11/17/06

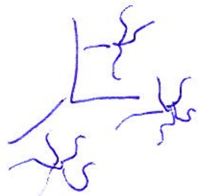
① finish off linear analysis $\boxed{\leftarrow \text{slope}}$

② fluctuation: quenched noise
   a) simple 1-D model
   b) 2-D model - balance
   3) temporal fluctuations

} emphasize: fluctuations smooth out gain function

③ Structured connectivity - $\frac{1}{K}$ scaling

④ Hopfield network
- pattern completion
- associative memory
- what weight matrix will give us that?
- <u>Very</u> simple model

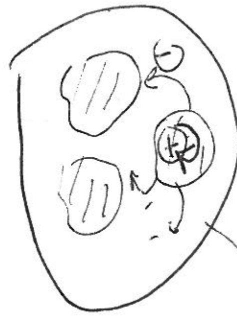


$w \to j$

$$X_i(t+1) = f\left(\sum_j W_{ij} X_j\right)$$

$$\to \text{sign}\left(\sum_j W_{ij} X_j\right)$$

Similar to mexican hat



want a nonlinear equation w/ lots of fixed points

$$\xi_i^\mu = \text{sign}\left(\sum_j W_{ij} \xi_j^\mu\right) \qquad \mu = 1, \dots, p$$

Genius: $W_{ij} = \frac{1}{N} \sum_\mu \xi_i^\mu \xi_j^\mu$

random vector ½ 1's + ½ -1's

$$\xi_i^\mu = \begin{cases} 1 & \text{prob } 0.5 \\ -1 & \text{prob } 0.5 \end{cases}$$

— why it works:

$$\sum_j W_{ij} \xi_j^\nu = \frac{1}{N} \sum_{\mu,j} \xi_i^\mu \xi_j^\mu \xi_j^\nu$$

$$= \frac{1}{N} \sum_j \xi_i^\nu \xi_j^\nu \xi_j^\nu + \sum_{\mu \neq \nu} \xi_i^\mu \frac{1}{N} \underbrace{\sum_j \xi_j^\mu \xi_j^\nu}_{\sim \sqrt{N}}$$

$$= \xi_i^\nu + \sum_{\mu \neq \nu} \xi_i^\mu \frac{1}{\sqrt{N}} \xi_i^\ell$$

$$+ \sqrt{\frac{P}{N}} \; \eta_i$$

quenched noise just like in randomly connected network case.

$\sqrt{\frac{P}{N}}$

$$p(0) = \frac{e^{-\frac{1}{2P/N}}}{\sqrt{2\pi P/N}} = \frac{e^{-\frac{N}{2P}}}{\sqrt{2\pi P/N}} \to , \text{ if } P \ll$$

$$\xi_i^\mu = \text{sign}\left(\xi_i^\mu\right)$$

Stability???

$$X_i(t+1) = \text{sign}\left(\frac{1}{N} \sum_j W_{ij} X_j\right) \qquad W_{ij} = W_{ji} \quad W_{ii} =$$

$$H^{(1)} = -\frac{1}{2} \sum_{ij} X_i(t) W_{ij} X_j(t)$$

asynchronous update:

$$H(t+1) - H(t) = -\frac{1}{2} \sum_{ij} X_i(t+1) W_{ij} X_j(t+1) - X_i(t) W_{ij} X_j(t)$$

unit k update:

$X_k(t+1) \neq X_k$ (maybe)

$X_{i \neq k}(t+1) = X_{i \neq k}$

$$\Delta H = -\frac{1}{2} \sum_j \left[X_k(t+1) W_{kj} X_j + X_j(t) W_{jk} X_k(t+1) \right]$$

$$- \quad (t) \qquad\qquad (t) \quad]$$

$$= -\left[X_k(t+1) - X_k(t)\right] \sum_j J_{kj}$$

$$= \mp \left[\text{sign}\left(\sum_j J_{kj} x_j\right) - x_i\right] \text{sign}($$

~~Problems ... violates~~

Capacity : $\frac{P}{N} = 0.138$ (hard!!)

Problems: Dale's law - fixable - capacity ~ $\frac{0.138}{\sqrt{2}}$

↑ clip

Saturation : not ~~fixable~~ so easily fixable

~~=~~

Neurons: $W_{ij}^{EE} \to W_{ij}^{EE} + \cancel{\frac{\beta}{N} \sum_{\mu} \xi_i^{\mu} \xi_j^{\mu}} \frac{C_{ij}}{K} \frac{\beta}{Nf(1-f)} \sum_{\mu} \xi_i^{\mu}(\xi_j^{\mu} - f)$

