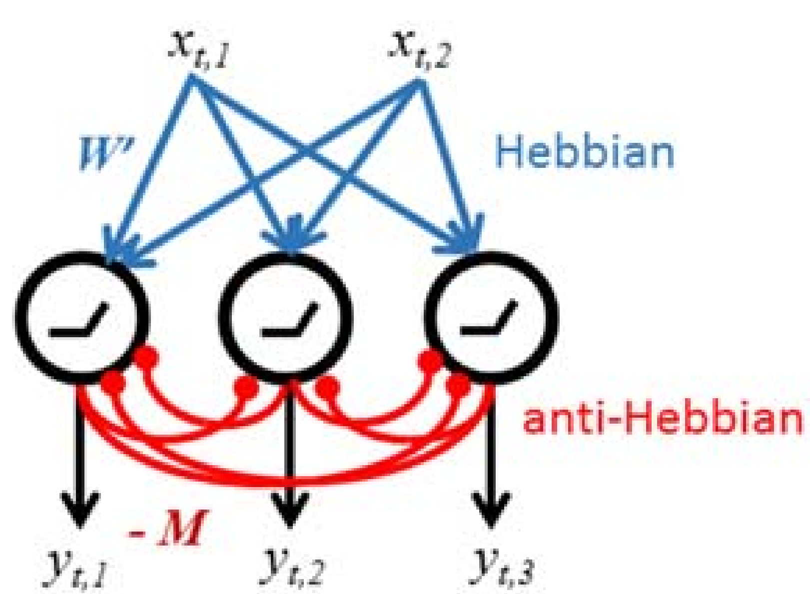
$$\boldsymbol{y}_T = \underset{\boldsymbol{y}_T \geq 0}{\arg\min} \left\| \boldsymbol{X}'\boldsymbol{X} - \boldsymbol{Y}'\boldsymbol{Y} \right\|_F^2 + \lambda \, \mathrm{rank}\left(\boldsymbol{Y}\right)$$

⋮ algebra

$$y_{T,i} \approx \underset{y_{T,i} \geq 0}{\arg\min} \left(\boldsymbol{W}_{T,i} \boldsymbol{x}_T - \boldsymbol{M}_{T,i} \boldsymbol{y}_T - y_{T,i} \right)^2$$

$$W_{T,i,j} = \frac{\sum_{t=1}^{T-1} y_{t,i} x_{t,j}}{\sum_{t=1}^{T-1} y_{t,i}^2} \qquad M_{T,i,k \neq i} = \frac{\sum_{t=1}^{T-1} y_{t,i} y_{t,k}}{\sum_{t=1}^{T-1} y_{t,i}^2}$$

$$y_{T,i} = \max\left(\boldsymbol{W}_{T,i} \boldsymbol{x}_T - \boldsymbol{M}_{T,i} \boldsymbol{y}_T , 0 \right)$$



SNMF network discovers sparse features in natural images