Classification of the Local Symmetry of an image using a Gaussian Derivative Filter Bank

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Measurement of an image by linear filters is known to be a useful first step for machine vision, and is a good model of simple cells in V1. What the optimal family of filters is has been much studied. However, whatever family is used, the measurements fail fully to determine the image that they measure. In other words spatial vision, like colour vision, suffers inevitably from metamerism. There is thus a puzzle as to what aspects of the image are actually determined by filter measurement.

As a possible answer, we have considered whether and when filters are able to detect if an image has a particular symmetry. The question can be formalized in only one way. Given a family of filters ($\{F_1,\ldots,F_n\}$), and some potential symmetry group (G) of isometries, is there a non-zero filter in the span of the family that gives a null response to any image with the symmetry group? We have proven a condition that decides the issue. The filter exists if and only if

$$\left|\begin{pmatrix} \left\langle F_1 \middle| \sum_{g\in G} F_1(g\circ \_) \right\rangle & \cdots & \left\langle F_n \middle| \sum_{g\in G} F_1(g\circ \_) \right\rangle \\ \vdots & & \vdots \\ \left\langle F_1 \middle| \sum_{g\in G} F_n(g\circ \_) \right\rangle & \cdots & \left\langle F_n \middle| \sum_{g\in G} F_n(g\circ \_) \right\rangle \end{pmatrix}\right| = 0$$

where the $\langle U | V \rangle = \int_{\vec{x}\in\mathbb{R}^2} U(x)V(x)$ is an inner product between filters, and $F_i(g\circ \_)$ is a copy of the filter F_i rigidly moved by the isometry g.

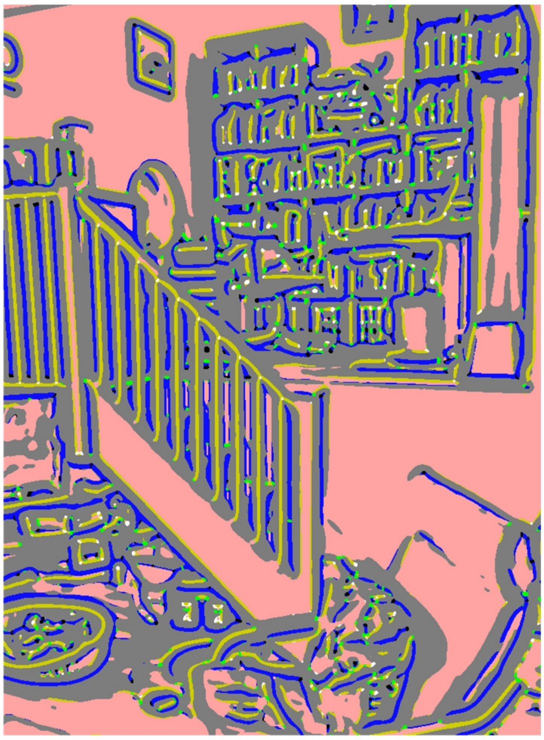
We say that such a null-responding filter is *sensitive* to the symmetry. A special case, which is easier to understand, is the test for whether a particular filter (F) is sensitive to the symmetry. It is, if and only if $\sum_{g\in G} F(g\circ \_) \equiv 0$.

These two results are for image symmetries defined as invariance to a group of planar isometries. Image transformations with more latitude can be considered. In particular *image isometries,* which we define as a planar isometry combined with an isometry of the intensity axis, are very natural. The tests above can be modified for symmetries of this wider class of transformations.

We have shown that families of Gaussian Derivative (DtG) filters are sensitive to several different groups of image isometries. This symmetry-sensitivity of DtG filters adds to their many already-know good properties; and since families of DtG filters are a particularly

good model of the simple cells of V1, is suggestive of a novel interpretation of their role i.e. as classifiers of local symmetry type.
We have found that families of DtGs can fulfil this property very well. Filter combinations that are sensitive to a symmetry, signal with their non-null responses the magnitude of failures of the symmetry to which they are sensitive. Thus, the degrees of failure of symmetry, of many different symmetry groups, can all be read from one family of DtGs.

In particular, consider when the image is being measured with the six filters of a 2^{nd} order DtG family (one 0^{th} order measurement, two 1^{st} derivative measurements, and three 2^{nd} derivative). It turns out that this filter family is sensitive to five different types of image symmetry, with two of them coming in light and dark flavours, thus seven types altogether. Deciding which symmetry type most approximately holds, amounts to deciding which of seven quantities is the largest, where each quantity is a simple algebraic combination of the DtG responses. The figure shows this scheme on an example image. The symmetry classification is at a particular scale: different results are obtained if different scale DtG filters are used, as different aspects of the multiscale image structure are revealed. Otherwise, there is only one free parameter. It controls the amount of image that is classified into the pink category which denotes a maximal symmetry – complete constancy.



Our scheme is completely general, so it will be possible to extend the analysis to higher order families of DtGs and more types of symmetry. Back-of-the-envelope estimates suggest that sensitivity to all the possible groups of image isometries will be possible at 4^{th} or 5^{th} order, which matches where human vision seems to cutoff under the DtG interpretation of V1 simple cells, and would result in 30 or so symmetry classes.

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