Microsoft Research

LETOR
Learning to Rank
for Information Retrieval

Learning to Rank

# Learning to Rank: from Pairwise Approach to Listwise Approach

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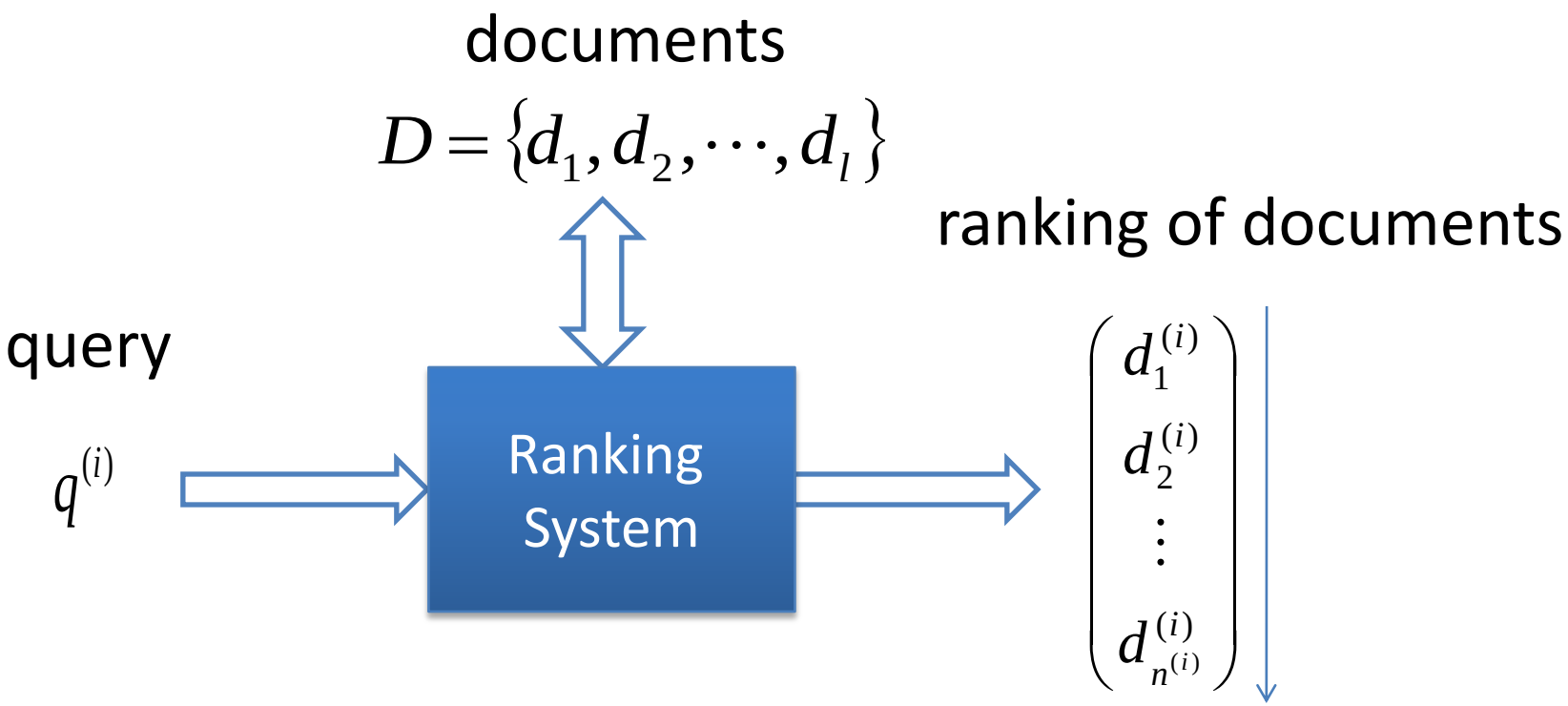
# Talk Outline

- Introduction to Learning to Rank
- Previous work: Pairwise Approach
- Our proposal: Listwise Approach
  - ListNet
  - Relational Ranking
- Summary

# Ranking Problem: Example = Information Retrieval




*Ranking is also important in NLP applications, such as first-pass attachment disambiguation, and reranking alternative parse trees generated for the same sentence by a statistical parser, etc.*

