12/7/04

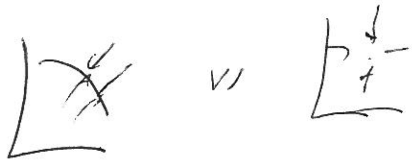
Line (and higher dimension) attractors.

~~Equilibria~~:

— motivation:
Can be used to store continuous variables. Working memory + head direction.

$$\dot{X}_i = \phi\left(\sum_j W_{ij} X_j\right) - X_i$$

X_{0i} is equilibrium:



$$X_{0i} = \phi\left(\sum_j W_{ij} X_{0j}\right)$$

Let $W_{ij} = W_{i-j}$

$y_{0i} = X_{0,i+k}$ also a solution

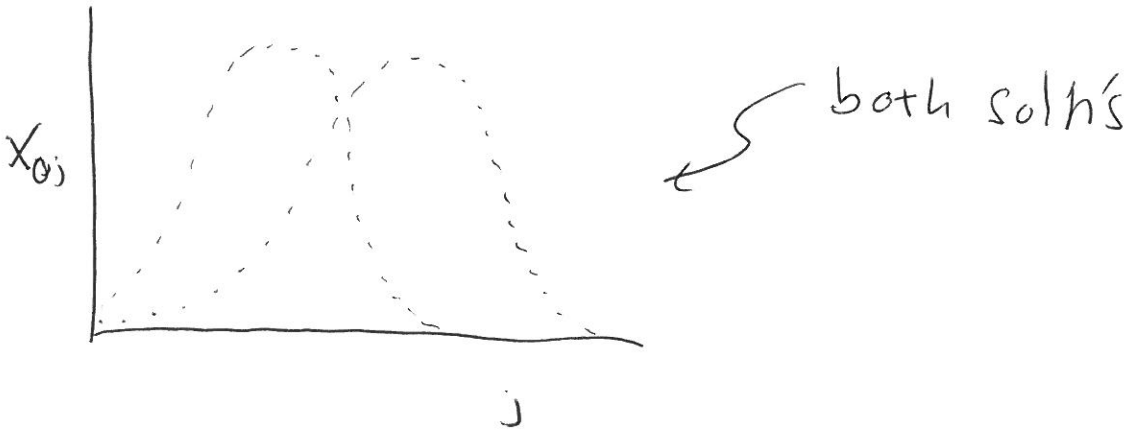
$$X_{0,i+k} = \phi\left(\sum_j W_{i-j} X_{0,j+k}\right)$$

$i = i' - k$
$j = j' - k$

$$X_{0,i'} = \phi\left(\sum_j W_{i'-k-(j'-k)} X_{0j'}\right)$$

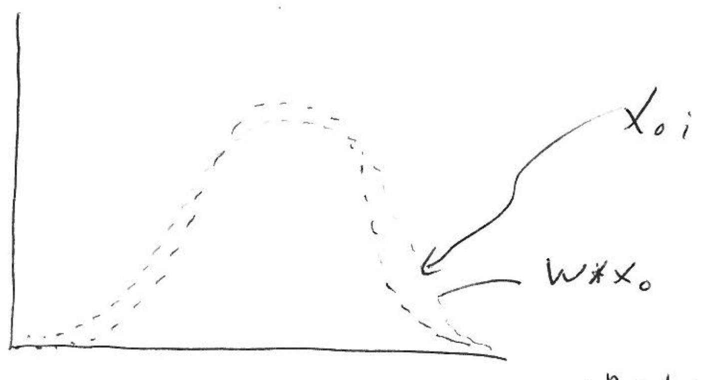
$$= \phi\left(\sum_j W_{i'-j'} X_{0j'}\right)$$

or $X_{0i} = \phi\left(\sum_j W_{i-j} X_{0j}\right)$

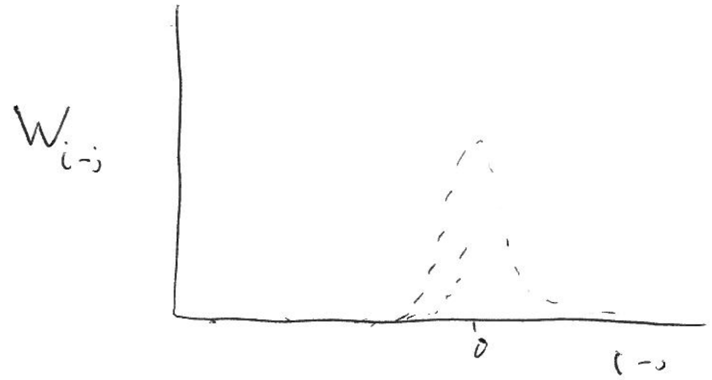


What's happening: W_{i-j} is a translation invariant operator (convolution).

