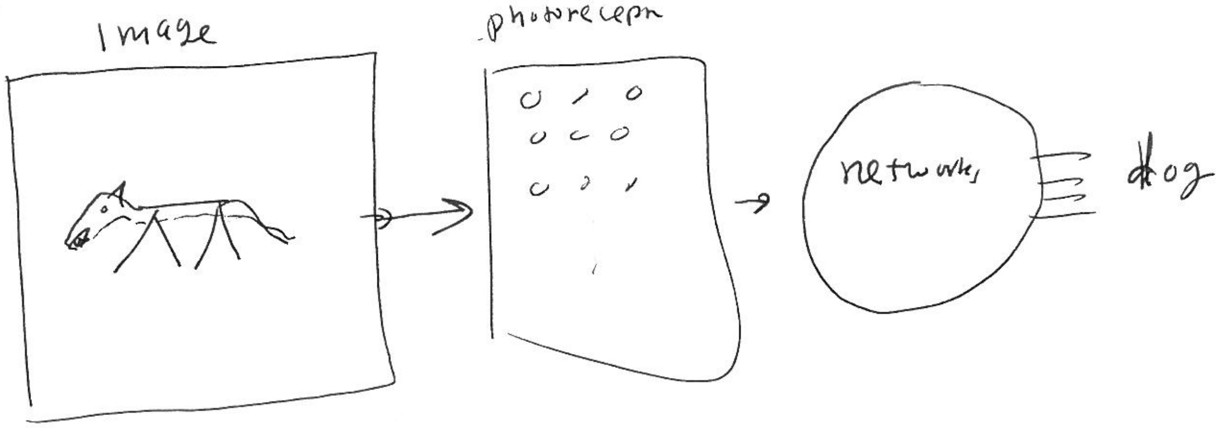
12/3/04

Be sure to state the problem!!
- At several levels!
- Also, why use the param Hopfield network [because it's neuron-like]

Non-random connectivity.

Feed-forward vs recurrent.

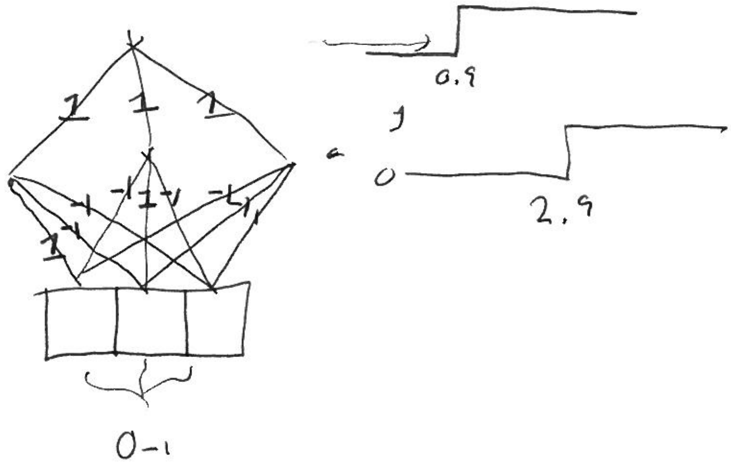


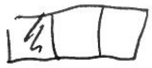
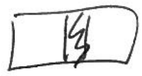
- lots of inputs map to the same output (lots of different types of dogs). Invariance

- all computations can be thought of as finding invariants (addition).

- how do we build a network to do this?

Method 1: matched filters



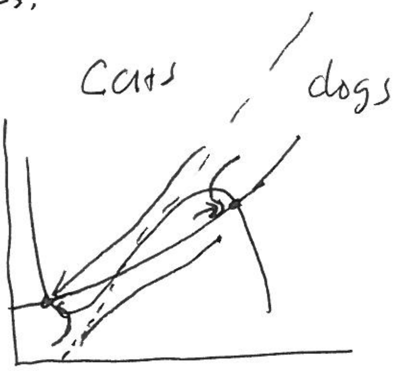
recognizes  or  or 

- can do the same thing w/ ~~do~~ images: one matched filter for each dog, and they all feed into the dog unit. In same network, can have one matched unit for each cat which feeds into the cat unit.