# IR Evaluation Measures

- Precision at position n

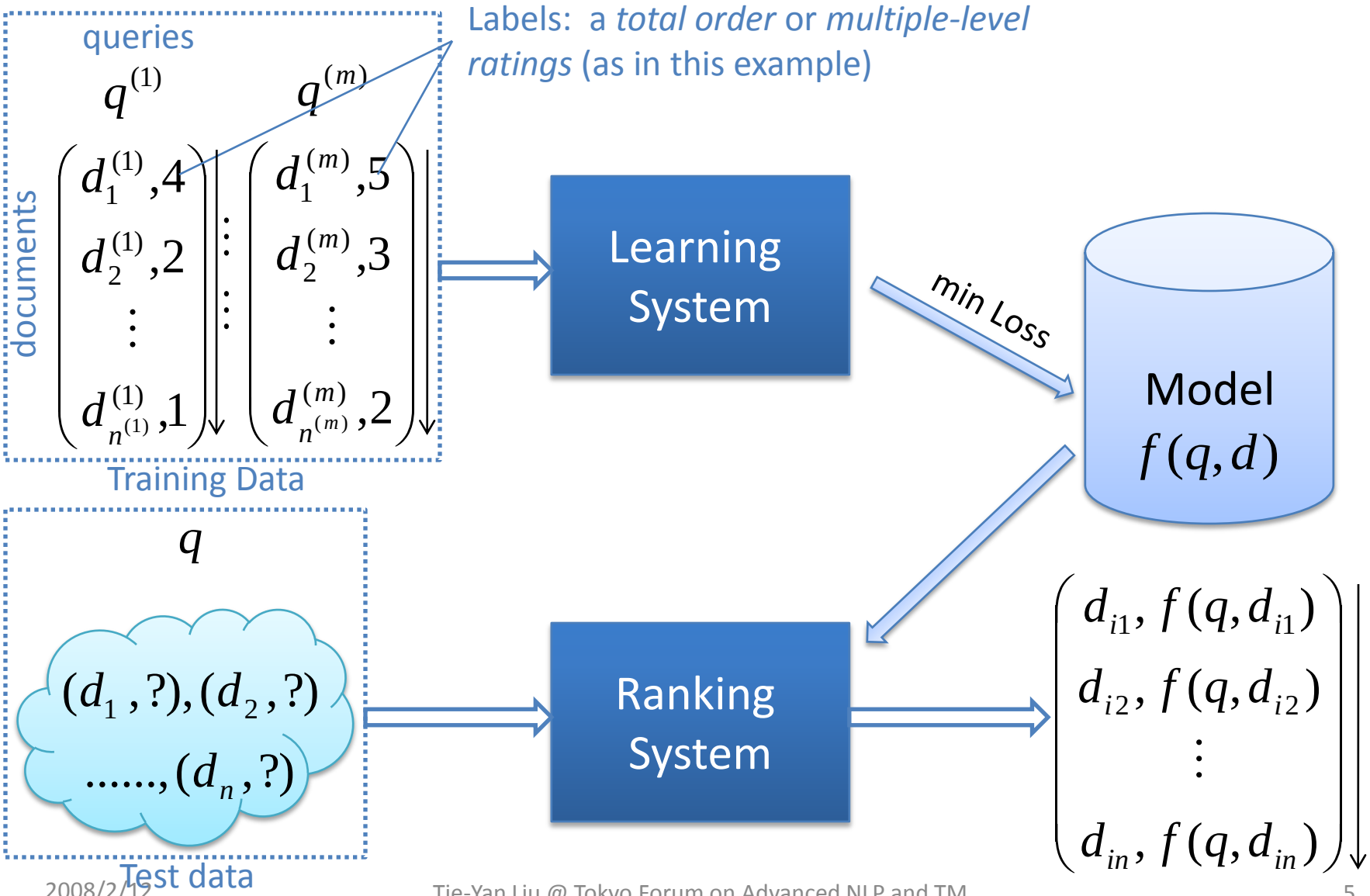
$$P@n = \frac{\#\{\text{relevant documents in top } n \text{ results}\}}{n}$$

- Average precision

$$AP = \frac{\sum_n P@n \cdot I\{\text{document } n \text{ is relevant}\}}{\#\{\text{relevant documents}\}}$$

- MAP: averaged over all queries in the test set
- NDCG at position *n*: $N@n = Z_n \sum_{j=1}^{n} (2^{r(j)} - 1) / \log(1 + j)$