Picture



- nonlinearities can narrow the hill again



Example:

$$X_i = \left[\sum_k \exp\left[- \frac{[\Delta(i-k)]^2}{2\sigma^2} \right] X_k \right]^2$$

Sol'n $\quad X_i = A \exp\left[- \frac{(i\Delta)^2}{2\sigma^2} \right]$

$$\sum_k \exp\left[- \frac{\Delta^2 (i-k)^2}{2\sigma^2} \right] A \exp\left[- \frac{k\Delta^2}{2\sigma^2} \right]$$

$$= A \sum_k e^{-\frac{\Delta^2}{2\sigma^2}\left((i-k)^2 + k^2\right)}$$

$$= A \sum_k e^{-\frac{\Delta^2}{2\sigma^2}\left[i^2 + 2k^2 - 2ik \right]}$$

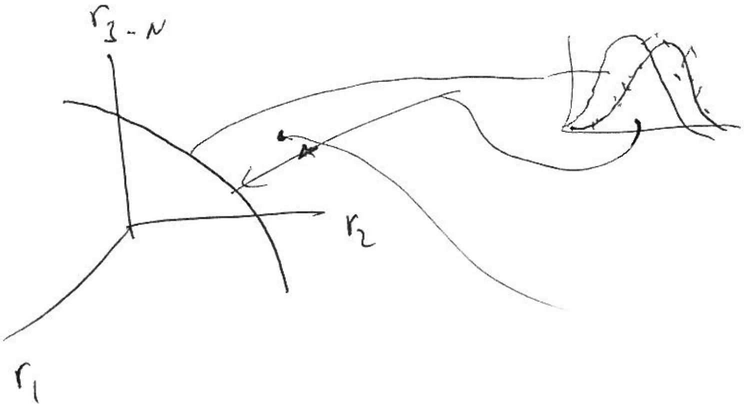
$$= A \sum_k e^{-\frac{\Delta^2}{2\sigma^2}\left[i^2 + 2\left(k^2 - ik + \frac{i^2}{4}\right) - \frac{i^2}{2} \right]}$$

$$= A\, e^{-\frac{\Delta^2}{4\sigma^2} i^2} \sum_k e^{-\frac{\Delta^2}{\sigma^2}\left(k - \frac{i}{2}\right)^2}$$

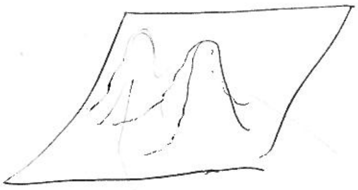
$$= A \sqrt{\frac{\pi\sigma^2}{\Delta^2}}\; e^{-\frac{\Delta^2}{4\sigma^2} i^2}$$

$$A e^{-\frac{\Delta^2}{2\sigma^2} i^2} = A^2 \frac{\pi\sigma^2}{\Delta^2} e^{-\frac{\Delta^2}{2\sigma^2} i^2} \implies A = \frac{\Delta^2}{\pi\sigma^2}$$

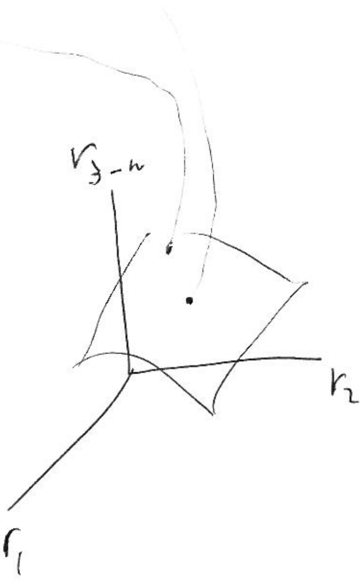
Abstract represen



D-dim attractors w/ symmetr



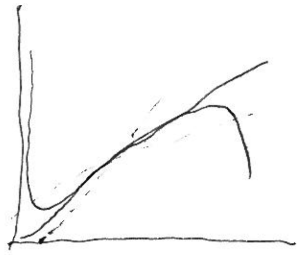
Abstra



- used to represent params in working / associative memory tasks
- Used for optimal computati

Also: firing rate attractor.

