

Generative Adversarial Networks

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Gatsby, UCL → TTIC → UBC



(from thispersondoesnotexist.com)

MLCC 2019

Generative models

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- Want a model for the data: $Q \approx \mathbb{P}$

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Why produce samples?



Is artificial intelligence set to become art's next medium?

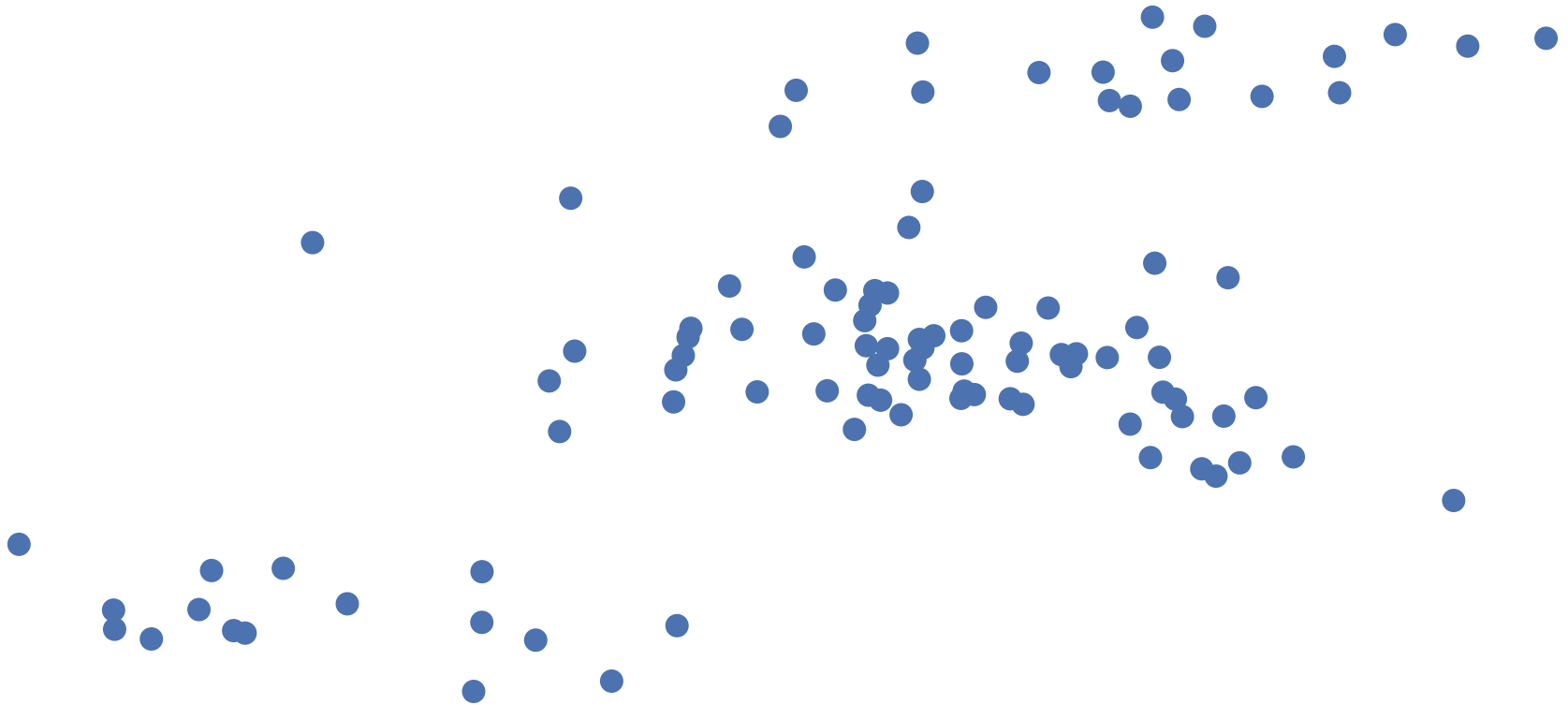
AI artwork sells for \$432,500 — nearly 45 times its high estimate — as Christie's becomes the first auction house to offer a work of art created by an algorithm

The portrait in its gilt frame depicts a portly gentleman, possibly French and — to judge by his dark frockcoat and plain white collar — a man of the church. The work appears unfinished: the facial features are somewhat indistinct and there are blank areas of canvas. Oddly, the whole composition is displaced slightly to the north-west. A label on the wall states that the sitter is a man named Edmond Belamy, but the giveaway clue as to the origins of the work is the artist's signature at the bottom right. In cursive Gallic script it reads:

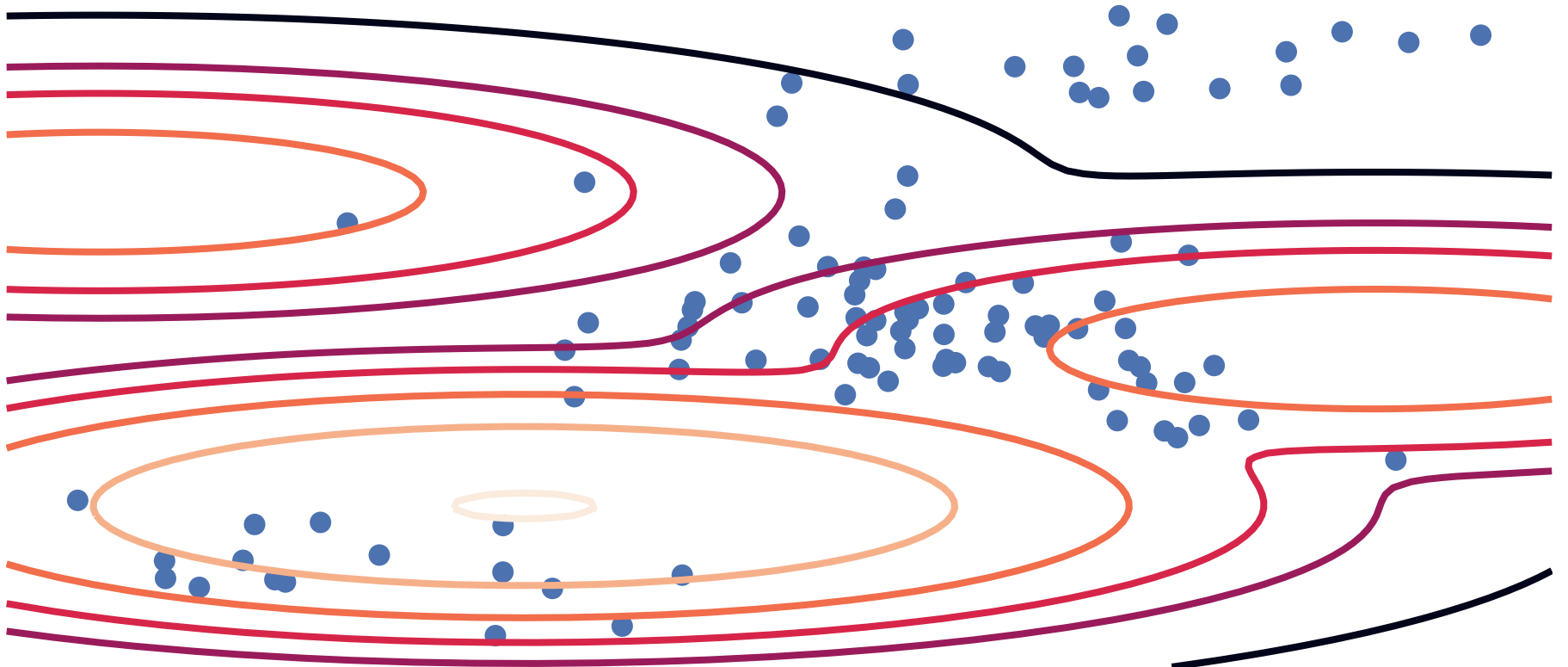
$$\min_G \max_D \mathbb{E}_x [\log(D(x))] + \mathbb{E}_z [\log(1 - D(G(z)))]$$

