Assignment 8
Theoretical Neuroscience

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All figure and equation numbers refer to *Theoretical Neuroscience*.

1. **Oja's rule convergence**

   Show that the averaged form of the single-trial Oja rule in equation 8.16 is given by

   $$\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{Q} \cdot \mathbf{w} - \alpha(\mathbf{w} \cdot \mathbf{Q} \cdot \mathbf{w})\mathbf{w}\,.$$

   Prove that if it converges, the averaged learning rule produces a set of weights proportional to an eigenvector of the correlation matrix $\mathbf{Q}$, normalized so that $|\mathbf{w}|^2 = 1/\alpha$.

2. **Ocular dominance model**

   Simulate the ocular dominance model of figure 8.7 using a subtractively normalized version of equation 8.31 (i.e. equation 8.14) with saturation limits at 0 and 1, and cortical interactions generated as in figure 8.8 from

   $$\mathbf{K}_{aa'} = \exp\left(-\frac{(a-a')^2}{2\sigma^2}\right) - \frac{1}{9}\exp\left(-\frac{(a-a')^2}{18\sigma^2}\right)\,,$$

   where $\sigma = 0.066$ mm. Use 512 cortical cells with locations a spread evenly over a nominal 10 mm of cortex, **and periodic boundary conditions** (this means that you can use Fourier transforms to calculate the effect of the cortical interactions).

   Also use the discrete form of equation 8.31:

   $$\mathbf{W} \to \mathbf{W} + \epsilon \mathbf{K} \cdot \mathbf{W} \cdot \mathbf{Q}$$

   with a learning rate of $\epsilon = 0.01$.

   Plot $\mathbf{w}_-$ as it evolves from near $\mathbf{0}$ to the final form of ocular dominance. Calculate the magnitude of the discrete Fourier transform of $\mathbf{w}_-$. Repeat this around 100 times, work out the average of the magnitudes of the Fourier transforms. Why might one expect this average to resemble the Fourier transform of $\mathbf{K}$? Does it, in fact, do so?

3. **Vectorised value function**

   Consider a finite Markov chain with transition matrix P_{xy}. If a subject gets reward r_x at state x, and we define the long-run discounted value function V_x as

   $$V_x = \mathcal{E}\left[\sum_{t=0}^{\infty} \gamma^t r_{x(t)} | x(0) = x\right]$$

   following the stochastic dynamics of the chain, write down a matrix equation that the vector of all values, $\mathbf{V}$, satisfies.

4. **Equilibrium of partial reinforcement**

   Consider the case of partial reinforcement (studied in figure 9.1) in which reward $r = 1$ is provided randomly with probability p on any given trial. Assume that there is a single stimulus with $u = 1$, so that $\epsilon\delta u$, with $\delta = r - v = r - wu$, is equal to $\epsilon(r - w)$. Calculate the self-consistent equilibrium values of the mean and variance of the weight w. What happens to your expression for the variance if $\epsilon = 2$ or $\epsilon > 2$? To what features of the learning rule do these effects correspond?

5. **Kalman filter**

   (a) Implement a Kalman filter model for classical conditioning, and apply it to the cases of forwards and backwards blocking. On what parameter does the relative strength of these depend?

   (b) What happens if you use an assumed density filter in which only the diagonal elements of the covariance matrix of uncertainty about the weights are retained?