Drawing is much nicer than algebra

May 24, 2019
(Tea Talk)

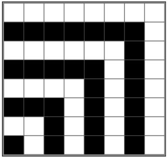
Proof without words - Definition

Wikipedia:

In mathematics, a proof without words is a proof of an identity or mathematical statement which can be demonstrated as self-evident by a diagram without any accompanying explanatory text. Such proofs can be considered more elegant than formal or mathematically rigorous due to their self-evident nature. When the diagram demonstrates a particular case of a general statement, to be a proof, it must be generalisable.

Proof without words - Simple Examples

- Sum of odd numbers is a perfect square



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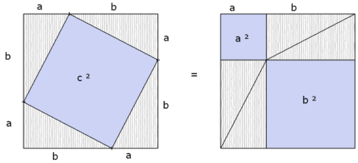

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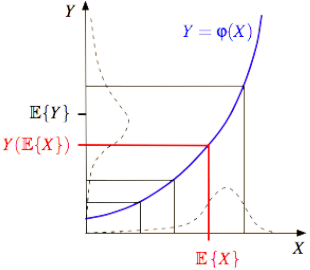

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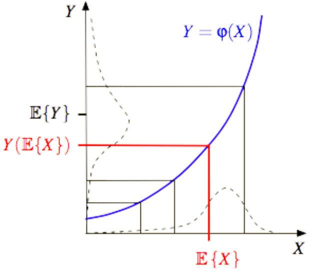

Wikipedia: `When the diagram demonstrates a particular case of a general statement, to be a proof, it must be generalisable.`

Proof without words - Several others

On Wikipedia, `Category:Proof without words` has a few more:

- Archimedes' inifinite geometric series
- Triangular number

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Feynman Diagrams

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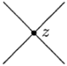
The rules for associating analytic expressions with pieces of diagrams are called the *Feynman rules*. In ϕ^4 theory the rules are:

1. For each propagator, 

$= D_F(x-y)$;

2. For each vertex, 

$= (-i\lambda)\int d^4z$;

3. For each external point, 

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4. Divide by the symmetry factor.

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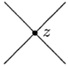
2. For each vertex, 

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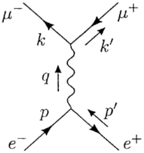


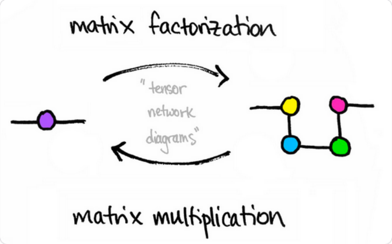
$$= \bar{v}^{s'}(p')(-ie\gamma^{\mu})u^{s}(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^{r}(k)(-ie\gamma^{\nu})v^{r'}(k').$$

Tensor Network Diagrams

Tai-Danae Bradley
@math3ma

Following

A while ago, I blogged about a simple way to think about matrices, namely as bipartite graphs. Now I'd like to share yet another way to think about matrices: tensor network diagrams! Here, familiar things have nice pictures. New blog post! math3ma.com/blog/matrices- ...



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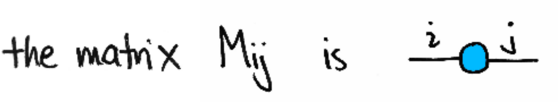
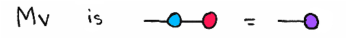
Tensor Network Diagrams

A matrix $M : \mathbb{R}^n \to \mathbb{R}^m$ can be represented by



https://www.math3ma.com/blog/matrices-as-tensor-network-diagrams

Tensor Network Diagrams



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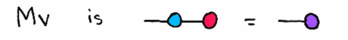

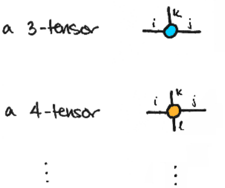

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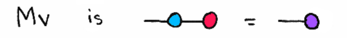

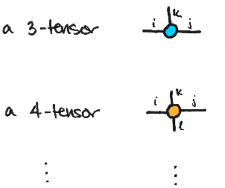

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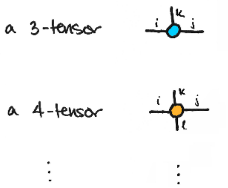

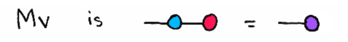

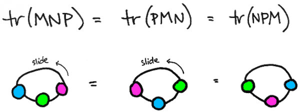

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Tensor Network Diagrams

- Matrix Product States (quantum mechanics)

A Practical Introduction to Tensor Networks: Matrix Product States and Projected Entangled Pair States

Roman Orus

(Submitted on 10 Jun 2013 (v1), last revised 10 Jun 2014 (this version, v3))

Tensor Network Diagrams

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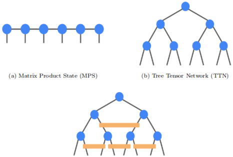


(a) Matrix Product State (MPS)

(b) Tree Tensor Network (TTN)

(c) Multiscale Entanglement Renormalization Ansatz (MERA)

Tensor Network Diagrams

- Matrix Product States (quantum mechanics)
- TensorFlow library
- Penrose graphical notation or tensor diagram notation
  - Kronecker delta
  - Levi-Civita antisymmetric tensor
  - Determinant, inverses, $\cdots$

String/Wiring Diagrams

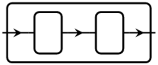
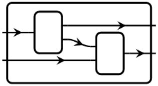
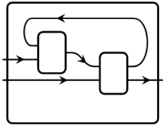
- Not just mapping between vector spaces, but any *monoidal category*

Examples of categories:

category's name:	its objects:	its morphisms:
Set	sets	functions
Group	groups	group homomorphisms
Top	topological spaces	continuous functions
Vect_k	vector spaces over a field, k	linear transformations
Meas	measurable spaces	measurable functions
Poset	partially ordered sets	order-preserving functions
Man	smooth manifolds	smooth maps
$\mathbb{R}$	the real numbers	the (total) order, $\leq$

String/Wiring Diagrams

- Not just mapping between vector spaces, but any *monoidal category*
- Seems to be quite useful in Category Theory

String/Wiring Diagrams

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- Actually, the whole idea of Algebra $\leftrightarrow$ Geometry comes from Category Theory