- ROI (strong ~~evidence~~ exp. evidence)
- integrate evidence (weak exp. evidence)



- Structurally unsound
- slow drift

Lin. analysis?

$$\lambda = \frac{T \pm \sqrt{T^2 - 4D}}{2}$$

$D = 0 \Rightarrow$ param

~~not~~ par $\Rightarrow \lambda = 0$

What about bumps? Also stream unstable:

$$\dot{\underset{\sim}{\nu}} = \phi(\underline{\underline{W}} \cdot \underset{\sim}{\nu} + \delta \underline{\underline{W}} \cdot \underset{\sim}{\nu}) - \underset{\sim}{\nu} \qquad \left[\dot{\nu}_i = \phi\left(\sum_j W_{i-j} \nu_j + \sum_j \delta W_{ij} \nu_j \right) - \nu_i \right]$$

$$\underset{\sim}{\nu} = \underset{\sim}{\nu}_0(\theta) + \delta \underset{\sim}{\nu} \quad \leftarrow f(\theta) \; !!$$

$$\dot{\underset{\sim}{\nu}} = \underset{\sim}{\nu}_0'(\theta)\dot{\theta} + \delta\dot{\underset{\sim}{\nu}} = \phi(\underline{\underline{W}} \cdot \underset{\sim}{\nu}_0(\theta)) + \phi' \underline{\underline{W}} \cdot \delta \underset{\sim}{\nu} + \phi' \delta \underline{\underline{W}} \cdot \underset{\sim}{\nu}_0 - \underset{\sim}{\nu}_0 - \delta \underset{\sim}{\nu}$$

$$= \underline{\underline{J}} \cdot \delta \underset{\sim}{\nu} + \phi' \delta \underline{\underline{W}} \cdot \underset{\sim}{\nu}_0$$

$$\underline{\underline{J}} \cdot \underset{\sim}{\nu}_0' = 0$$

$$\underline{\underline{J}} \cdot \underset{\sim}{\nu}_k = -\lambda_k \underset{\sim}{\nu}_k$$

$$\underset{\sim}{\nu}_k^{\dagger} \cdot \delta \dot{\underset{\sim}{\nu}} = -\lambda_k \underset{\sim}{\nu}_k^{\dagger} \cdot \delta \underset{\sim}{\nu} + \phi' \underset{\sim}{\nu}_k^{\dagger} \cdot \delta \underline{\underline{W}} \cdot \underset{\sim}{\nu}_0(\theta) = 0$$

$$\dot{\theta} = \phi' \underset{\sim}{\nu}_0^{\dagger} \cdot \delta \underline{\underline{W}} \cdot \underset{\sim}{\nu}_0(\theta)$$

$$= F(\theta) \qquad \text{slow (heuristic) drive}$$

Implicatn for multiple line in one net

$$W_{ij} = W^{*}_{i-j} + W^{*}_{p(i)-p(j)} + w_{p_2(j)-p_2(i)}$$

↑ record

$$W_{i-j} X_{0j} + W_{p(i)-p(j)} X_{0j} \qquad \bar{w} = 0$$

$\cdot\, \theta(N)$ $\qquad \theta(\sqrt{N})$ ..

$\theta(\sqrt{N \cdot p})$

$$\dot{V}_i = \phi\left(\sum_{\mu} \sum_{j} W^{\mu}_{ij} V_j \right) - V_i$$

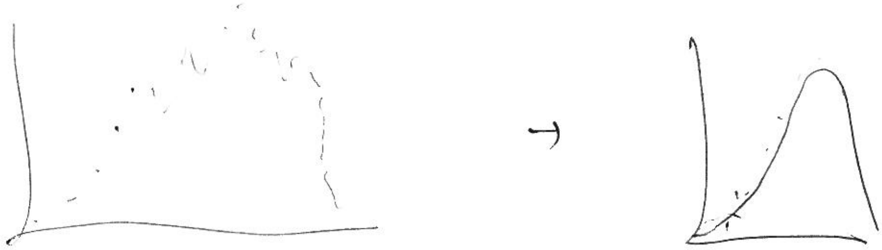
~~$V_i = V^{\mu}_{0i}(\theta)$~~

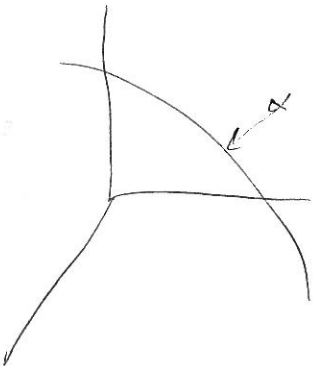
$$= \phi\left(\sum_j W^{0}_{i-j} V_j + \delta h_{i-j} \right) - V_i$$

