Assignment 3
Theoretical Neuroscience

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Question 1. Contrast saturation and nonspecific suppression.

1. Assume a V1 cell has a response kernel,

$$f_{\alpha,a,\psi}(x,y) = r_{max} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) \cos\left(a\cos(\psi)x + a\sin(\psi)y - \alpha\right)$$

where α is the preferred phase, ψ the preferred orientation and a the preferred frequency. We stimulate it with the following (static) stimulus:

$$s(x,y) = B\cos\left(b\cos(\phi)x + b\sin(\phi)y - \beta\right)$$

Compute analytically and plot the response

$$L_{\alpha,a,\psi}(x_0, y_0, \phi, \beta, b) = \iint_{-\infty}^{\infty} dx\, dy\, f_{\alpha,a,\psi}(x - x_0, y - y_0)s(x,y) \tag{1}$$

for $\alpha = \beta$ as a function of orientation ϕ for $\psi = 0$ and as a function of frequency b for $a = 1$, showing that this cell is tuned to both spatial frequency and orientation. Assume $x_0 = y_0 = 0$, $\sigma_x = \sigma_y = 1$.

Hint 1: Computing convolutions is much easier in the 2D Fourier transform space. Remember both the convolution theorem and the table of important Fourier transforms from Wikipedia. **Hint 2:** $\cos(x) = (e^{ix} + e^{-ix})/2$.

2. What should the limits of integration in Eq. (1) really be? In which case would it change anything?

3. Unlike the prediction from this model, responses of cells in visual cortex saturate at high contrasts and they also adapt. Furthermore, presentation of a grating at an orientation to which the cell shows no response prevents the cell from responding to a grating presented at its preferred stimulus orientation (nonspecific suppression). Heeger (1992) proposed a very simple modification of this model that accounts for these two effects. Let simple $SC_{\alpha,a,\psi}$ and complex cells $CC_{a,\psi}$ respond as:

$$SC = \frac{[L]_+^2}{F_{1/2}^2 + [L]_+^2}$$

$$CC = \frac{\sum_{\alpha=0,90,180,270}[L_\alpha]_+^2}{G_{1/2}^2 + \sum_{\alpha=0,90,180,270}[L_\alpha]_+^2}$$

where $[f]_+ = f$ if $f > 0$ and $= 0$ otherwise; $F_{1/2}$ and $G_{1/2}$ are constants, and we have suppressed the subscripts where possible. Show analytically that this formulation leads to saturation at high stimulus contrasts B. How might you modify the two models to produce nonspecific suppression? What might these expressions imply about cortical architecture?

Question 2. Images seen through visual receptive fields

1. Load the image provided into MATLAB/Python/etc.

2. Construct an on-centre difference-of-gaussians (DOG) centre-surround receptive field (RF):

$$D(x,y) = \frac{1}{2\pi\sigma_c^2}e^{-(x^2+y^2)/2\sigma_c^2} - \frac{1}{2\pi\sigma_s^2}e^{-(x^2+y^2)/2\sigma_s^2}.$$

   Make the receptive field on a 21-by-21 pixel grid, with a central Gaussian width of 1.5 pixels and a surround Gaussian width of 3 pixels. Note that the normalisation ensures that the RF has no DC component (i.e. the total influence of all pixels sums to zero for a constant input).

3. Suppose you had a cell with a receptive field like this centred at each pixel in the image. Show the image as represented by the activity of these cells, placing the cells in topographic order according to their centres.

   Hint: this is effectively a 2D convolution. Why? How would you pad missing values for the convolution?

4. Threshold the activity image (i.e. set all the values above some cutoff to 1, all below to 0). Does this look like the cells are detecting edges? Tune the parameters of the DOG RF and the threshold to improve the quality of the edge detection as much as you can.

5. Now construct a Gabor receptive field on the same 21-by-21 pixel grid:

$$D(\vec{x}) = \exp\left(-\frac{(\vec{k}(\theta)\cdot\vec{x})^2}{2\sigma_l^2} - \frac{(\vec{k}_\perp(\theta)\cdot\vec{x})^2}{2\sigma_w^2}\right)\cos\left(2\pi\frac{\vec{k}_\perp(\theta)\cdot\vec{x}}{\lambda} + \phi\right)$$

   where $\vec{x} \in \mathbb{R}^2$, $\vec{k}(\theta)$ is a unit vector with the orientation θ, $\vec{k}_\perp(\theta)$ is an orthogonal unit vector and θ, σ_l, σ_w, λ and ϕ parametrise the Gabor. Start with $\theta = \pi/2$, $\sigma_l = \sigma_w = 3$ pixels, $\lambda = 6$ pixels and $\phi = 0$.

6. Show the image as seen by through receptive fields of this sort, again with one centred at each pixel in the image. Threshold the activities. Does this picture match what you expected?

7. What parameter(s) determine(s) the orientation bandwidth of the Gabor (i.e. the range of orientations for which it will fire)?

   Derive a formal result using a cosine-grating stimulus with spatial frequency $\Omega_s = 2\pi/\lambda$ matching that of the Gabor, and orientation ϕ:

$$s(x,y) = \cos\left(\Omega_s\cos(\phi)x + \Omega_s\sin(\phi)y - \psi_s\right).$$

   Hint: by symmetry you can consider the case of $\vec{k}$ parallel to the y-axis and then use the result from Question 1.

8. Adjust the parameter(s) you identified in the part above to narrow the bandwidth, and look at the resulting image. Did it work?

9. Construct 3 Gabors with $\theta = 0, \pi/4, \pi/2$ and the other parameters as above. Sum the thresholded outputs of all three types of cell. Does this image look any better than that obtained by the RGC? Why do you think this is?

10. How might you improve the quality of the edge detection?

    Hint: one thing to think about is boosting responses of cells that line up consistently along an edge.