- theorem: this will work (any function can be computed)

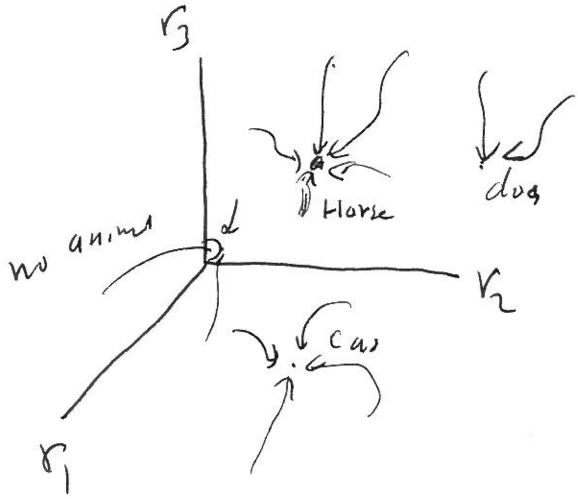
- problem: might need an exponentiall large number of weights.

Alternative: make use of network dynamics.



Very natural classifier

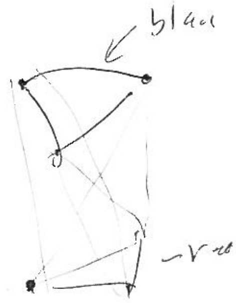
More sophisticated version:



- Associative memory / attractor network / Hopfield network

- How can we build a network w/ multiple stable states?

A possibility:



$$X_i(t+1) = \text{sign}\left(\sum_j J_{ij} X_j(t) \right)$$

$X_i = 1$: neuron is firing at high rate

$X_i = -1$: neuron is quiet

$J_{ij} = J_{ji}$ (symmetric)

$J_{ii} = 0$ (no autapses)

$$H(t) = -\frac{1}{2} \sum_{ij} X_i(t) J_{ij} X_j(t)$$

$$H(t+1) - H(t) = -\frac{1}{2} \sum_{ij} \left[X_i(t+1) J_{ij} X_j(t+1) - X_i(t) J_{ij} X_j(t) \right]$$

$i = k$ update $\quad X_k(t+1) \neq X_k(t)$ (maybe)

$\quad X_i(t+1) = X_i(t) \quad i \neq k$

$$H(t+1) - H(t) = -\frac{1}{2} \sum_j \Big[X_k(t+1) J_{kj} X_j(t) + X_j(t) J_{jk} X_k(t+1)$$

$$- \big(X_k(t) J_{kj} X_j(t) + X_j(t) J_{jk} X_k(t) \big) \Big]$$

~~legal~~ ← legal because $J_{jj} = 0$ $\qquad$ same because of symmetry

$$H(t+1) - H(t) = -\left[X_k(t+1) - X_k(t)\right]\sum_j J_{kj} X_j(t)$$

$$= -\left[\mathrm{sign}\left(\sum_j J_{kj} X_j(t)\right) - X_k(t)\right]\sum_j J_{kj} X_j(t)$$

$$\cancel{= -\left[\mathrm{sign}(h_k) - X_k(t)\right]}$$

$$\cancel{= -\left[\mathrm{sign}\right.}$$

$$= \left[X_k(t) - \mathrm{sign}(h_k)\right] h_k$$

$h_k > 0 \Rightarrow \le 0$ $\quad$ $> 0 \Rightarrow \le 0$

$h_k < 0 \Rightarrow \ge 0$ $\quad$ $< 0 \Rightarrow \le 0$

$h_k = 0 \Rightarrow$ $\quad$ 0

$$H(t+1) - H(t) \le 0$$

$$H(t+1) \le H(t)$$

How do we have lots of minima?

$$J_{ij} = \frac{\beta}{N} \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu$$

$$\xi_i^\mu = \begin{cases} 1 & \text{prob. } 1/2 \\ -1 & \text{prob } 1/2 \end{cases}$$

