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ORIGINAL ARTICLE

Taking Plateau into Microgravity: The Formation of an Eightfold Vertex in a System of Soap Films

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Abstract The microgravity phases of parabolic flights were used to perform experiments with soap films trapped in wire frames, a variation of the wire frame experiments originally designed by the nineteenth century Belgian scientist Joseph Plateau. We considered the formation of an eightfold vertex of Plateau borders within a cubic frame. In terrestrial experiments such a vertex can only be formed when liquid is forced through the Plateau borders, but in microgravity we found this vertex to be stable under equilibrium (non-flow) conditions once the liquid volume fraction exceeds 0.022±0.005. This is consistent with the theoretical value for the transition, which for our experiment we estimate to be 0.0192.

Keywords Plateau's rules · Eightfold vertex · Microgravity · Foam

Introduction and Background

Joseph Antoine Ferdinand Plateau's (1873) book "Statique Expérimentale et Théorique des Liquides soumis aux seules Forces Moléculaires" is generally regarded as the first landmark in the physics of foams (Weaire and Hutzler 1999). In his book Plateau describes detailed experiments on the formation and arrangements of soap films when wire frames are dipped into and then carefully withdrawn from a vessel containing soap solution. Plateau found that three such soap films always meet symmetrically at a line (now called a Plateau border), forming angles of 120°. Also, he found that four such lines meet symmetrically in a junction at an angle of $\cos^{-1}(-1/3)\approx 109.43°$ (Fig. 1a).

These observations, which are now called Plateau's rules, were not explained with full mathematical rigour until 1976 (Almgren and Taylor 1976), although earlier explanations date back to 1864 (Lamarle 1864). In the mathematical analysis of his experiments the Plateau borders are treated as infinitesimally thin lines.

One particular Plateau experiment is the forerunner of that described here. In this case he used a cubic frame, as shown in Fig. 1. He found that the equilibrium configuration broke the symmetry of the cube, and consisted of four fourfold vertices (Fig. 1a), in accordance with his rules. A variation of this experiment was carried out by in het Panhuis et al. (1998). In their experiment, films are formed in a cubic frame and liquid is injected at a constant flow rate into one of the Plateau borders at the corner of the frame, continuously replacing liquid lost due to gravitational drainage. This leads to a thickening of both Plateau borders and films, with most of the liquid confined to the Plateau borders. At a critical flow rate a transition to the eightfold symmetry shown in Fig. 1c is observed. A surprising result is that when decreasing the flow rate, the eightfold vertex remains stable down to very small flow rates. There was a question of whether this configuration survives to the dry limit or not. Since these were non-equilibrium experiments, the possibility that the stability of the eightfold vertex was partly due to the flow of liquid under gravity and viscous forces could not be ruled out. It is natural to describe such findings in terms of a critical liquid fraction (the fraction of volume in the frame occupied by liquid; see below) but this was not possible in the experiments of in het Panhuis et al. (1998).

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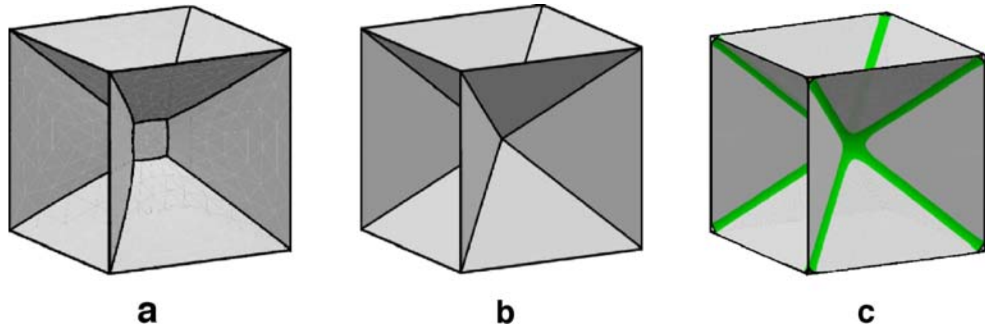

Fig. 1 **a** A stable configuration of soap films that satisfy Plateau's rules in a cubic wire frame. An eightfold vertex as shown in **b** is stable only for Plateau borders of sufficient thickness, as in **c**. Surface Evolver simulations reproduced with kind permission from Brakke (2005)

We are therefore confronted by a classic observation of a structural transition in a system of soap films, about which some questions and doubts remain. In such macroscopic systems there are usually no activated transitions: we are dealing with structural instabilities that lead to finite excursions to a new stable structure, as sketched in Fig. 2. Hence, hysteresis is always a strong feature in such a case (Weaire et al. 2007).