Image © Obvious

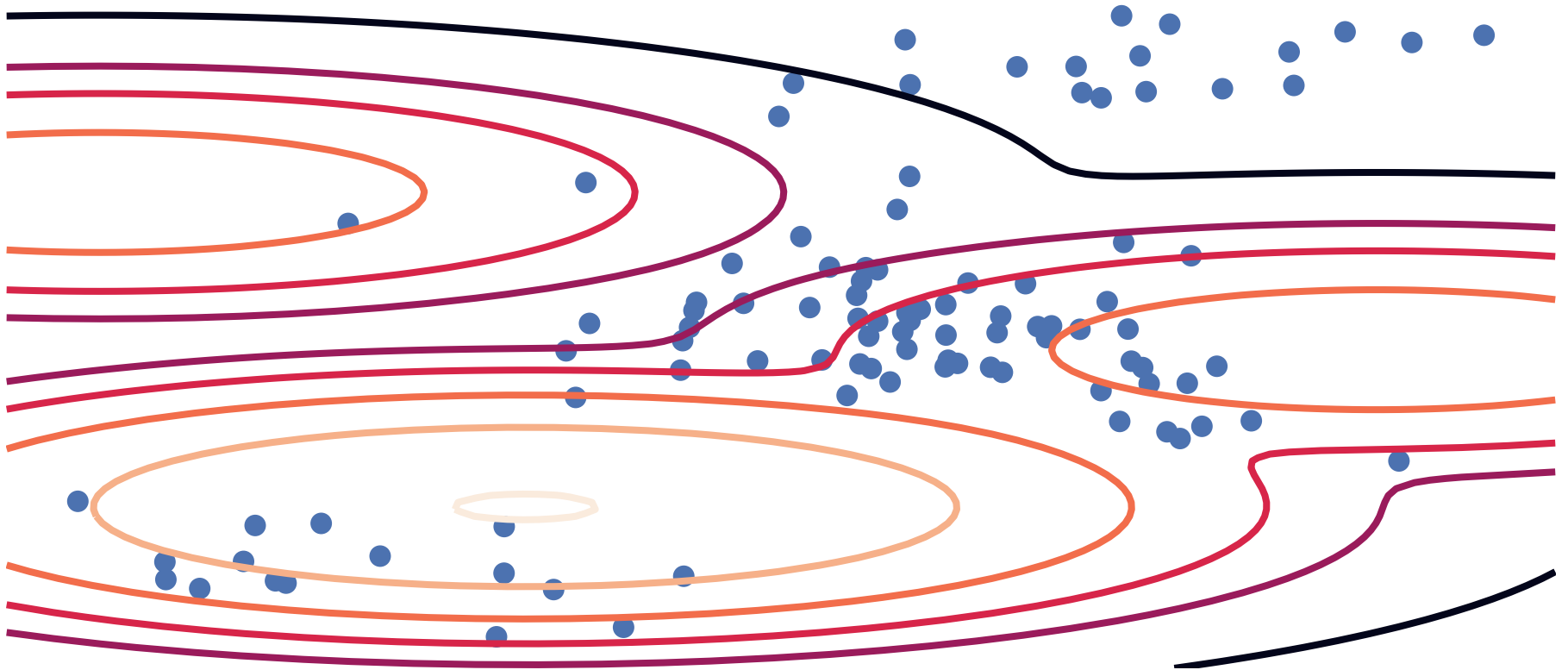
Generative models: a traditional way



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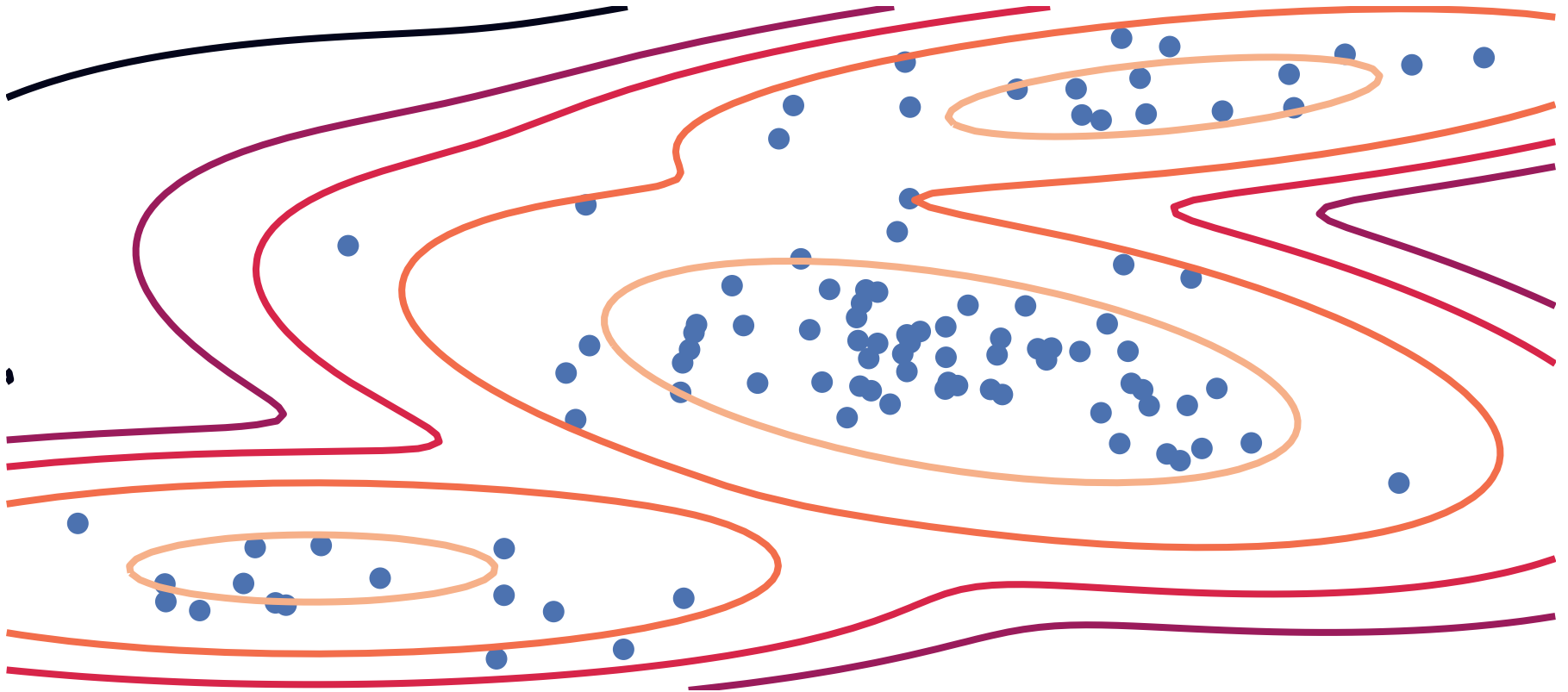


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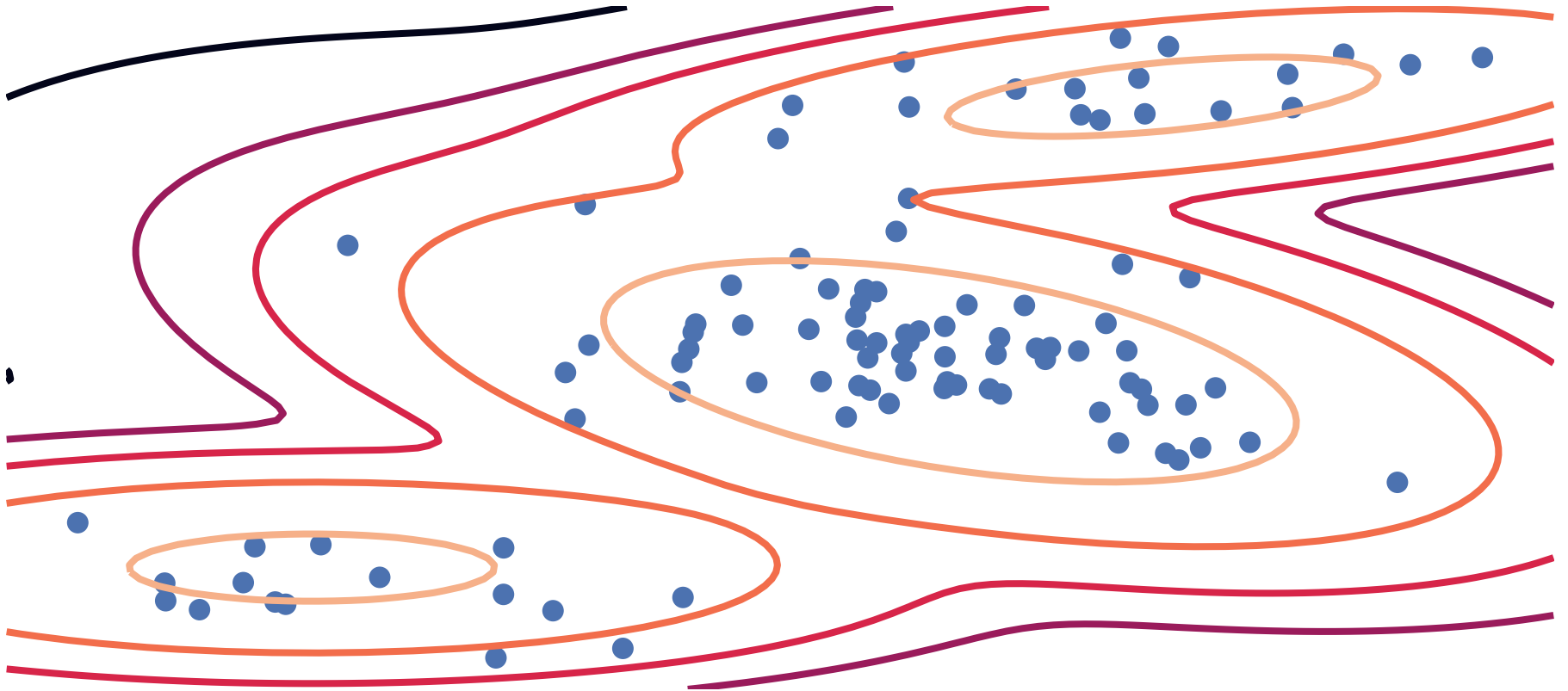
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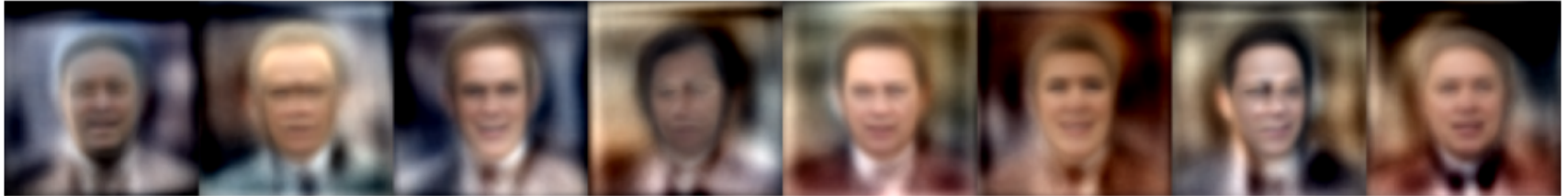
Generative models: a traditional way