# Learning to Rank



# Previous Work: Pairwise Approach

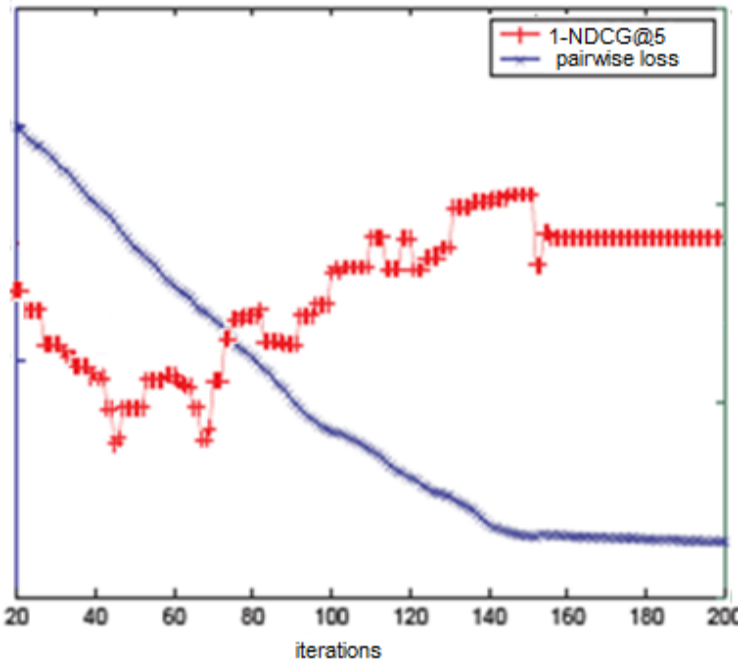
- Transforming ranked list into document pairs
- Formalizing ranking as classification on document pairs
- Ranking SVM, RankNet, RankBoost

$$q^{(i)} \begin{pmatrix} d_1^{(i)}, 5 \\ d_2^{(i)}, 3 \\ \vdots \\ d_{n^{(i)}}^{(i)}, 2 \end{pmatrix} \xrightarrow{\text{Transform}} q^{(i)} \left\{ \left(d_1^{(i)}, d_2^{(i)}\right), \left(d_1^{(i)}, d_{n^{(i)}}^{(i)}\right), \ldots, \left(d_2^{(i)}, d_{n^{(i)}}^{(i)}\right) \right\}$$

$$5 > 3 \qquad 5 > 2 \qquad 3 > 2$$

# Problems with Pairwise Approach

- Loss function is suboptimal
  - Not to optimize evaluation measures (e.g. NDCG and MAP)
  - Not to consider position in ranking and number of documents per query
- Ranking function cannot represent relational information
- Generalization theory is limited




Pairwise loss vs. (1-NDCG@5)
TREC Dataset

# Our Proposal: Listwise Approach

- Listwise Loss Function
  - Using permutation probability distribution to represent a ranked list
  - Using KL-divergence between permutation probability distributions to define loss function
- Learning to rank relational objects
  - Embed object relationship in the ranking function
  - Formalize ranking function as a new optimization problem.
- Query-level Generalization Analysis

# ListNet: A Listwise Loss Function for Learning to Rank

(ICML, 2007)

# Distance between Ranked Lists

- A Simple Example:
  - function $f$: $f(A)=3,\ f(B)=1,\ f(C)=0$ ABC
  - function $h$: $h(A)=5,\ h(B)=2,\ h(C)=3$ ACB
  - ground truth $g$: $g(A)=5,\ g(B)=3,\ g(C)=2$ ABC
- Question: which function is closer to ground truth?
- Euclidian distance between score vectors?
  - $f$: <3,1,0> $g$:<5,3,2> $h$:<5,2,3>

    Closer!

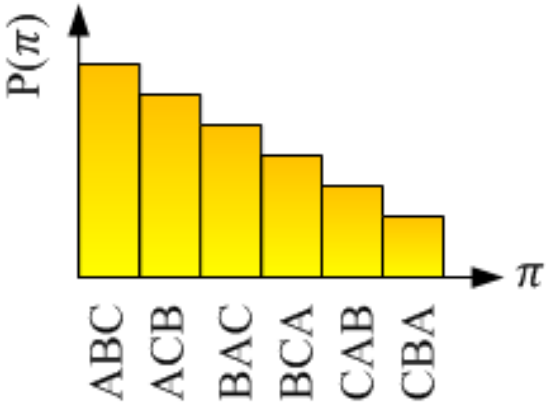
    $d(f,g)=3.5$ $d(g,h)=1.4$
  - However, ranked lists given by $f$ and $g$ are exactly the same!

# Representation of Ranked List

- Question:
  - How to represent a ranked list?
- Our proposal:
  - Ranked list ↔ Permutation probability distribution

$f$: $f(A)=3$, $f(B)=1$, $f(C)=0$;
Ranking by $f$: ABC

↔



# Permutation Probability

- Probability of permutation $\pi$ is defined as (Luce Model)

$$P_s(\pi) = \prod_{j=1}^{n} \frac{\exp(s_{\pi(j)})}{\sum_{k=j}^{n} \exp(s_{\pi(k)})}$$

- Example:

$$P_f(\mathrm{ABC}) = \boxed{\frac{\exp(f(\mathrm{A}))}{\exp(f(\mathrm{A})) + \exp(f(\mathrm{B})) + \exp(f(\mathrm{A}))}} \cdot \boxed{\frac{\exp(f(\mathrm{B}))}{\exp(f(\mathrm{B})) + \exp(f(\mathrm{C}))}} \cdot \boxed{1}$$