import

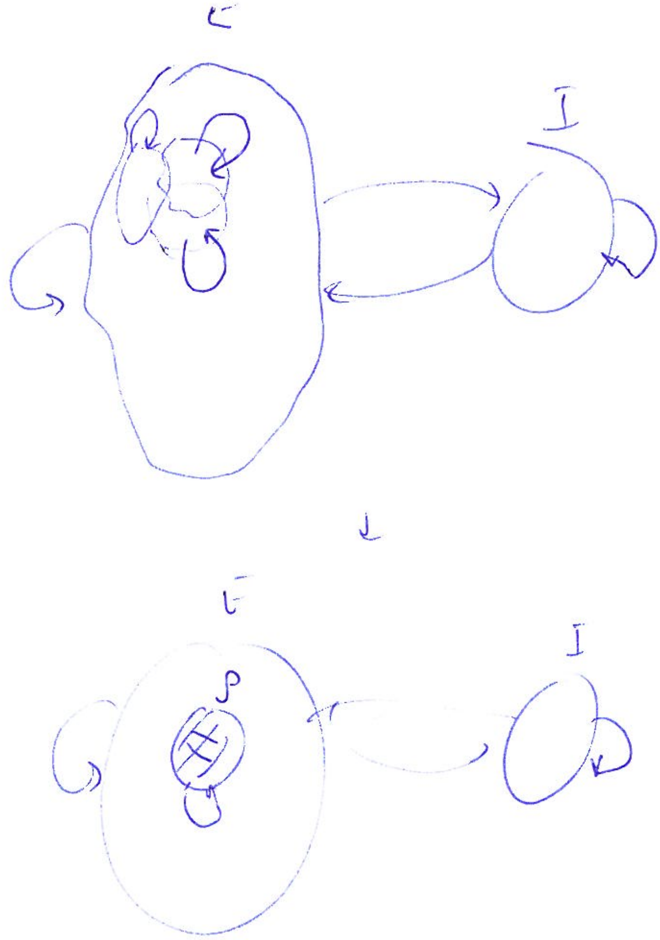
$\cancel{\sum_j W_{ij} \nu_j \to \sum_j W_{ij} \nu_j + \frac{\beta}{N} \sum}$ $\quad \xi_i : \begin{cases} 1 & \text{prob } f \\ 0 & \text{prob } 1-f \end{cases}$

$$\nu_{Ei} = \phi_E \left(\sqrt{K} \left(\tilde{W}_{EE} \nu_E - \tilde{W}_{EI} \nu_I \right) + \sigma_E \xi_{Ei} + \frac{\beta}{Nf(1-f)} \sum_{\mu, j} \xi_i^{\mu} (\xi_j^{\mu} - f) \nu_{Ej} \right)$$

$\nu_{Ii} = \ldots$

$\cancel{\ldots} = \frac{1}{Nf(1-f)}$

~~Memory~~ Pic



$$M_\rho = \cancel{\frac{1}{Nf(1-f)} \sum_j \xi_j^\rho} \quad \frac{1}{1-f}\left[\frac{1}{Nf}\sum_j \xi_j^\rho \nu_{Ej} - \frac{1}{N}\sum_j \nu_{Ej}\right]$$

$$= \frac{1}{Nf(1-f)} \sum_j (\xi_j^\rho - f)\,\nu_{Ej}$$

$\nu_E =$

$\nu_I =$

$f \to \xi$

$$\frac{1}{Nf(1-f)} \sum_{\mu j} \xi_i^\mu (\xi_j^\mu - f) V_{Ej}$$

$$= \xi_i^\rho m_\rho + \underbrace{\underbrace{\frac{1}{Nf(1-f)} \sum_{\mu \neq \rho, j} \xi_i^\mu (\xi_j^\mu - f) V_{Ej}}_{\sqrt{pN}}}_{\sqrt{\frac{p}{N}}}$$

$$V_{Ei} = \phi_E \left(\sqrt{k} \left(\qquad \right) + \beta \xi_i^\rho m_\rho + \sigma_E' \eta_i \right)$$

$$\cancel{V_E = \sum_i \to \int dz \frac{e^{-z^2/2}}{\sqrt{2\pi}} \quad f \quad (\beta = 1)}$$

$$(1-f) \quad (\beta = 0)$$

$$V_E = \frac{1}{N} \sum_i \phi_E \left(\quad \ldots \quad \xi_i^\rho \right)$$

$$= \frac{1}{N} \sum_i (1 - \xi_i^\rho + \xi_i^\rho) \quad \phi_E$$

$$= \frac{1}{N} \sum_i \phi_E \left(\qquad 0 \right)(1-f) + f \left(\qquad 1 \right)$$

$$V_E = \tilde{\phi}_E \left(\sqrt{k} \left(\qquad \right) + \beta m_\rho \right)(1-f) + (\quad) f$$

$\uparrow$ inh

(A) $f \to 0$

(B) $f > 0$