- Maximum likelihood: $\max_{\theta} \mathbb{E}_{X \sim \mathbb{P}} [\log q_{\theta}(X)]$
- Equivalent: $\min_{\theta} \text{KL}(\mathbb{P} \parallel \mathbb{Q}_{\theta}) = \min_{\theta} \int p(x) \log \frac{p(x)}{q_{\theta}(x)} dx$

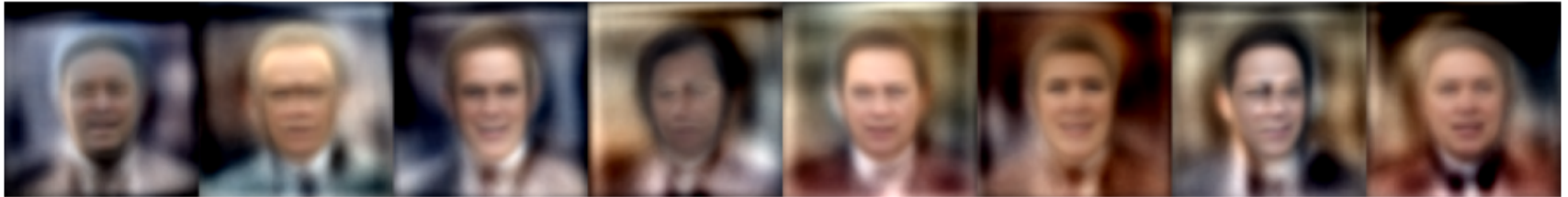
Traditional models for images

- 1987-style generative model of faces (Eigenface via [Alex Egg](#))



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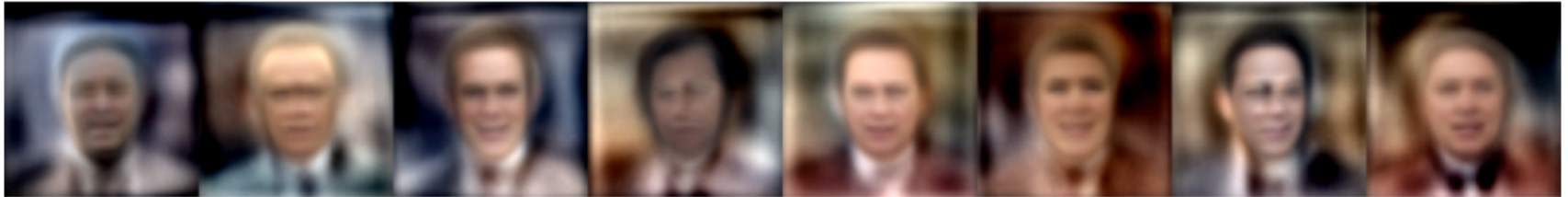
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- Usually based on Gaussian noise $\approx L_2$ loss

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- One use case of generative models is inpainting [[Harry Yang](#)]:



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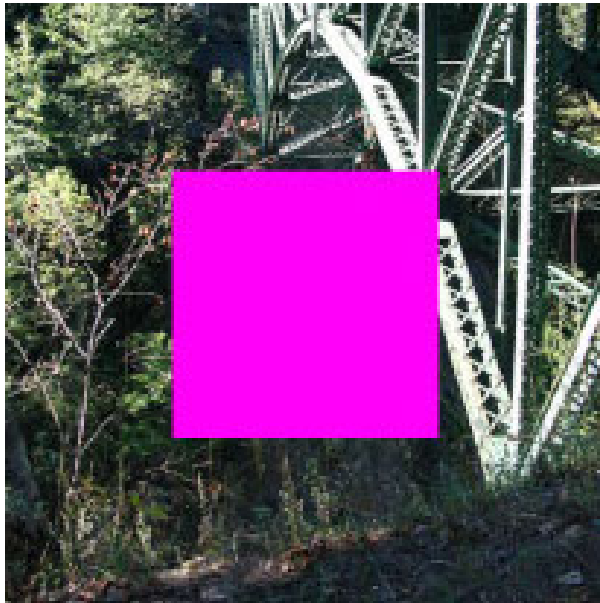
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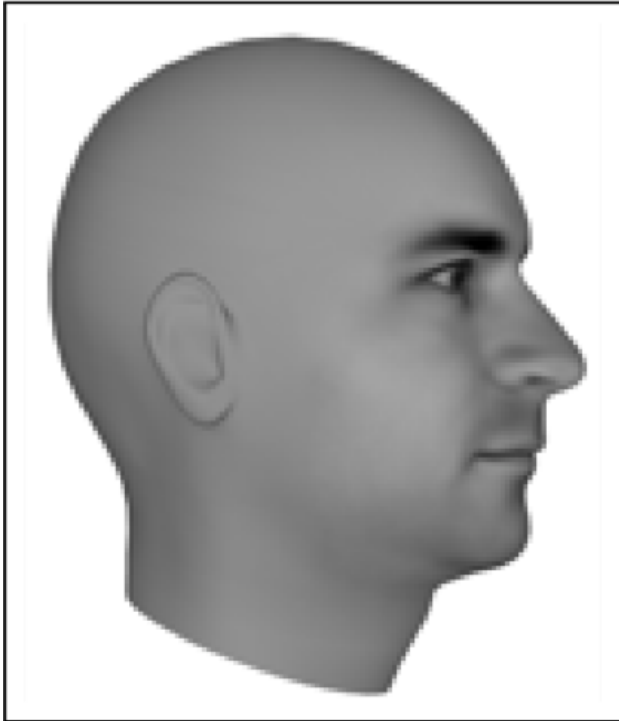
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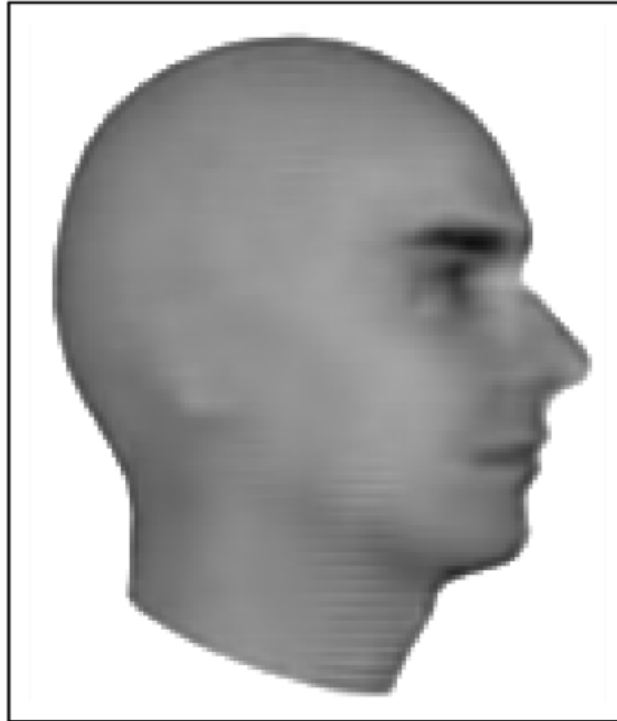
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Next-frame video prediction

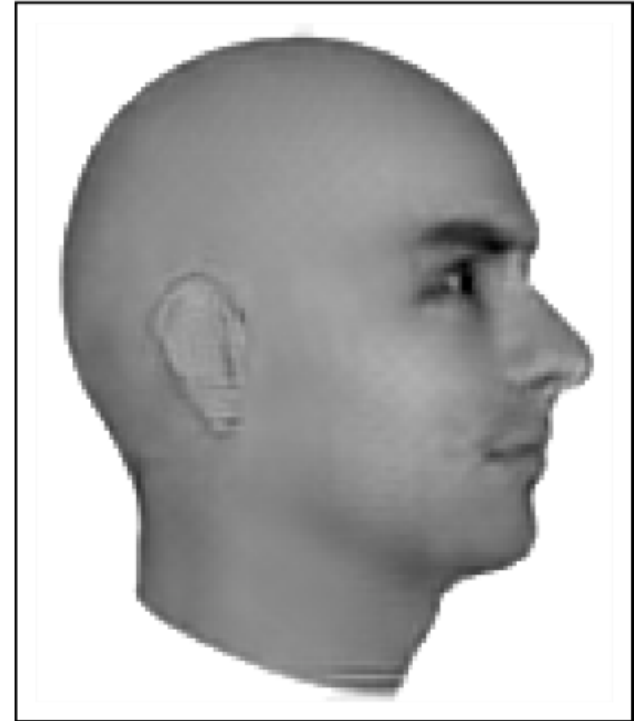
Ground Truth



MSE



Adversarial



[Lotter+ 2016]

Trick a discriminator [Goodfellow+ NeurIPS-14]

Generator (Q_{θ})



Discriminator



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Is this real?