Using the Surface Evolver software, Brakke (1992) numerically computed the equilibrium configurations of soap films spanning a cubic frame. In these calculations the films and frame wires have zero thickness. Thus there are no Plateau borders along the edges of the cube. The liquid fraction of the soap film configuration is then given by the liquid in the Plateau borders and their internal junctions, divided by the volume of the cube. Brakke found that once an eightfold vertex has been created, it is stable down to the very small liquid fraction of 0.000278 (Brakke 2005). Brakke also computed Φ_{th} the critical liquid fraction for the transition from four fourfold vertices to an eightfold vertex to be Φ_{th}=0.0103.

The stability of an eightfold vertex is directly relevant to the properties of an *fcc* foam which may be considered (when stable) to consist of eightfold vertices of the same kind. The bulk liquid fraction is roughly half that of the corresponding frame system (with Plateau borders of the same size) for a calculation such as that of Brakke. With this adjustment, his quantitative prediction of the limit of stability may be adduced to the case of fcc (Weaire 1994).

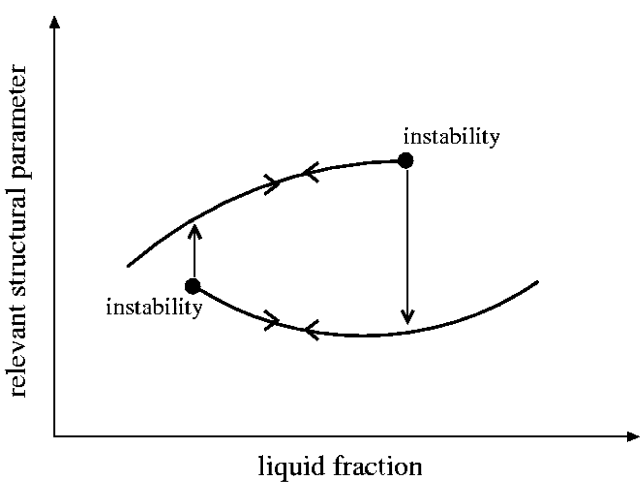

Fig. 2 In foams and soap film systems structural transitions are usually of the character sketched here (Weaire et al. 2007)

Plateau observed eightfold vertices by performing experiments with density-matched emulsions (Plateau 1873), where no gravitationally induced drainage occurs. This technique of density matching has very recently been used again in experiments on liquid foams (Péron et al. 2007). In our experiments described below we have resorted to the microgravity phase of a parabolic flight to study the formation and stability of an eightfold vertex in a cubic wire frame. In particular we have experimentally determined the critical liquid fraction for the transition from four fourfold vertices to one eightfold vertex. While recent years saw a number of microgravity experiments on foams, for example (Saint-Jalmes et al. 2006, 2007; Brunke et al. 2005), we are not aware of any experiments making use of Plateau's wire frames.

Experimental Set-up and Procedure

In designing our microgravity experiment we used the simplest design possible for creating and wetting soap films while fulfilling all safety requirements for performing a parabolic flight. To this end, we constructed a rectangular box consisting of two compartments, one compartment full of soap solution (water, glycerol and commercial detergent Fairy Liquid, approximately mixed in the ratio 110:10:1 respectively), the other supporting a cubic wire frame of edge length D=2 cm (Figs. 3 and 4). The compartments are connected by two copper pipes. By turning the box upside down in gravity we can drain liquid from one compartment to the other, allowing soap solution to pass through one pipe while air escapes through the other. The soap films then form in the wire frame in the configuration shown in Fig. 1a.