**P(A ranked No.1)**

**P(B ranked No.2 | A ranked No.1)**
**= P(B ranked No.1)/(1- P(A ranked No.1))**

**P(C ranked No.3 | A ranked No.1, B ranked No.2)**

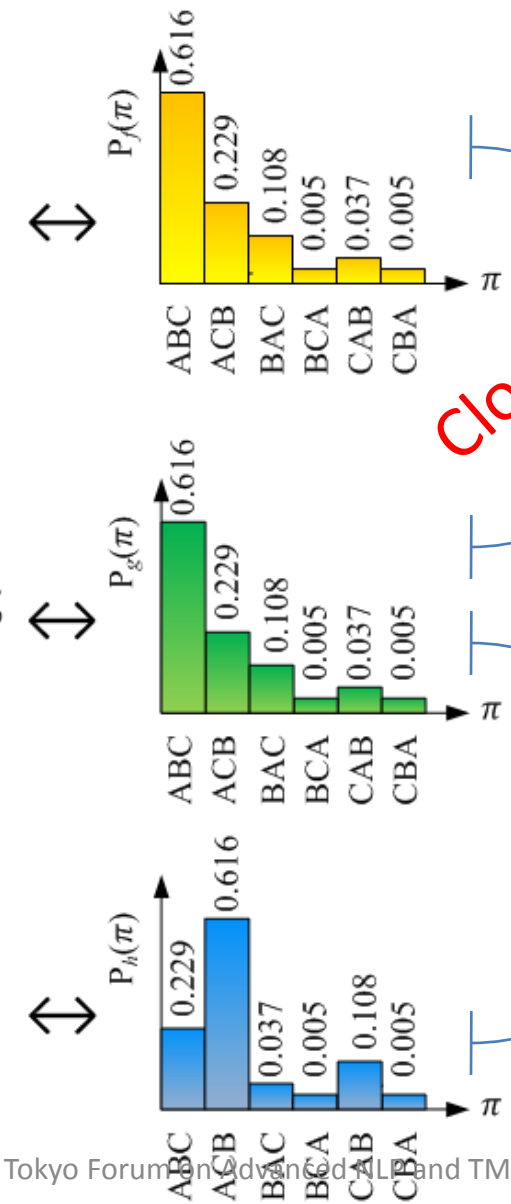
# Distance between Ranked Lists

$\varphi = exp$

$f$: $f(A) = 3, f(B)=1, f(C)=0;$
Ranking by $f$: ABC

$g$: $g(A) = 5, g(B)=3, g(C)=2;$
Ranking by $g$: ABC

$h$: $h(A) = 5, h(B)=2, h(C)=3;$
Ranking by $h$: ACB




*Using **KL-divergence** to measure difference between distributions*

Closer!

$d(f,g) = 0$

$d(g,h) = 0.5$

# Permutation Probability Loss

- Formulation

$$L(w) \propto -\sum_{q \in Q} \sum_{G(j_1,\ldots,j_k)} \left( \prod_{t=1}^{n} \frac{\exp(s_{y(j_t)})}{\sum_{u=t}^{n} \exp(s_{y(j_u)})} \right) \log \left( \prod_{t=1}^{n} \frac{\exp(w \cdot X_{y(j_t)})}{\sum_{u=t}^{n} \exp(w \cdot X_{y(j_u)})} \right)$$

- Properties
  - Continuous and Differentiable
  - Convex
    - Log of a convex function is still convex, and the set of convex functions is closed under addition.
  - Consistent
    - In the large sample limit, minimizing the proposed listwise loss can achieve the optimal Bayes error rate with respect to 0-1 loss.

# Top-k Probability

- Correctly ranking top-$k$ documents is more critical
- Computation of Permutation Probability is intractable
- Top-$k$ Probability
  - Defining Top-$k$ subgroup $G(j_1, \ldots, j_k)$ containing all permutations whose top-$k$ documents are $j_1, \ldots, j_k$
  - $P_s(G(j_1,\ldots, j_k)) = \prod_{t=1}^{k} \frac{\exp(s_{\pi(j_t)})}{\sum_{u=t}^{n} \exp(s_{\pi(j_u)})}$
  - Time complexity of computation : from $n!$ to $n!/(n-k)!$