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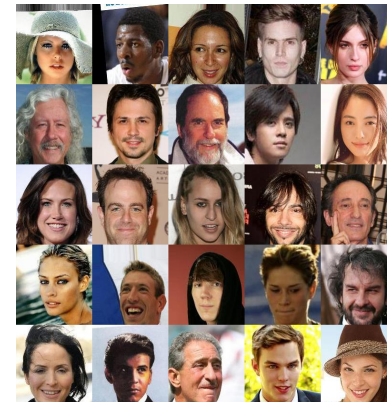
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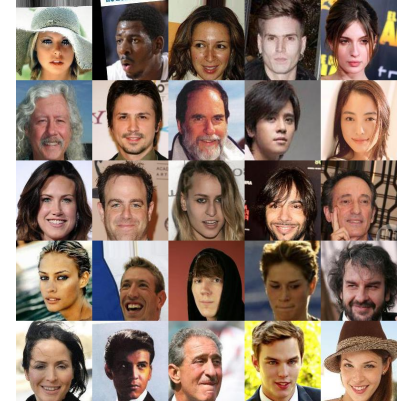
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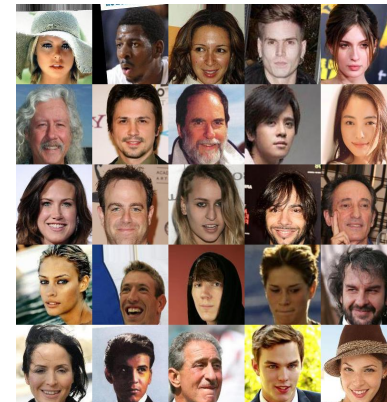


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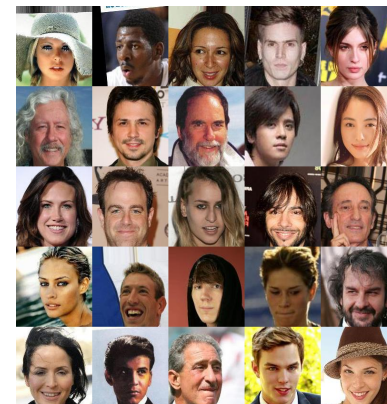
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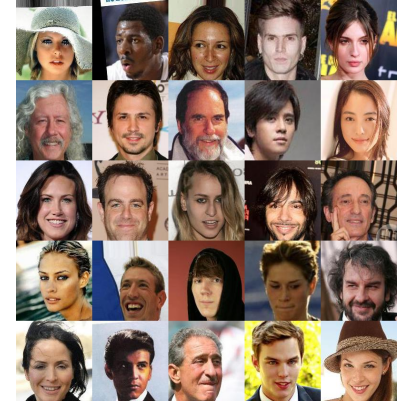
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Umm... $\Pr(\text{real}) = 0.48$

Aside: deep learning in one slide

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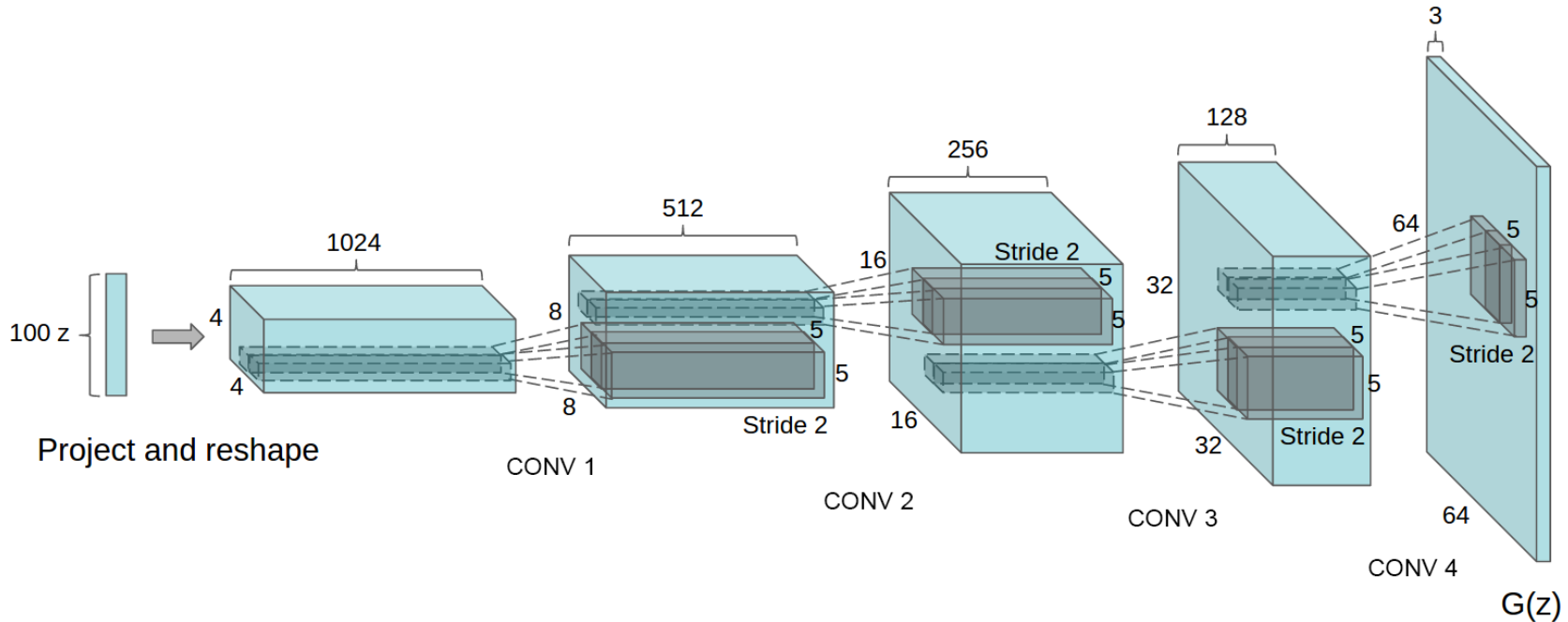
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- Optimize with gradient descent

Generator networks

- How to specify Q_θ ?



[Radford+ ICLR-16]

- $Z \sim \mathbb{Z} = \text{Uniform}([-1, 1]^{100})$
- $G_\theta : [-1, 1]^{100} \rightarrow \mathcal{X}, G_\theta(Z) \sim Q_\theta$

GANs in equations

- Tricking the discriminator:

$$\min_{\theta} \max_{\psi} \frac{1}{2} \mathbb{E}_{X \sim \mathbb{P}} [\log D_{\psi}(X)] + \frac{1}{2} \mathbb{E}_{Y \sim Q_{\theta}} [\log(1 - D_{\psi}(Y))]$$

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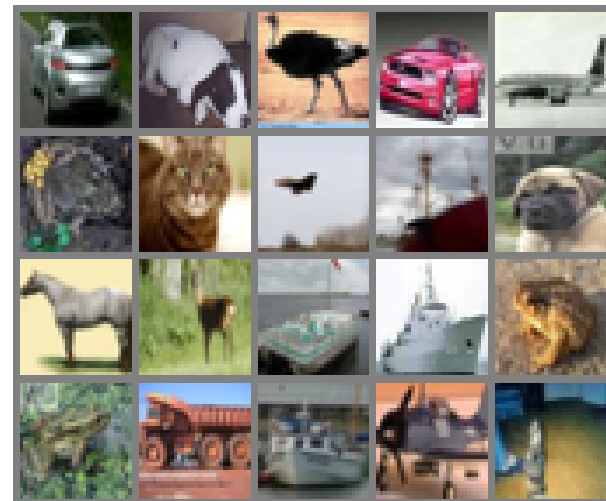
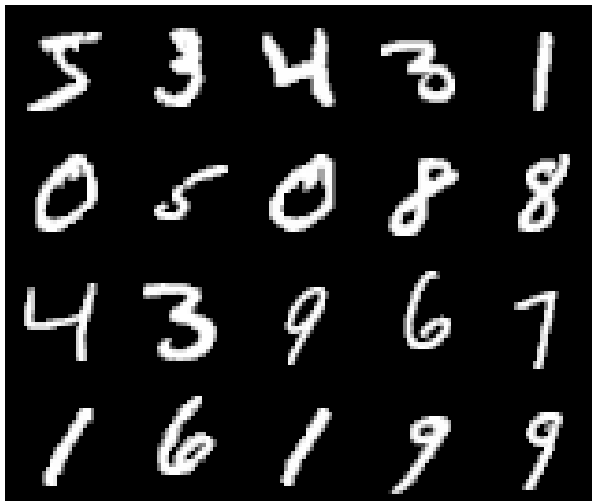
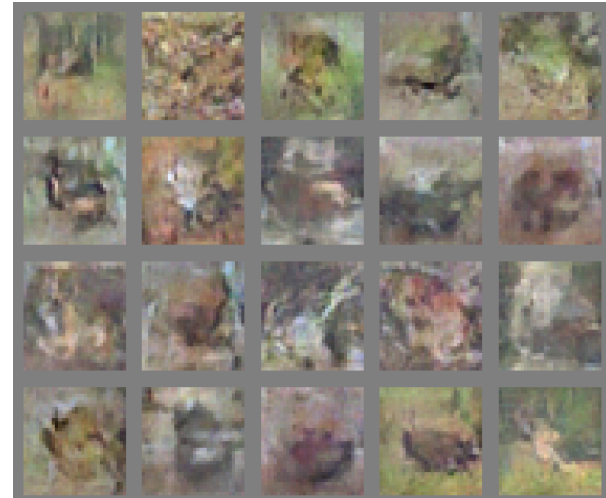
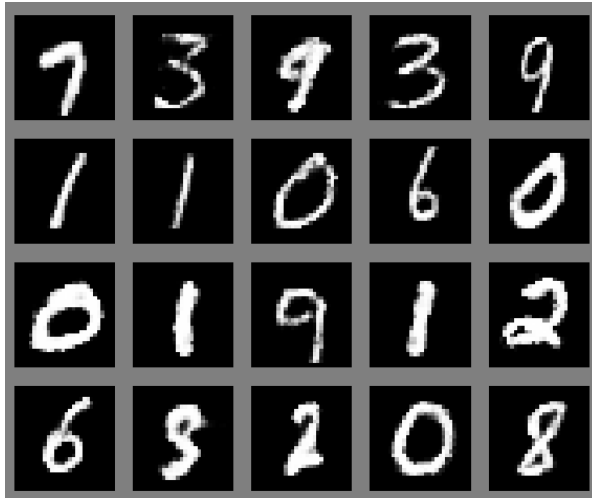
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- Can do alternating gradient descent!