A syringe was attached to the outside of our box. This enabled us to deliver soap solution through a pipe and needle directly into the Plateau border at one of the corners

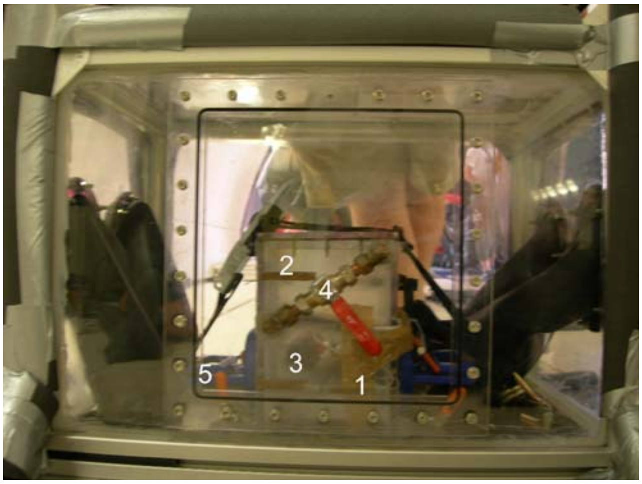

Fig. 3 The wire frame box is contained in the aluminium rack and is fixed to the base plate using four small clamps (*5*). During the gravity phase, soap solution was drained from the wire-frame compartment (*2*) to the second compartment (*3*) using a tap (*4*). During microgravity, the wire-frame soap film was wetted using a syringe (*1*). The equipment was built in the workshop of the School of Physics, Trinity College Dublin under the supervision of Mick Reilly

of the wire frame. In the microgravity phase this leads to the establishment of an eightfold vertex, once a sufficient amount of liquid has been added. In between the different flight parabolas the syringe is refilled from the liquid in the compartment hosting the wire frame, thus conserving the total amount of liquid in the box.

Our experiment was required to withstand a 9 g force (crash conditions). To this end we designed a reinforced aluminium rack consisting of a 10 mm thick aluminium base plate, four vertical 'structs' (45×45 Bosch Aluminium Structural Framing) and eight horizontal 'structs' which enclose our wire frame box and hold the video camera in place (Fig. 4). The camera, a Sony Camcorder, is held in place at the top of the rack, looking directly down at the wire frame box. During microgravity, the wire frame box is secured to the aluminium base plate using four small clamps.

Double waterproof sealing of our experiment was achieved by fixing Lexan® sheets to the walls of the rack. Four gloves are connected to the Lexan sheets through which we can access our wire frame box and accommodate any pressure differentials during the flight. Since the soap films are formed in 1 g during the flight, the wetting apparatus (syringe, small tube and needle) could be tested under gravity on earth. Also, the rack and wire frame box were tested for their load capabilities, waterproofing and impact resistance.

The microgravity experiments were performed by four undergraduate students from Trinity College Dublin (DGT Barrett, EJ Daly, MJ Dolan and S Kelly) as part of the Student Parabolic Flight Campaign of the European Space Agency. This campaign involved 30 different projects from European and Canadian students. The experiments were carried out on board NovaSpace's Zero-G Plane (a modified Airbus A300) which performed a series of 30 parabolic manoeuvres on the 26th and 27th of June 2005, departing from Bordeaux Airport, France.

We were able to make use of six microgravity phases to perform six experiments. In each of these the wetting of a freshly made four fourfold vertex configuration (Fig. 1a) resulted in a transition to the eightfold vertex configuration of Fig. 1c. However, only three experiments lend themselves to a detailed analysis since in the other cases small bubbles were created and trapped in the films during the experiment.

In one of these three experiments soap solution was added in a series of ten 0.05 ml injections, using the syringe. In the other two experiments the injections consisted of series of ten 0.1 ml volumes. Each injection lasted approximately 1 s. We used the video camera, which was fixed perpendicular to the square face of the initial soap film configuration to record the wetting of the soap films.

Results and Analysis

We videotaped the soap film configurations during the entire parabolic flight. Some still-frames are shown in Fig. 5. Figure 5a was taken in microgravity. It shows the four fourfold vertex configuration having just undergone