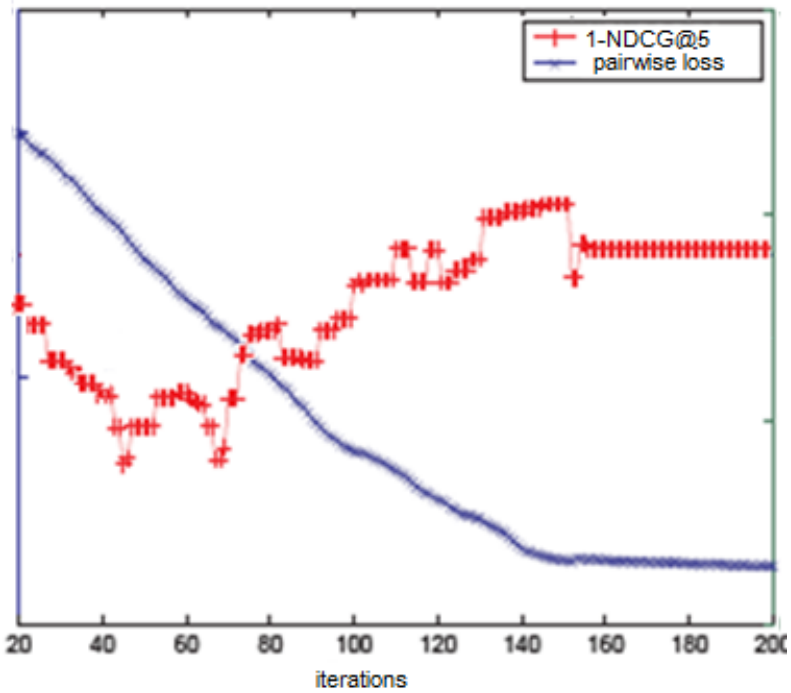
# ListNet Method

- Loss function = KL-divergence between two Top-$k$ probability distributions

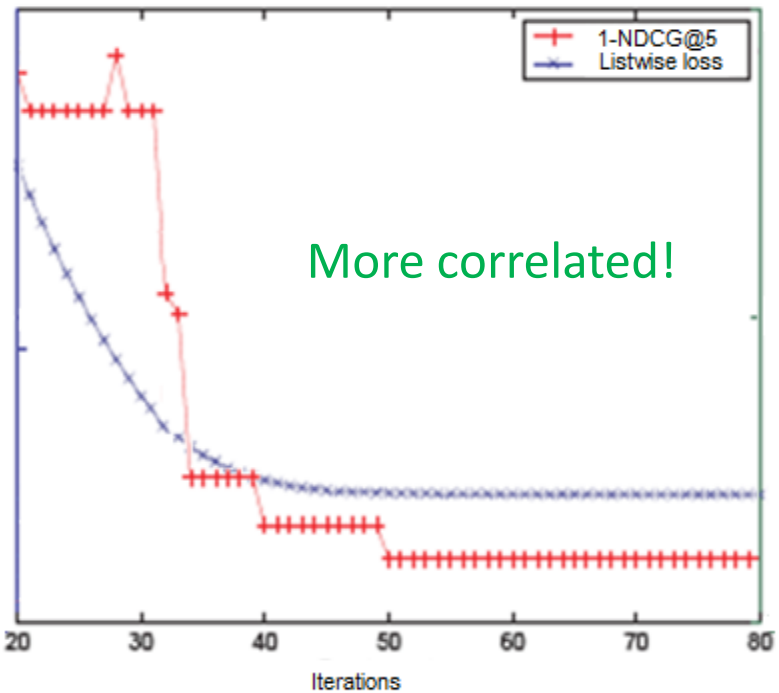
$$L(w) \propto -\sum_{q \in Q} \sum_{G(j_1,\ldots,j_k)} \left( \prod_{t=1}^{k} \frac{\exp(s_{y(j_t)})}{\sum_{u=t}^{n} \exp(s_{y(j_u)})} \right) \log \left( \prod_{t=1}^{k} \frac{\exp(w \cdot X_{y(j_t)})}{\sum_{u=t}^{n} \exp(w \cdot X_{y(j_u)})} \right)$$