Original paper's results [[Goodfellow+ NeurIPS-14](#)]

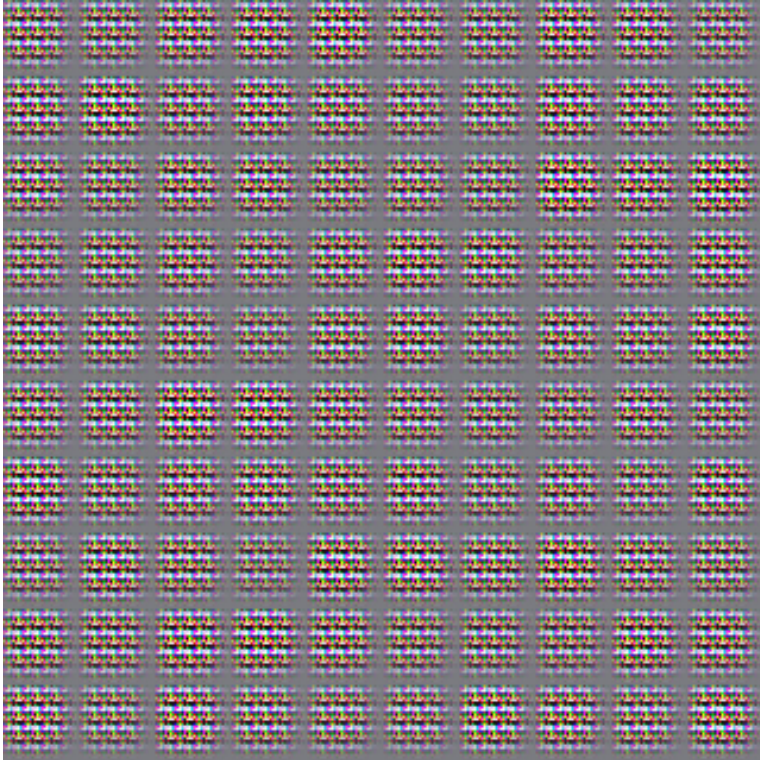


DCGAN results [Radford+ ICLR-16]



Training instability

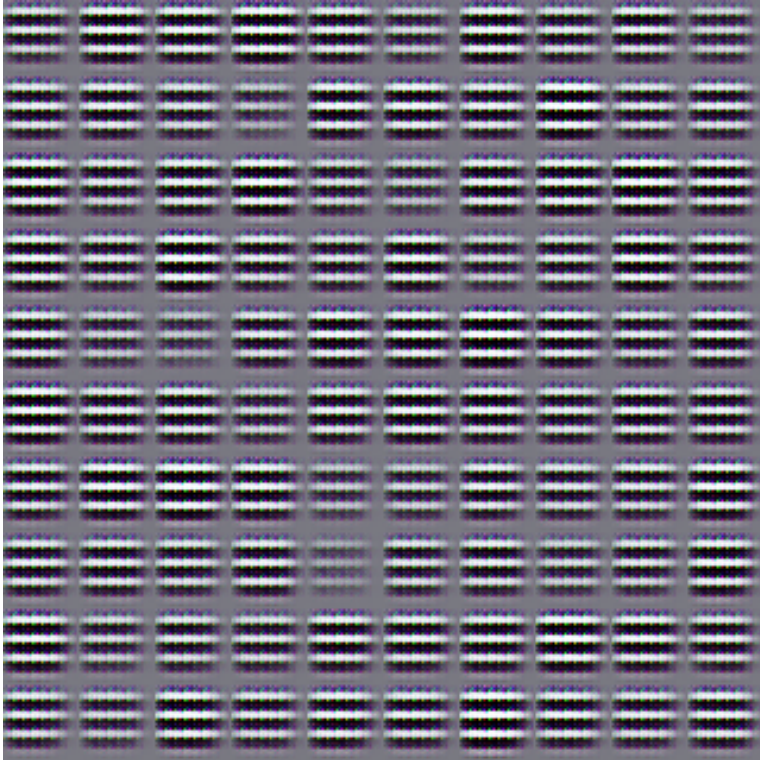
Running code from [Salimans+ NeurIPS-16]:



Run 1, epoch 1

Training instability

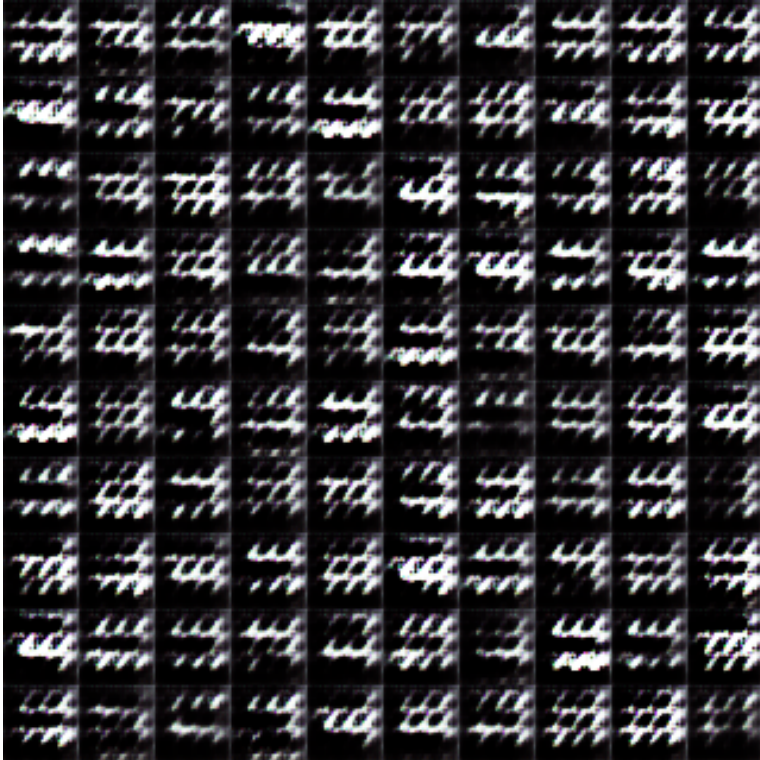
Running code from [[Salimans+ NeurIPS-16](#)]:



Run 1, epoch 2

Training instability

Running code from [Salimans+ NeurIPS-16]:



Run 1, epoch 3

Training instability

Running code from [\[Salimans+ NeurIPS-16\]](#):



Run 1, epoch 4

Training instability

Running code from [\[Salimans+ NeurIPS-16\]](#):



Run 1, epoch 5

Training instability

Running code from [Salimans+ NeurIPS-16]:



Run 1, epoch 6

Training instability

Running code from [\[Salimans+ NeurIPS-16\]](#):



Run 1, epoch 11

Training instability

Running code from [[Salimans+ NeurIPS-16](#)]:



Run 1, epoch 501

Training instability

Running code from [\[Salimans+ NeurIPS-16\]](#):



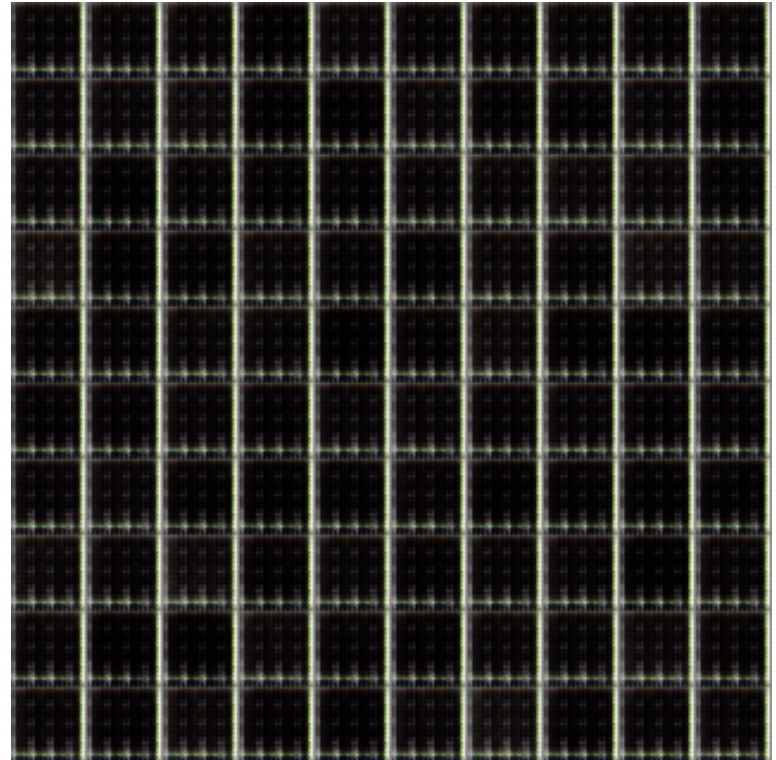
Run 1, epoch 900

Training instability

Running code from [\[Salimans+ NeurIPS-16\]](#):