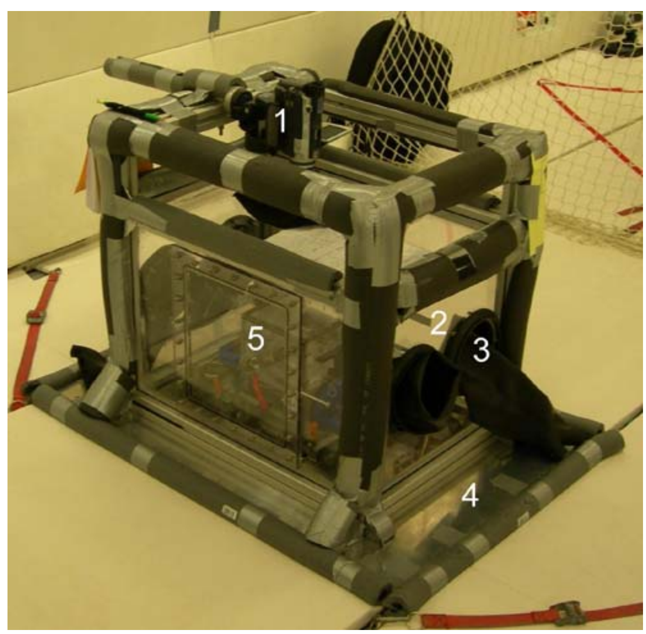

Fig. 4 The aluminium rack base plate (*4*) is bolted in place on the Zero-G plane. The soap films spanning the wire frame were recorded with a camcorder (*1*). During the flight, the experiment was accessed using gloves (*3*) attached to the waterproof seal (*2*). A sealed door (*5*) was used to access the experiment before and after the flight

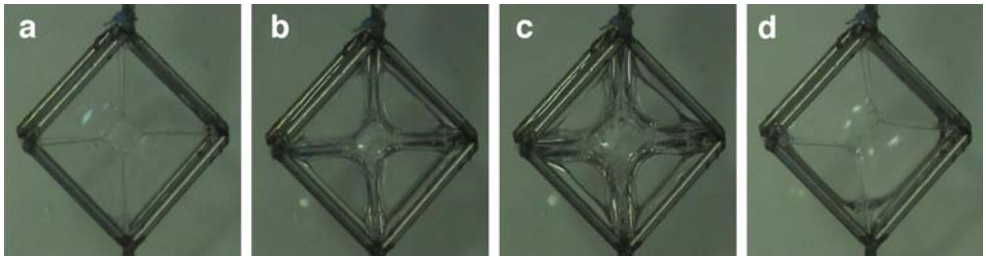

Fig. 5 Photographs obtained during our parabolic flight experiment. **a**–**c** Taken under microgravity. The values of liquid fraction are **a** $\Phi <$ 0.003, **b** $\Phi = 0.075 \pm 0.003$, **c** $\Phi = 0.125 \pm 0.003$, **d** $\Phi < 0.003$ (the values of Φ for **b** and **c** are computed from the number of liquid injections, the values for **a** and **d** are estimates (Hutzler et al. 2007)). The dry film configuration of **a** features a square soap film at the centre of the cubic wire frame. The wet soap film configurations of **b** and **c** show that beyond a critical liquid fraction an eightfold vertex forms in the centre of the frame. On return to gravity (**d**), drainage occurs and the wet configuration returns to the dry configuration of **a**. Note that at the end of this experiment the square soap film is oriented perpendicular to the camera

gravitational induced drainage in a high gravity 1.8 g phase (compare with Fig. 1a). It has the lowest value of liquid fraction that we can achieve in our experiments, and from similar previous experiments (Hutzler et al. 2007) we estimate this value to be less than 0.003.

The still frame of Fig. 5b shows the eightfold vertex that was formed in microgravity, after the addition of controlled amounts of soap solution using the syringe (to be compared with Fig. 1c). Continued addition of soap solution leads to a swelling of the eightfold vertex, as is shown in Fig. 5c. As gravity returns at the end of a parabolic loop, drainage occurs and we again obtain the initial dry four fourfold vertex configuration, featuring a square face (Fig. 5d, compare with Fig. 1a). Note that at the end of the particular experiment shown, the square face is oriented perpendicular to its original orientation. There are a total of three energetically equivalent mutually perpendicular orientations available for this face. We find the same change in orientation in all of our experiments.

Using image analysis we can determine a size L of the square soap film or eightfold vertex by measuring the shortest distance between two opposite sides of the square film or vertex respectively. Figure 6 shows L for different times as liquid is injected during the microgravity phase.