- Model = Neural Network
- Algorithm = Stochastic Gradient Descent

# Experimental Results



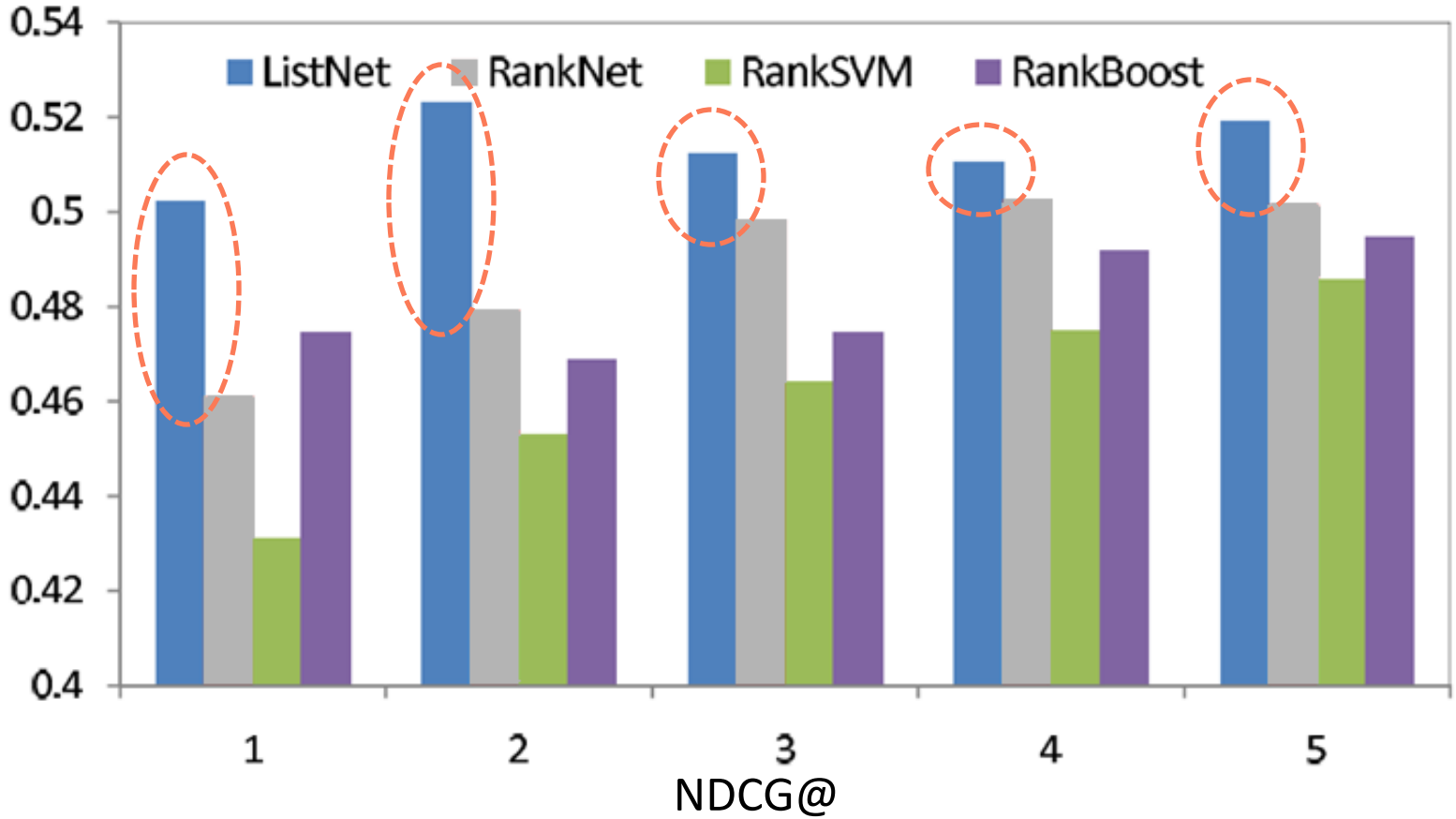

Pairwise (RankNet)




Listwise (ListNet)

Training Performance on TREC Dataset

# Experimental Results



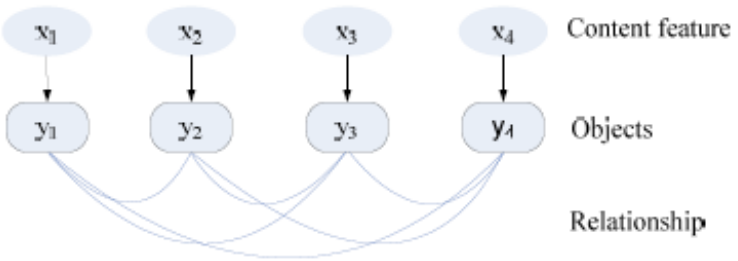

Testing Performance on TREC Dataset

# Learning to Rank Relational Objects using a Listwise Ranking Function

(WWW, 2008)

# Motivation

- Traditional ranking function models **independent relevance**
  - Working on features of single documents
- Many applications in IR are beyond independent relevance
  - Subtopic retrieval
  - Representative page retrieval
  - Pseudo relevance feedback
  - Topic distillation



# Relational Ranking Model

- Generic formulation

$$y = f(X, R) = f(h(X;\omega), R).$$

$$\min_{\omega} \sum_{i=1}^{N} L(f(h(X_i;\omega), R_i), y_i)$$

- Practical formulation

$$f(h(X;\omega), R) = \arg\min_{z} \{l_1(h(X;\omega), z) + \beta l_2(R, z)\}$$

$$l_1(h(X;\omega), z) = \|h(X;\omega) - z\|^2$$

Application-dependent

# Pseudo Relevance Feedback

- A very common type of relationship between objects is similarity.
  - Similarity relationship can be represented in an undirected graph.
  - Such kind of graph is widely used in the learning tasks of clustering and semi-supervised learning.

$$l_2(R, z) = 1/2 \sum_i \sum_j R_{i,j}(z_i - z_j)^2$$

$$f(h(X;\omega), R) = \arg\min_z \{\|X\omega - z\|^2 + \beta/2 \sum_i \sum_j R_{i,j}(z_i - z_j)^2\}$$