= random binary vector

approximate minimum:

$$X_i = \xi_i^\nu$$

$$\sum_j \left(\frac{\beta}{N} \sum_\mu \xi_i^\mu \xi_j^\mu \right) \xi_j^\nu$$

$$\xi_i^\nu \overset{?}{=} \mathrm{sign}\left(\frac{\beta}{N} \sum_\mu \xi_i^\mu \sum_j \xi_j^\mu \xi_j^\nu \right)$$

$$= \mathrm{sign}\left(\frac{\beta}{N} \left(\xi_i^\nu \sum_j \xi_j^\nu \xi_j^\nu + \sum_{\nu \neq \mu, j} \xi_i^\mu \xi_j^\mu \xi_j^\nu \right) \right)$$

$$= \mathrm{sign}\left(\underbrace{\beta \xi_i^\nu + \frac{\beta}{N} \underbrace{\sum_{\mu \neq \nu, j} \xi_i^\mu \xi_j^\mu \xi_j^\nu}_{O(\sqrt{(p-1)N})}}_{\beta \sqrt{\frac{p-1}{N}}} \right)$$

$$\approx \mathrm{sign}(\beta \xi_i^\nu) = \xi_i^\nu$$

OK if $p \ll n$

actually, OK if $p < 0.138N$

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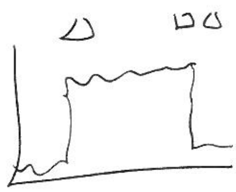
A problem: neurons fire at saturation. Can we fix this by having a more mellow gain function?

Exp. fact: ment in working memo

$$X_i = \tanh\Big[\beta \sum_\mu \xi_i^\mu \sum_j \xi_j^\nu X_j\Big]$$

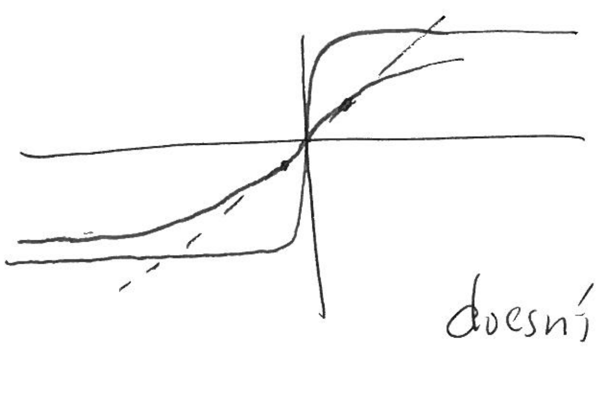


Ansatz: $X_i = a\,\xi_i^\mu$

Same analysis, $p \ll N$

$$a\,\xi_i^\mu = \tanh \beta a \xi_i^\mu$$

$$\Rightarrow \quad a = \tanh \beta a$$



doesn't help much!!!

Slightly more neurally plausible model:

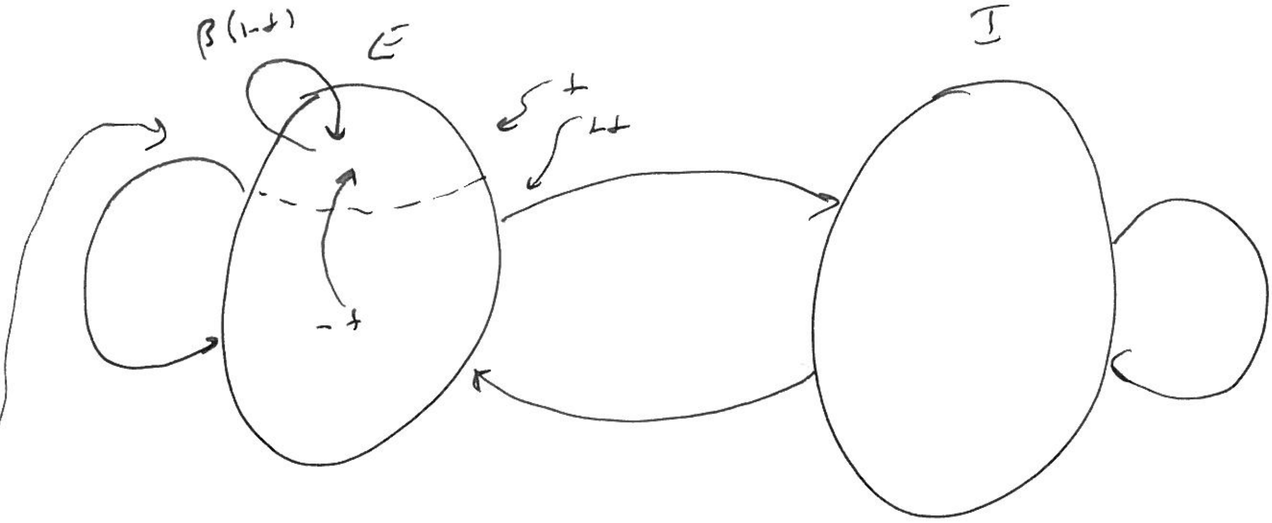
$$\dot{\nu}_{Ei} = \phi_E\left(\nu_E + \frac{\beta}{Nf(1-f)}\sum_j \eta_i^{\mu}(\eta_j^{\mu}-f)\,\nu_{Ej},\ \nu_I\right) - \nu_{Ei}$$