The first injection, at about 5.8 s (Fig. 6) results in a decrease in L due to suction of the added liquid into the Plateau borders. After the second injection at about 8 s the transition to the eightfold configuration occurs. Every further injection leads to an increase in L, but note that L stays constant in the intervals between two injections. This indicates that the configuration is stable. The microgravity regime ends after 24.5 s (Fig. 6). As gravity again begins to increase we see a dramatic drop in L which corresponds to gravity driven drainage of the eightfold configuration. This is quickly followed by a large increase in L to a maximum value at 25.5 s as the wet eightfold configuration returns to a fourfold configuration. The large value of L is due to the large amount of liquid that still remains in the configuration at the onset of gravity. The square face then continues to dry, as L decreases to its initial size (as during the initial 1.8 g phase) to which it finally arrives after 29.5 s. These variations in L during the change from microgravity to the

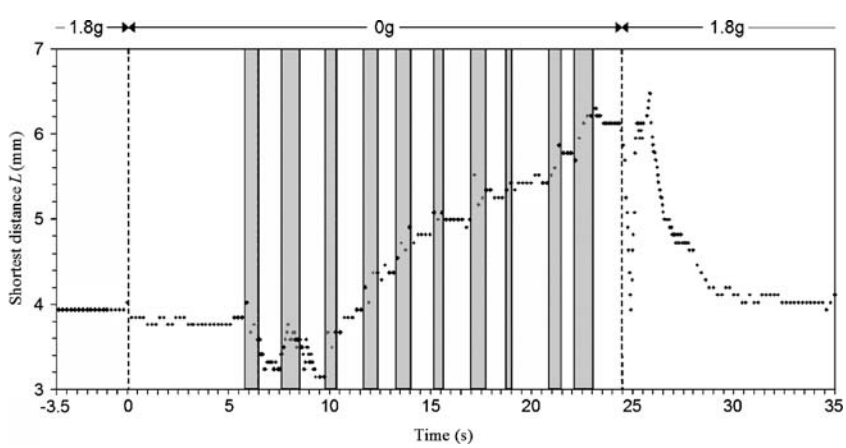

Fig. 6 Variation of the shortest distance L between two opposite sides of the square film or the vertex (see Fig. 5) during a single parabolic flight loop. In microgravity, injections of soap solution occur during the time intervals shaded in grey. In the data shown, the transition to the eightfold vertex occurs after two such 0.1 ml injections (after about 8 s). The vertex increases in size due to subsequent injections of soap solution. It remains stable until gravity returns, and the soap films rapidly return to the dry configuration (after 24.5 s)

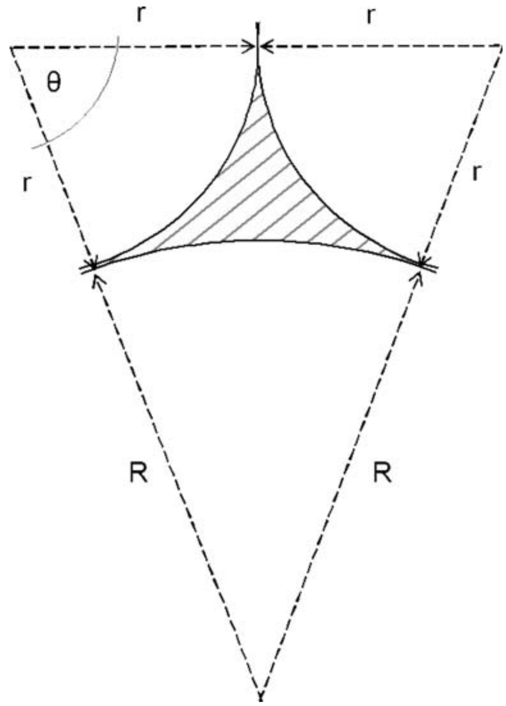

Fig. 7 Diagram of Plateau border cross section (*shaded area*) of radius r attached to a cylindrical wire of radius R. The area of the shaded region can be found by subtracting the area of the segments of radius r and radius R from the area of the equilateral triangle of base $2r$. This gives $A_{\mathrm{w}} = r(R^2 + 2rR)^{1/2} - \theta r^2 - (\pi/2 - \theta)R^2$, where $\cos\theta = r/(r + R)$. The cross sectional area of a Plateau border of radius r which is not touching a wire is obtained by setting R=r, giving $A_{\mathrm{pb}} = (3^{1/2} - \pi/2)r^2$

1.8 g phase reflect the dynamic character of this transition under a changing strength of gravity. These non-equilibrium conditions do not allow us to measure a critical liquid fraction for the transition from an eightfold to a four fourfold vertex configuration.