# Topic Distillation

- Topic distillation prefers parent objects to be ranked before child objects.
  - Preference relationship can be represented by directed graph.

$$R_{i,j} = \begin{cases} 1 & \text{if instance } i \text{ is the parent of } j, \\ 0 & \text{other.} \end{cases}$$

$$l_2(R, z) = \sum_i \sum_j R_{i,j} \exp(z_j - z_i)$$

$$f(h(X;\omega), R) = \arg\min_z \{\|X\omega - z\|^2 + \beta \sum_i \sum_j R_{i,j} \exp(z_j - z_i)\}$$

# Learning of Relational Ranking Model

- Substitute the ranking model into conventional optimization problems for model learning

$$\min_{\omega} \sum_{i=1}^{N} L(f(h(X_i;\omega), R_i), y_i)$$

$$s.t. \quad f(h(X;\omega), R) = \arg\min_{z} \{l_1(h(X;\omega), z) + \beta l_2(R, z)\}$$

- Conventional Ranking SVM is a special case of relational ranking

$$\min_{\omega,\xi_{i,j}} \tfrac{1}{2}\omega^T\omega + c\mathbf{1}^T[\mathbf{1} - Cf(X;\omega)]_+$$

$$s.t. f(X;\omega) = \arg\min_z \|X\omega - z\|^2$$

# Challenges

- The optimization problem for relational ranking is a constrained optimization problem, and the constraint itself is also an optimization problem.
- No existing method can be directly applied to the problem.
- We propose solving the problem in two steps.
  - Solve the optimization problem in constraint and obtain an explicit form for the constraint
  - Solve the original optimization problem with explicit constraint

# Explicit Model for Pseudo Relevance Feedback

$$l(z) = \|X\omega - z\|^2 + \frac{\beta}{2}\sum_i \sum_j R_{i,j}(z_i - z_j)^2$$

$$= \|X\omega - z\|^2 + \beta z^T (D - R) z.$$

$$\frac{\partial l(z)}{\partial z} = 2(z - X\omega) + 2\beta(D - R)z = 0.$$

$$f(X, G; \omega) = (I + \beta(D - R))^{-1} X\omega$$

# Explicit Model for Topic Distillation

$$l(z) = \|X\omega - z\|^2 + \beta \sum_i \sum_j R_{i,j} \exp(z_j - z_i)$$

$$\exp(z_j - z_i) \approx 1 + (z_j - z_i) + \frac{1}{2}(z_j - z_i)^2.$$

$$\begin{aligned} l(z) &\approx \|X\omega - z\|^2 + \beta \sum_i \sum_j R_{i,j} \left\{ 1 + (z_j - z_i) + \frac{1}{2}(z_j - z_i)^2 \right\} \\ &= \|X\omega - z\|^2 + \beta(g_0 + g_1^T z + z^T (D - R) z) \end{aligned}$$

$$\frac{\partial l(z)}{\partial z} = 2(z - X\omega) + \beta g_1 + \beta(2D - R - R^T) z = 0$$

$$f(X, R; \omega) = (2I + \beta(2D - R - R^T))^{-1}(2X\omega - \beta g_1)$$

# Relational Ranking SVM

Ranking SVM

$$\min_{\omega,\xi_q} \frac{1}{2}\omega^T\omega + c\sum_q \mathbf{1_q^T}\xi_q$$

$$\text{s.t.} \quad C_q f(X_q;\omega) \geq \mathbf{1}_q - \xi_q, f(X_q;\omega) = X_q\omega, \xi \geq 0$$

