Assignment 6

Probabilistic and Unsupervised Learning

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Due: Thursday Dec 17, 2009

1. **[40 marks] Expectation-Propagation**

   Derive an EP algorithm for inference in the binary latent factor model of Assignment 4.

   (a) First, write down the log-joint probability $\log(p(\mathbf{s}, \mathbf{x}))$. Rearrange the terms to form a sum of log-factors $\sum_i \log f_i(\mathbf{s})$, where each f_i collects any terms that depend on s_i alone, as well as a "symmetric" share of the cross-terms. Remember that $s_i^2 = s_i$.

   (b) Next, find the expected value (with respect to q_{jn} for $j \neq i$) that minimises the partial KL:

   $$\mathsf{KL}\left[\prod_{j\neq i} q_{jn}(s_j^{(n)}) f_i(\mathbf{s}^{(n)}) \middle\| \prod_{j\neq i} q_{jn}(s_j^{(n)}) q_{in}(s_i^{(n)})\right]$$

   Here $q_{jn}(s_j^{(n)})$ is the jth factor in $q_n(\mathbf{s}^{(n)})$. It might help to reparameterise in terms of the natural parameter ρ_{jn} instead of λ_{jn} as above.

   (c) Finally, find the update rule for ρ_{in}. It might to differentiate the expected value derived above with respect to s_i.

2. **[10 marks]** Describe a Bayesian method for selecting K, the number of hidden binary variables in this model. Does your method pose any computational difficulties and if so how would you tackle them?

3. **[Bonus: 50 marks]** Implement the EP algorithm you derived above, and compare your results to those of the variational mean-field algorithm of Assignment 4.