However, from our knowledge of the amount of liquid that we add in each injection we can determine the critical liquid fraction at which the fourfold configuration changes to the eightfold configuration. In our two successful series of 0.1 ml liquid injections the transition occurred after the second injection and in the one successful series of 0.05 ml liquid injections it occurred after the fifth injection. Taking into account the discrete nature of our increments we then place the point of transition at (1.5±0.5) injections of 0.1 ml and at (4.5±0.5) injections of 0.05 ml. Our three successful experimental runs thus result in the following values for the critical liquid fraction, $\Phi = 0.019 \pm 0.006$, $\Phi = 0.019 \pm 0.006$ and $\Phi = 0.028 \pm 0.003$. We average these to give for our experimental value of the fourfold to eightfold transition $\Phi_{\mathrm{exp}} = 0.022 \pm 0.005$.

This value is considerable higher than Brakke's result of Φ_{th}=0.0103 (Brakke 2005), but this can be attributed to the following. In Brakke's simulations the soap films are attached to wire frames of zero thickness. However, in our experiments the wires have a diameter of $2R$=1.57 mm. Our measured value of liquid fraction will thus include the additional volume of the 12 Plateau borders (each of frame edge length D=2 cm) that are attached to the frame. In order to estimate their contribution, which we shall then add to Brakke's result Φ_{th}, we must first evaluate their radius of curvature.

Ignoring the volume of the junction of Plateau borders (which is small for the low value of liquid fraction at the transition), Φ_{th} is given by the ratio of the volume of the eight Plateau borders to the volume of the cube. The Plateau border length is $3^{1/2}/2D$ with cross-sectional area $A_{\mathrm{pb}} = (3^{1/2} - \pi/2)r^2$, where r is the Plateau border radius. From $\Phi_{\mathrm{th}} = A_{\mathrm{pb}} 4 \times 3^{1/2}/D^2$ and using Brakke's value of the critical liquid fraction Φ_{th} we can readily compute r/D=0.096 and thus obtain r for a given frame edge length D. (Note that r/D=0.096 holds only at the critical liquid fraction Φ_{th}.)

The cross-sectional area of a Plateau border attached to one of our wires (radius R) is given by $A_{\mathrm{w}} = r(R^2 + 2rR)^{1/2} - \theta r^2 - (\pi/2 - \theta)R^2$, where $\cos\theta = r/(r + R)$. (See the diagram in Fig. 7 for details). The additional volume of liquid in our cubic frame is then given by $12DA_{\mathrm{w}}$, corresponding to an increase in liquid fraction by $12A_{\mathrm{w}}/D^2$. For D=2 cm and R=0.785 mm, the increase in liquid fraction is 0.00885. Adding this to Brakke's theoretical value Φ_{th} gives a corrected result for the predicted liquid fraction Φ'_{th} at which the transition from four fourfold vertices to one eightfold vertex occurs as Φ'_{th}=0.0192.

Our value of $\Phi_{\mathrm{exp}} = 0.022 \pm 0.005$ is thus consistent with the corrected theoretical result Φ'_{th}. In our theoretical analysis we considered neither the volume of the vertex nor

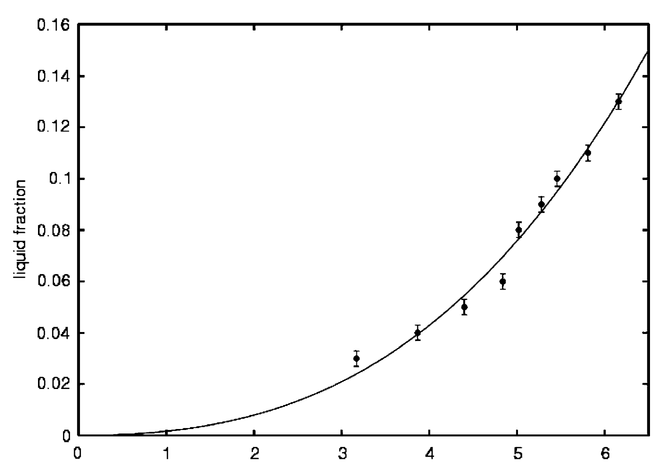

Fig. 8 Variation of liquid fraction as a function of the shortest distance L between two opposite sides of the eightfold vertex. The distance L was determined after each injection of soap solution during the microgravity phase (see Fig. 6). The *solid line* is a fit to the functional form $\Phi = c_1 L^2 + c_2 L^3$, which is obtained from scaling arguments. The fit parameters are $c_1 = 0.0013 \pm 0.0006$ and $c_2 = 0.0004 \pm 0.0001$

the volume of the liquid contained in the corners of the frame. Both contributions would result in an increase of Φ'_{th}, leading to a further convergence of theoretical and experimental estimates of the critical liquid fraction at which the transition occurs. The contribution of non-zero film thickness at equilibrium under gravity was neglected in the computation of both Φ'_{th} and Φ_{exp}.