Relational Ranking SVM for Pseudo Relevance Feedback

$$\min_{\omega,\xi_q} \frac{1}{2}\omega^T\omega + c\sum_q \mathbf{1}_q^T\xi_q$$

$$s.t. C_q f(X_q, R_q;\omega) \geq \mathbf{1}_q - \xi_q, \xi_q \geq 0$$

$$f(X_q, R_q;\omega) = (I - \beta(D_q - R_q))^{-1}X_q\omega$$

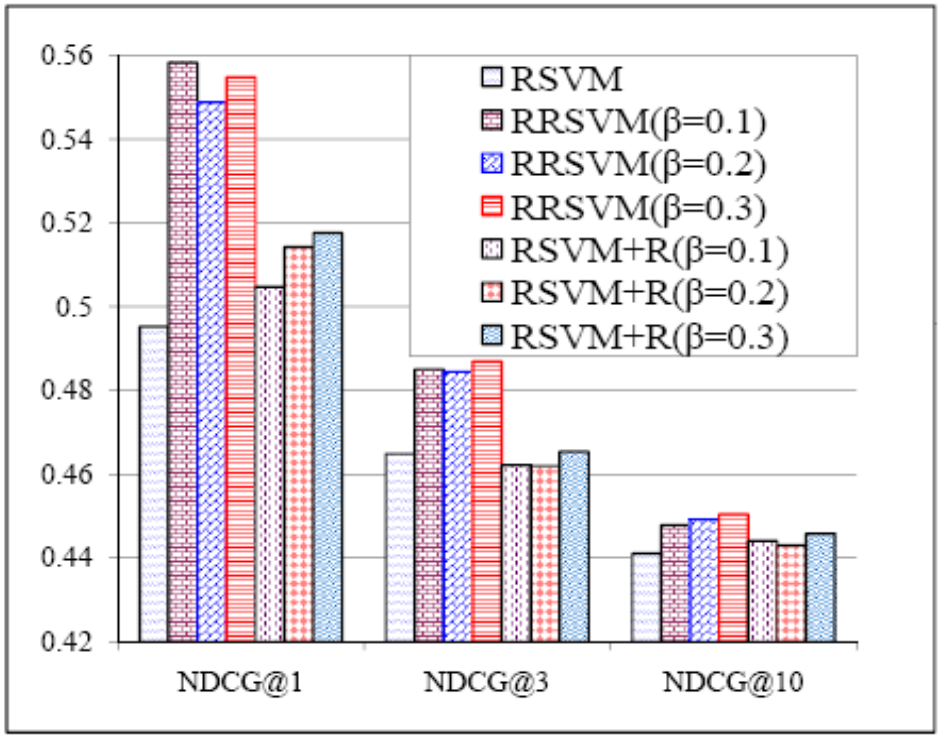
Relational Ranking SVM for Topic Distillation

$$\min_{\omega,\xi_q} \frac{1}{2}\omega^T\omega + c\sum_q \mathbf{1}_q^T\xi_q$$

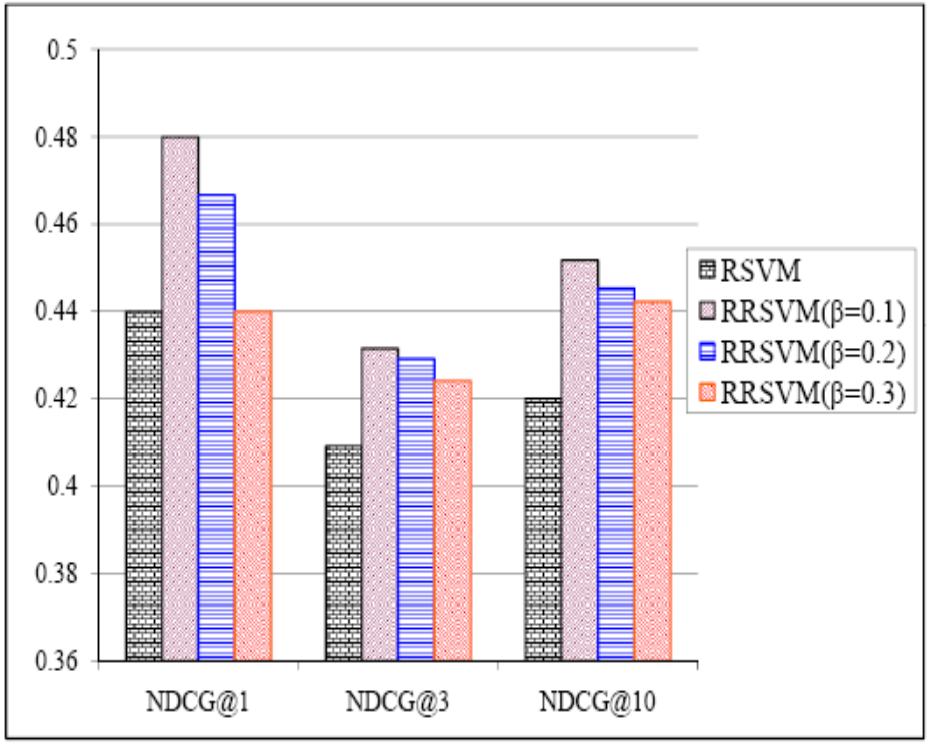
$$s.t. C_q f(X_q, R_q;\omega) \geq \mathbf{1}_q - \xi_q, \xi_q \geq 0$$

$$f(X_q, R_q;\omega) = (2I + \beta(2D_q - R_q - R_q^T))^{-1}(2X_q\omega - \beta g_{q,1})$$

# Experimental Results




Pseudo Relevance Feedback on OHSUMED




Topic Distillation on TD 2004

# Summary

- Listwise approach is more effective than pairwise approach

- Future Work
  - Combination of listwise loss function and listwise ranking function
  - Investigation of generalization ability of listwise approach

# References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# Thanks!

Benchmark Dataset for Learning to Rank:
http://research.microsoft.com/users/LETOR/