Run 1, epoch 900



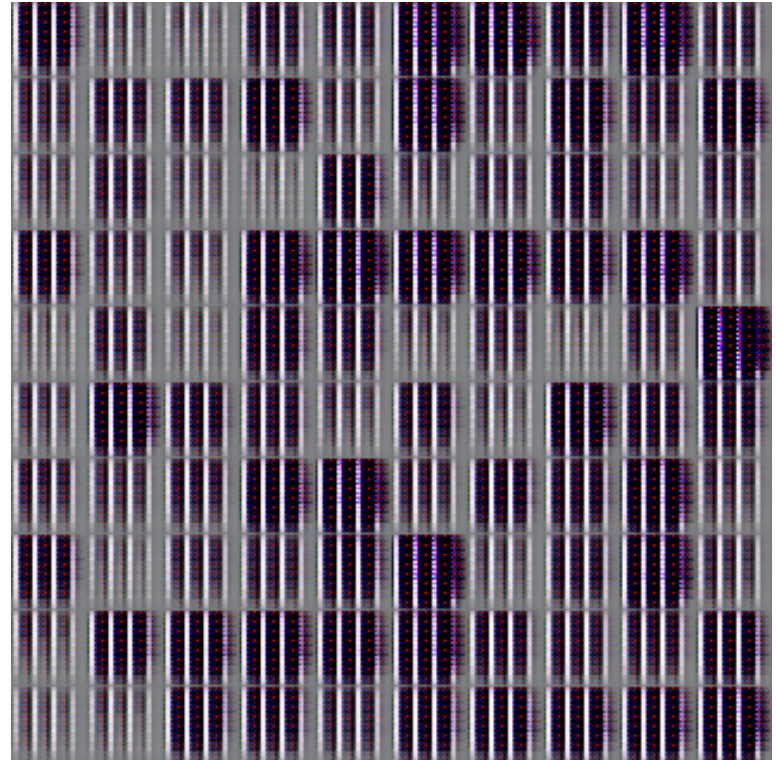
Run 2, epoch 1

Training instability

Running code from [\[Salimans+ NeurIPS-16\]](#):



Run 1, epoch 900



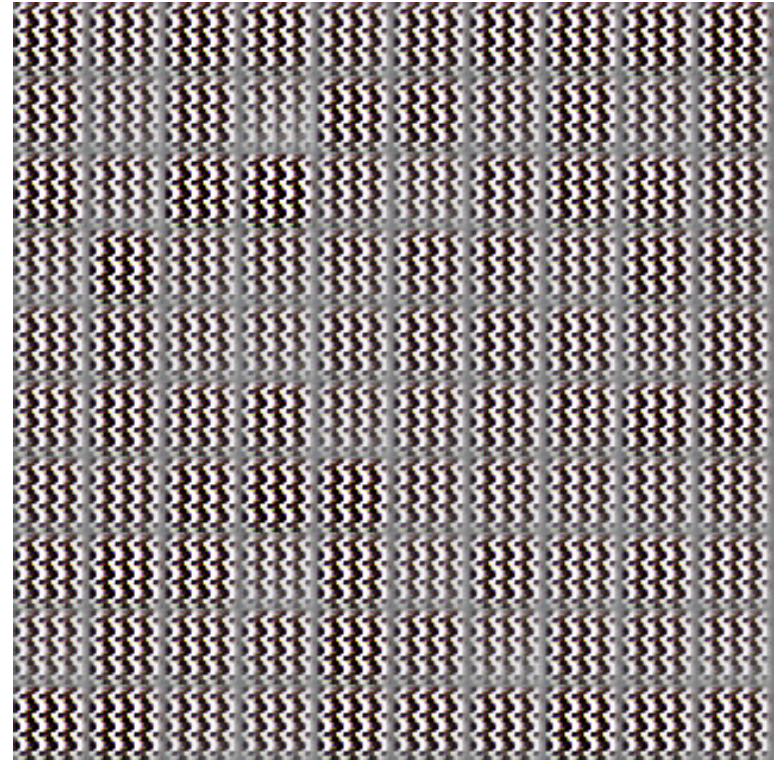
Run 2, epoch 2

Training instability

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Run 1, epoch 900



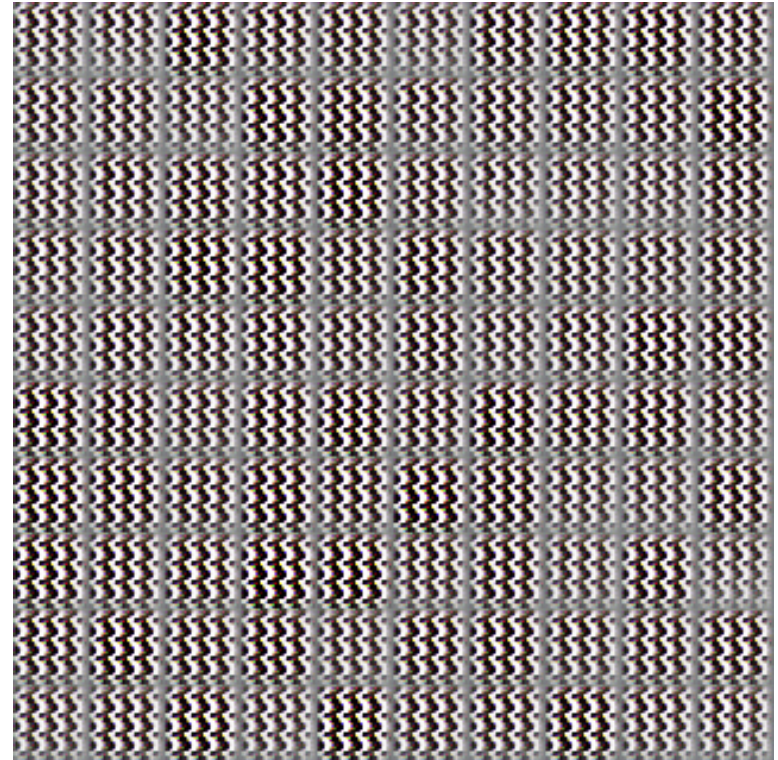
Run 2, epoch 3

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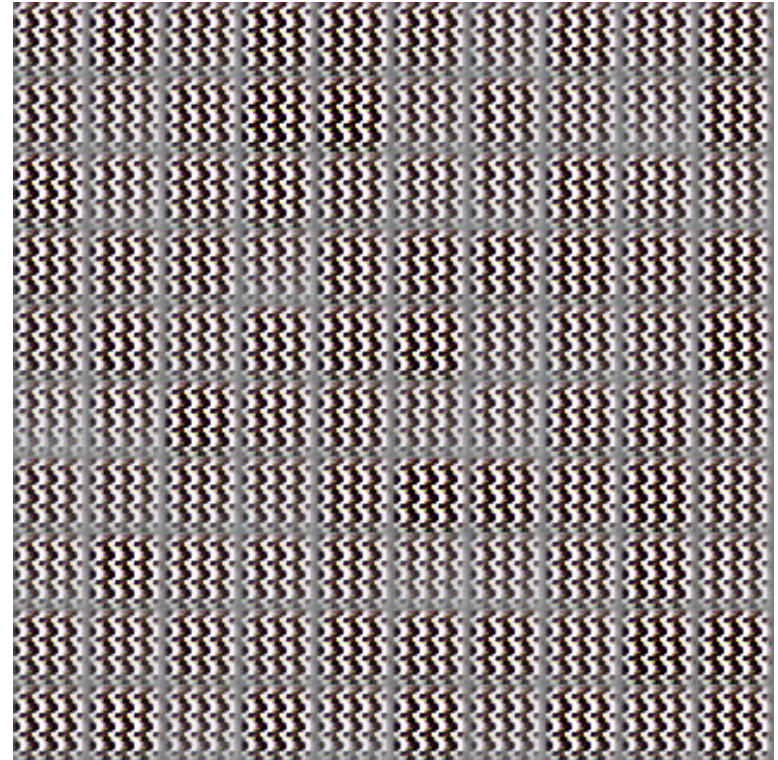
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Run 2, epoch 5

One view: distances between distributions

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One view: distances between distributions

- What happens when D_ψ is at its optimum?
- If distributions have densities, $D_\psi^*(x) = \frac{p(x)}{p(x)+q_\theta(x)}$
- If D_ψ stays optimal throughout, θ tries to minimize

$$\frac{1}{2} \mathbb{E}_{X \sim \mathbb{P}} \left[\log \frac{p(X)}{p(X) + q_\theta(X)} \right] + \frac{1}{2} \mathbb{E}_{Y \sim \mathbb{Q}_\theta} \left[\log \frac{q_\theta(X)}{p(X) + q_\theta(X)} \right]$$

which is

$$\text{JS}(\mathbb{P}, \mathbb{Q}_\theta) - \log 2$$

Jensen-Shannon divergence

$$\begin{aligned} \text{JS}(\mathbb{P}, \mathbb{Q}_\theta) &= \frac{1}{2} \int p(x) \log \frac{p(x)}{\frac{1}{2}p(x) + \frac{1}{2}q_\theta(x)} dx \\ &+ \frac{1}{2} \int q_\theta(x) \log \frac{q_\theta(x)}{\frac{1}{2}p(x) + \frac{1}{2}q_\theta(x)} dx \end{aligned}$$

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JS with disjoint support [Arjovsky/Bottou ICLR-17]

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$$\text{so } \text{JS}(\mathbb{P}, \mathbb{Q}_\theta) = \log 2$$

Discriminator point of view

Generator (Q_{θ})



Discriminator



Discriminator point of view

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Discriminator



Is this real?