From our video images we can also determine how the size of the eightfold vertex, characterised by the shortest distance L between two opposite sides of the vertex, varies with liquid fraction. The variation, shown in Fig. 8, can be understood using the following scaling argument. The liquid fraction may be written as $\Phi = A_{\mathrm{pb}} 4 \times 3^{1/2} / D^2 + 12 A_{\mathrm{w}} / D^2 + c_{\mathrm{v}} (L/D)^3$, where we now have taken into account the volume of the vertex (c_{V} is an undetermined constant, D is the edge length of the frame). Since the Plateau border radius r scales with L (there is no pressure gradient between them), this gives $\Phi = c_1 L^2 + c_2 L^3$, where c_1 and c_2 are constants. A least-square fit to this functional form is shown as a solid line in Fig. 8 and describes the data well, while we find a simple quadratic variation to be inadequate.

Conclusion

Our experiments have shown that the dry soap film configuration of Fig. 1a becomes unstable once a critical liquid fraction is exceeded, leading to the configuration shown in Fig. 1c. We find the liquid fraction at this transition to be 0.022±0.006 which is consistent with the theoretical value for the transition, estimated to be 0.0192. Factors such as non-zero film thickness and volume corrections for Plateau borders contacting the edges of the wire frame could in principle be included in further Surface Evolver calculations, possibly giving a more accurate theoretical prediction. This appears to be the first report of the formation of an eightfold vertex in foams under equilibrium (non-flow) conditions.

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References

Almgren, F.J., Taylor, J.E.: The geometry of soap films and soap bubbles. Sci. Am. **235**, 82(1976)

Brakke, K.: The surface evolver. Exp. Math. **1**, 141(1992)

Brakke, K.: Instability of the wet cube cone soap film. Colloids Surf., A Physicochem. Eng. Asp. **263**, 4(2005)

Brunke, O., Hamann, A., Cox, S.J., Odenbach, S.: Experimental and numerical analysis of the drainage of aluminium foams. J. Phys.: Condens. Matter **17**, 6353–6362 (2005)

Hutzler, S., Weaire, D., Cox, S.J., van der Net, A., Janiaud, E.: The nature of topological transitions in foam. Europhys. Lett. **77**, 28002(2007)

in het Panhuis, M., Hutzler, S., Weaire, D., Phelan, R.: New variations on the soap film experiments of Plateau. I. Experiments under forced drainage. Phila. Mag. B **78**, 1(1998)

Lamarle, E.: Sur la stabilité des systèmes liquides en lames minces. Mem. Acad. R. Belg. **35**, 3–104 (1864)

Péron, N., Cox, S., Hutzler, S., Weaire, D.: Steady drainage in emulsions: corrections for surface Plateau borders and a model for high aqueous volume fraction. Eur. Phys. J. E **22**, 341(2007)

Plateau, J.A.F.: Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires, 2 vols. Ghent, Paris (1873)

Saint-Jalmes, A., Marze, S., Ritacco, H., Langevin, D., Bail, S., Dubail, J., Guingot, L., Roux, G., Sung, P., Tosini, L.: Diffusive liquid propagation in porous and elastic materials the case of foams under microgravity conditions. Phys. Rev. Lett. **98**, 058303(2007)

Saint-Jalmes, A., Marze, S., Safouane, M., Langevin, D.: Foam experiments in parabolic flights: development of an ISS facility and capillary drainage experiments. Microgravity Sci. Technol. **18**, 22(2006)

Weaire, D.: Structural transformations in foam? Philos. Mag. Lett. **69**, 99–105 (1994)

Weaire, D., Hutzler, S.: The Physics of Foams. Clarendon Press, Oxford, Oxford (1999)

Weaire, D., Vaz, M.V., Teixeira, P.I.C., Fortes, M.A.: Instabilities in liquid foams. Soft Matter **3**, 47(2007)