$+ \sum_j \delta w_{ij}\, \nu_{Ej}$

$+ \sum_{\rho\neq\mu} \eta_i^{\rho}(\eta_j^{\rho}-f)\,\nu_{Ej}$ ...

$$\dot{\nu}_I = \phi_I(\nu_E, \nu_I) - \nu_I$$

$$\eta_j^{\mu} = \begin{cases} 1 & \text{prob. } f \\ 0 & \text{prob } 1-f \end{cases} \quad \Rightarrow \quad \langle \eta \rangle = f$$



Three variables:

$$\nu_E = \frac{1}{N}\sum_i \nu_{Ei} \quad = \text{avg. exc. rate}$$

$$\nu_I \quad = \text{avg. inh rate}$$

$$m = \frac{1}{1-f}\left[\frac{1}{Nf}\sum_i (\eta_i - f)\,\nu_{Ei}\right]$$

$$= \frac{1}{1-f}\left[\frac{1}{Nf}\sum_i \eta_i \nu_{Ei} - \nu_E\right]$$

~~overlap~~ mean firing rate of sub populat

$$\cancel{\dot{\nu}_E = \frac{1}{N} \sum_i \phi_E(\nu_E + }$$

$$\frac{\beta}{Nf(1-f)} \sum_j \eta_i (\eta_j - f) \nu_{E_j} = \beta \eta_i m$$

$$\dot{\nu}_E = \frac{1}{N} \sum_i \phi_E(\nu_E + \beta \eta_i m, \nu_I) - \nu_E$$

$$= (1-f)\, \phi_E(\nu_E, \nu_I) + f\, \phi_E(\nu_E + \beta m, \nu_I) - \nu_E$$

$$= \phi_E(\nu_E, \nu_I) - \nu_E \quad + f \left[\phi_E(\nu_E + \beta m, \nu_I) - \phi_E(\nu_E) \right]$$

$$\dot{m} = \frac{1}{1-f} \left[\frac{1}{Nf} \sum_i \eta_i\, \phi_E(\nu_E + \beta m \eta_i, \nu_I) - \frac{1}{N} \sum_i \phi_E(\nu_E + \beta m \nu_i, \nu_I \right] - m$$

$$= \frac{1}{1-f} \left[\phi_E(\nu_E + \beta m, \nu_I) - \phi_E(\nu_E, \nu_I) - f \left[\phi_E(\nu_E + \beta m, \nu_I) - \phi_E(\nu_E, \nu_I) \right] \right] - m$$

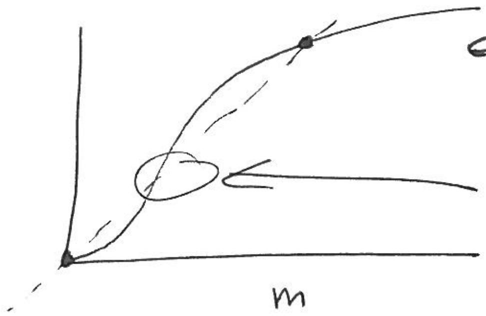
$$= \phi_E(\nu_E + \beta m, \nu_I) - \phi_E(\nu_E, \nu_I) - m$$

$$\dot{\nu}_I = \phi_I(\nu_E, \nu_I)$$

$f \to 0$

$$\left.\begin{aligned} \dot\nu_E &= \phi_E(\nu_E,\nu_I) - \nu_E \\ \dot\nu_I &= \phi_I(\nu_E,\nu_I) - \nu_I \end{aligned}\right\} \quad \nu_E, \nu_I \to \nu_{E0}, \nu_{I0}$$

$$\dot m = \underbrace{\phi_E(\nu_{E0}+\beta m, \nu_{I0}) - \phi_E(\nu_E,\nu_E) - m}$$



still at high ν

can be stabiliz
if f large enough