Discriminator point of view

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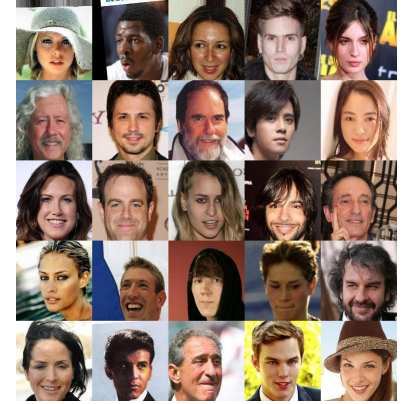
Is this real?



Discriminator



Target (\mathbb{P})



Discriminator point of view

Generator (Q_{θ})

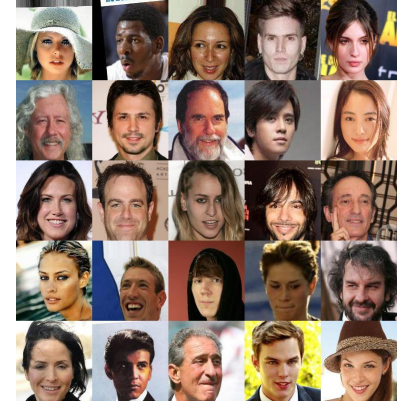


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Target (\mathbb{P})

Discriminator



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Discriminator point of view

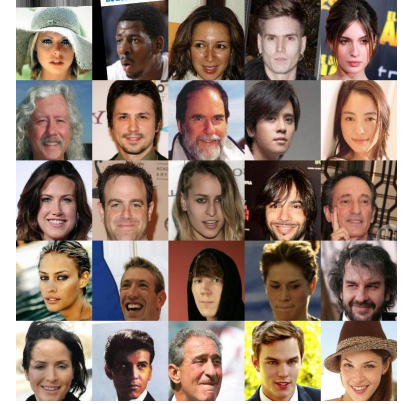
Generator (Q_{θ})



Discriminator



Target (\mathbb{P})



Is this real?

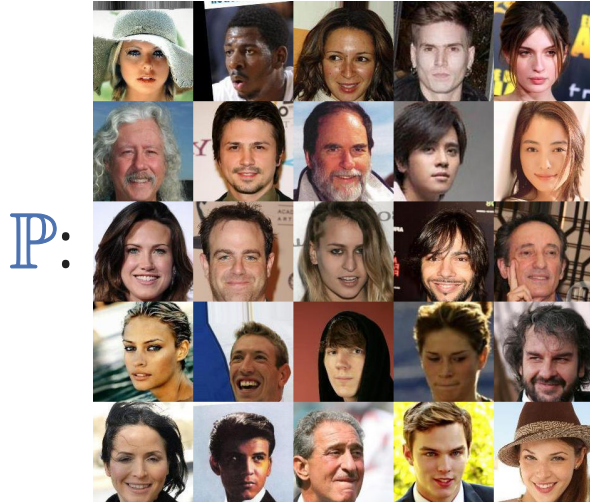


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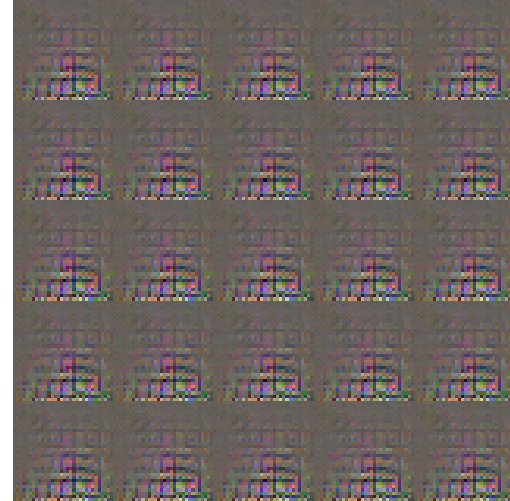
:(I don't know how to do any better...

How likely is disjoint support?

- At initialization, pretty reasonable:

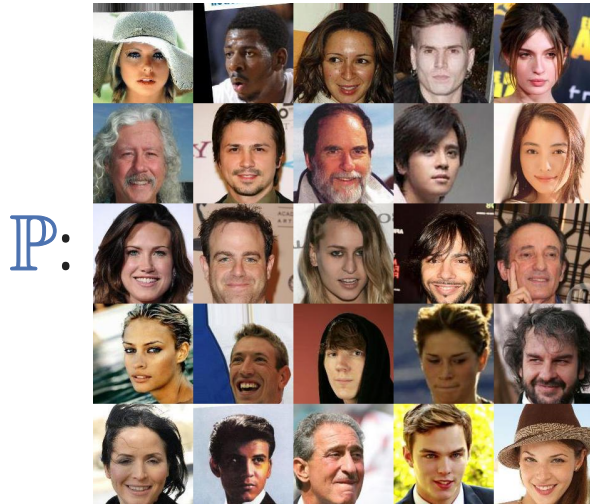


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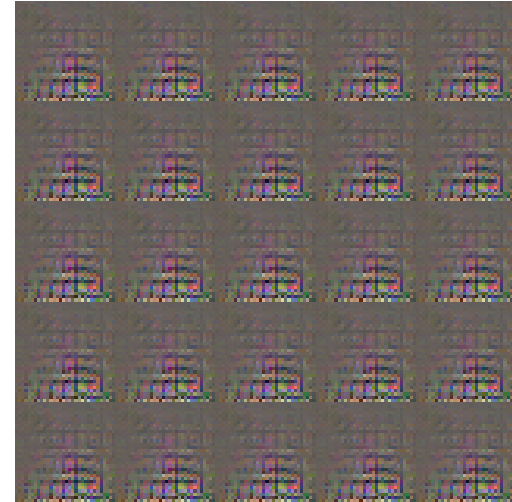


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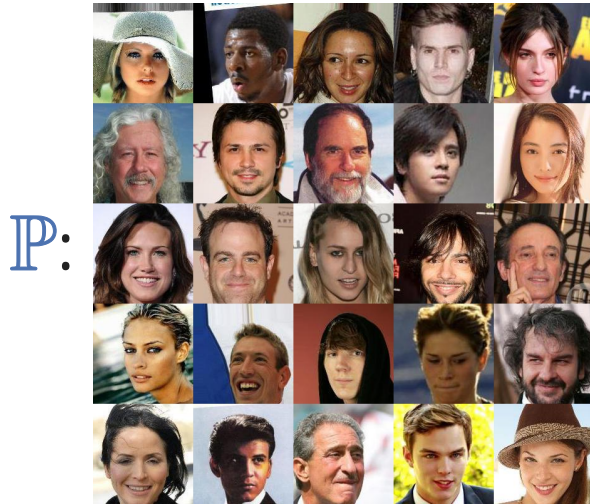
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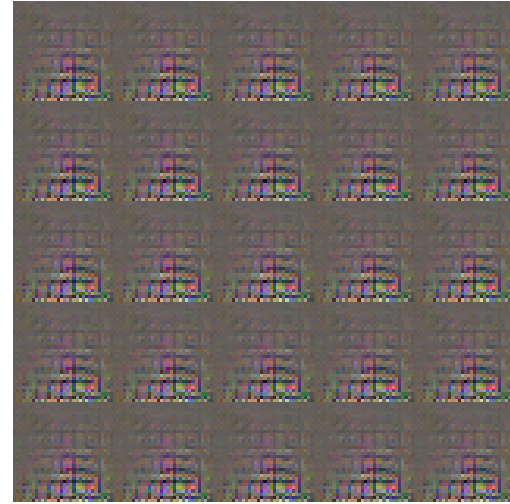
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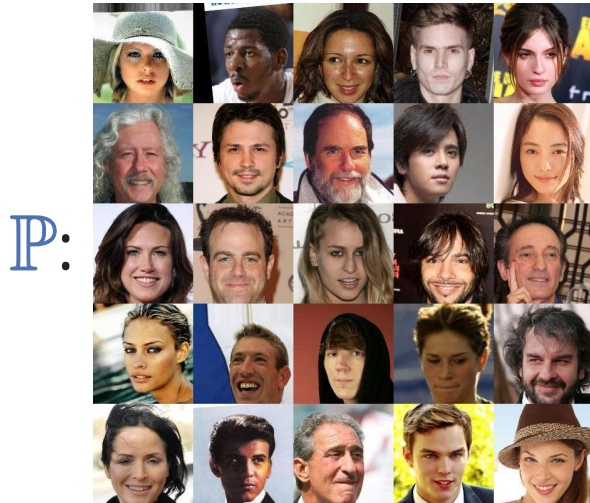
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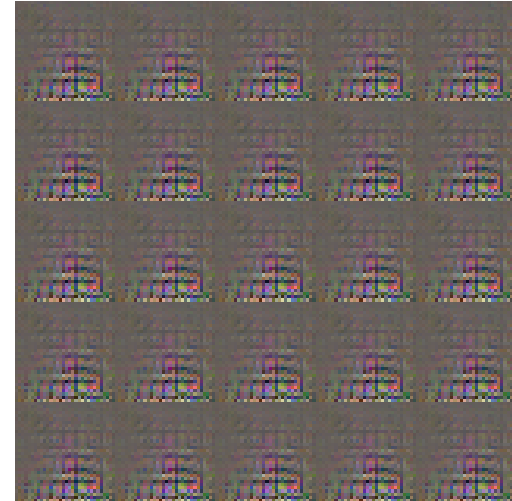
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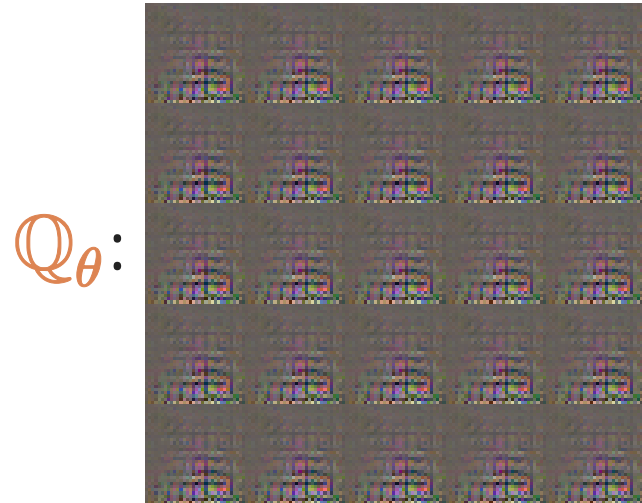
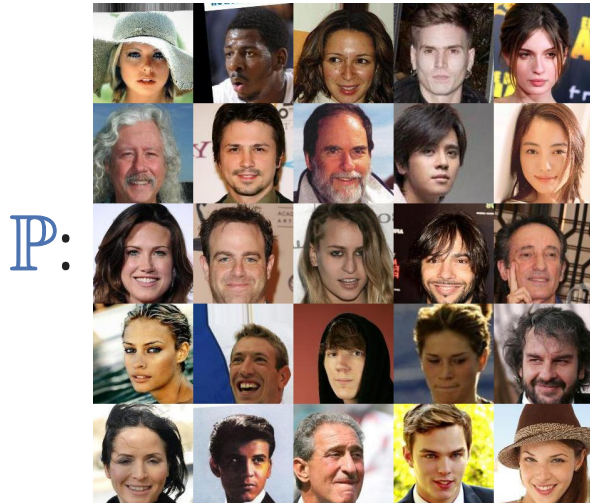
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- For usual G_θ , Q_θ is supported on a countable union of manifolds with $\dim \leq 100$
- “Natural image manifold” usually considered low-dim
- No chance that they'd align at init, so $\text{JS}(\mathbb{P}, Q_\theta) = \log 2$

