

# Assignments for TN lectures 23th and 27th Feb Gatsby Spring 2017

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1. In ICA, we can assume without loss of generality that the variances of the independent components  $s_i$  are equal to one, and their means are zero.  
Consider a general linear combination  $y = \sum_{i=1}^n q_i s_i$  of the independent components.
  - (a) Compute the variance of  $y$ .
  - (b) Compute the kurtosis of  $y$ .
2. Assume for simplicity  $\text{kurt}(s_i) = 1$  for all  $i$ .
  - (a) Let's constrain the variance of  $y$  to unity; what does this mean geometrically for the  $q_i$ ?
  - (b) Set  $n = 2$  for simplicity; sketch the isocontours of the kurtosis of  $y$  in the space of  $q_1, q_2$ .
  - (c) Can you see geometrically why the extrema of kurtosis correspond to the independent components?
3.
  - (a) Derive the score matching objective for the multivariate Gaussian distribution parametrized by the mean and the covariance matrix.
  - (b) Find its minimum, i.e. the score matching estimator for the mean and the covariance matrix (simple formulae are possible!).
4. Derive the score matching objective function for an exponential family.