$\dot{\theta}^{\mu} = V^{\dagger}_{0} \cdot \phi' \, V_i \implies$ drift

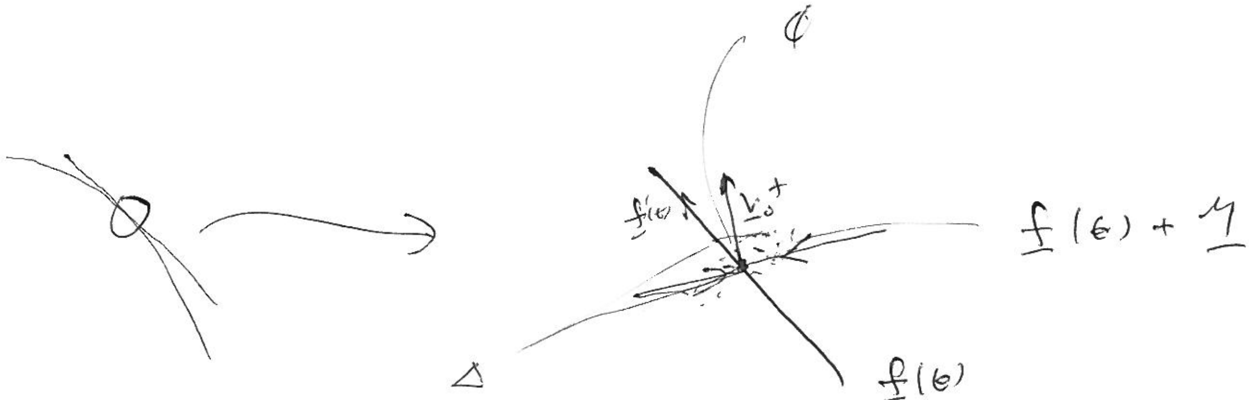
fruitu limit !!!

Noise cleanup





how do we choos
netwch dyn?



$$\Delta \quad \cos\phi = \frac{\underline{v}_0^+ \cdot \underline{\eta}}{\Delta}$$

$$\Delta = \frac{\underline{v}_0^+ \cdot \underline{\eta}}{\cos\phi} = \frac{\underline{v}_0^+ \cdot \underline{\eta}}{\underline{f}' \cdot \underline{v}_0^+} \, |\underline{f}'|$$

$$\langle \Delta^2 \rangle = \frac{\underline{v}_0^+ \cdot \underline{\underline{R}} \cdot \underline{v}_0^+ \; |\underline{f}'|^2}{|\underline{f}' \cdot \underline{v}_0^+|^2}$$

min: $\quad |\underline{f}' \cdot \underline{v}_0^+|^2 \, \underline{v}_0^+ \cdot \underline{\underline{R}} - \underline{v}_0^+ \cdot \underline{\underline{R}} \cdot \underline{v}_0^+ \; \underline{f}' \cdot \underline{v}_0^+ \, \underline{f}'$

$$\underline{f}' \sim \underline{v}^+ \cdot \underline{\underline{R}}$$

$$\boxed{\underline{v}_0^+ \sim \underline{\underline{R}}^{-1} \cdot \underline{f}'}$$

10

$$\delta \dot{\underline{v}} = \underline{\underline{J}} \cdot \delta \underline{v}$$

$$\uparrow$$

$$\sum_k \lambda_k \underline{v}_k \underline{v}_k^T$$

$$\delta \dot{\underline{v}} = \sum_k$$

$$\delta \underline{v} = \sum_k a_k \underline{v}_k$$

$$\sum \dot{a}_k \underline{v}_k = \sum_k \lambda_k \underline{v}_k \underline{v}_k^T \cdot \sum_\ell a_\ell \underline{v}_\ell$$

$$= \sum_k a_k \lambda_k \underline{v}_k$$

$$\dot{a}_k = \lambda_k a_k$$

$$\frac{d}{dt} \underline{v}_0^T \cdot \delta \underline{x} = \underline{v}_0^T \cdot \sum_k \lambda_k \underline{v}_k \underline{v}_k^T \cdot \delta \underline{x}$$

$$= \lambda_0 \underline{v}_0^T \delta \underline{x}$$

$$= 0$$