A heuristic partial workaround

- Original GANs almost never use the minimax game

$$\min_{\theta} \max_{\psi} \frac{1}{2} \mathbb{E}_{X \sim \mathbb{P}} [\log D_{\psi}(X)] + \frac{1}{2} \mathbb{E}_{Y \sim Q_{\theta}} [\log(1 - D_{\psi}(Y))]$$

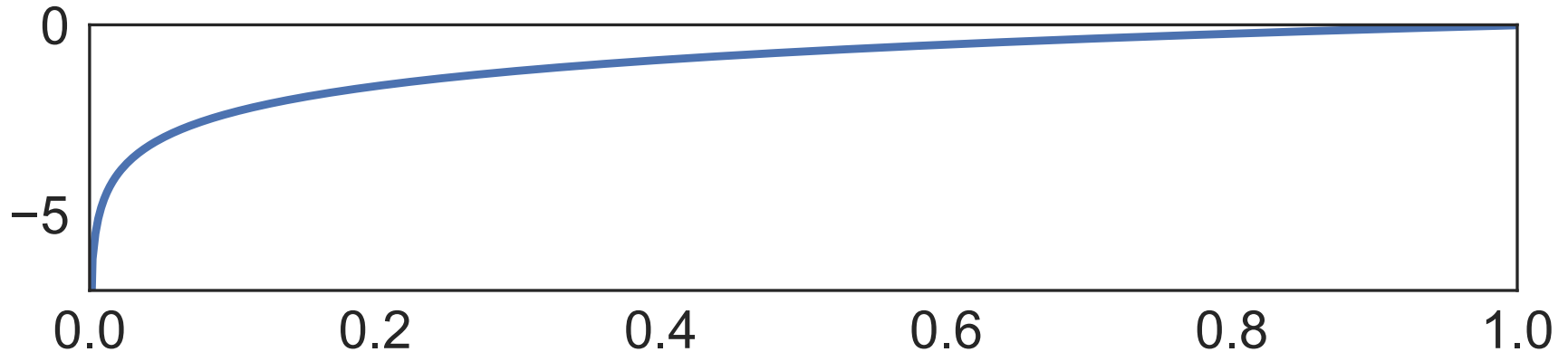
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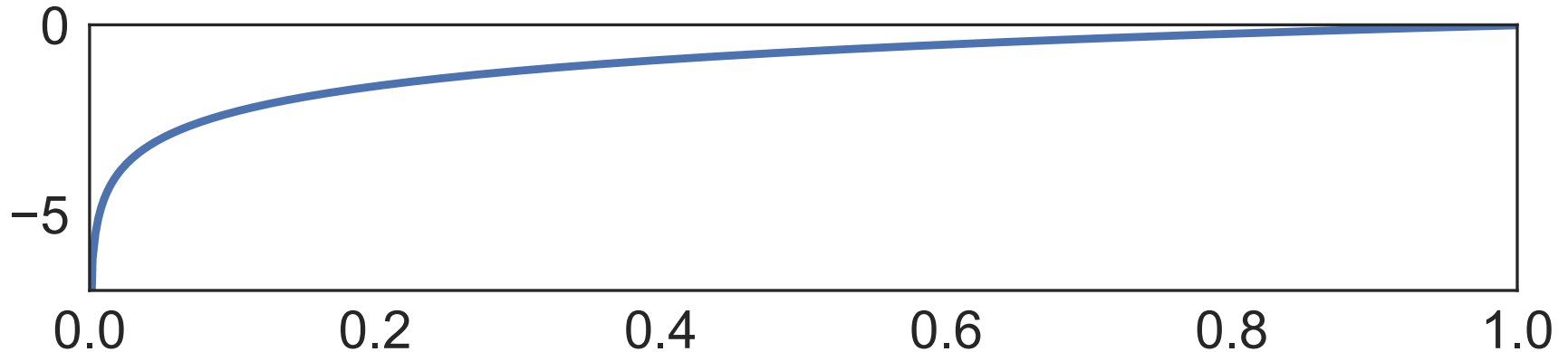


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- When D_{ψ} is near-perfect, makes it unstable instead of stuck

Solution 1: the Wasserstein distance

$$\mathcal{W}(\mathbb{P}, \mathbb{Q}) = \sup_{f: \|f\|_{\text{Lip}} \leq 1} \mathbb{E}_{X \sim \mathbb{P}} [f(X)] - \mathbb{E}_{Y \sim \mathbb{Q}} [f(Y)]$$

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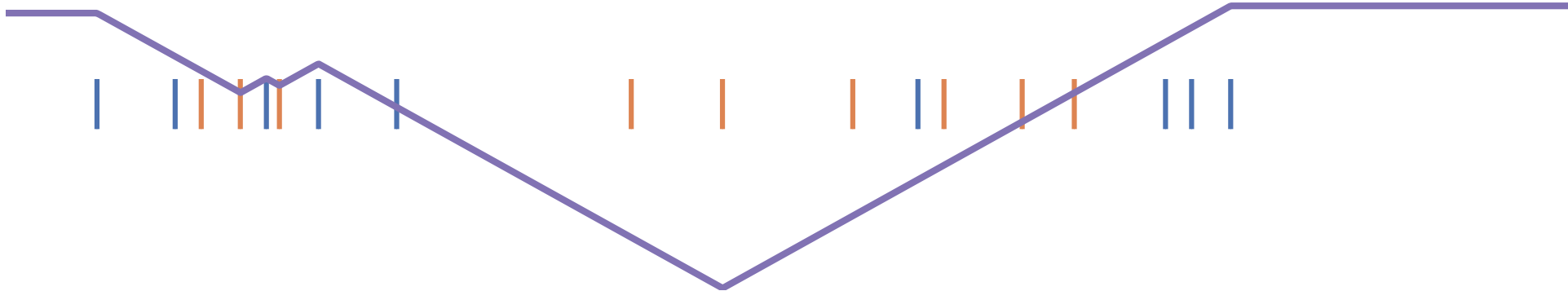
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- Specifically: $\tilde{X} = \theta X + (1 - \theta)Y, \theta \sim \text{Uniform}([0, 1])$

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Solution 3: Spectral normalization [Miyato+ ICLR-18]

- Regular deep nets: $f_\ell = \sigma(W_\ell f_{\ell-1}(x) + b_\ell)$
- Spectral normalization: $f_\ell = \sigma\left(\frac{1}{\|W_\ell\|_2} W_\ell f_{\ell-1}(x) + b_\ell\right)$
- $\|W\|_2 := \sup_{x \neq 0} \frac{\|Wx\|_2}{\|x\|_2} = \sigma_{\max}(W)$ is the *spectral norm*
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New samples [Mescheder+ ICML-18]



How to evaluate?



FID [Heusel+ NeurIPS-17] and KID [Bińkowski+ ICLR-18]

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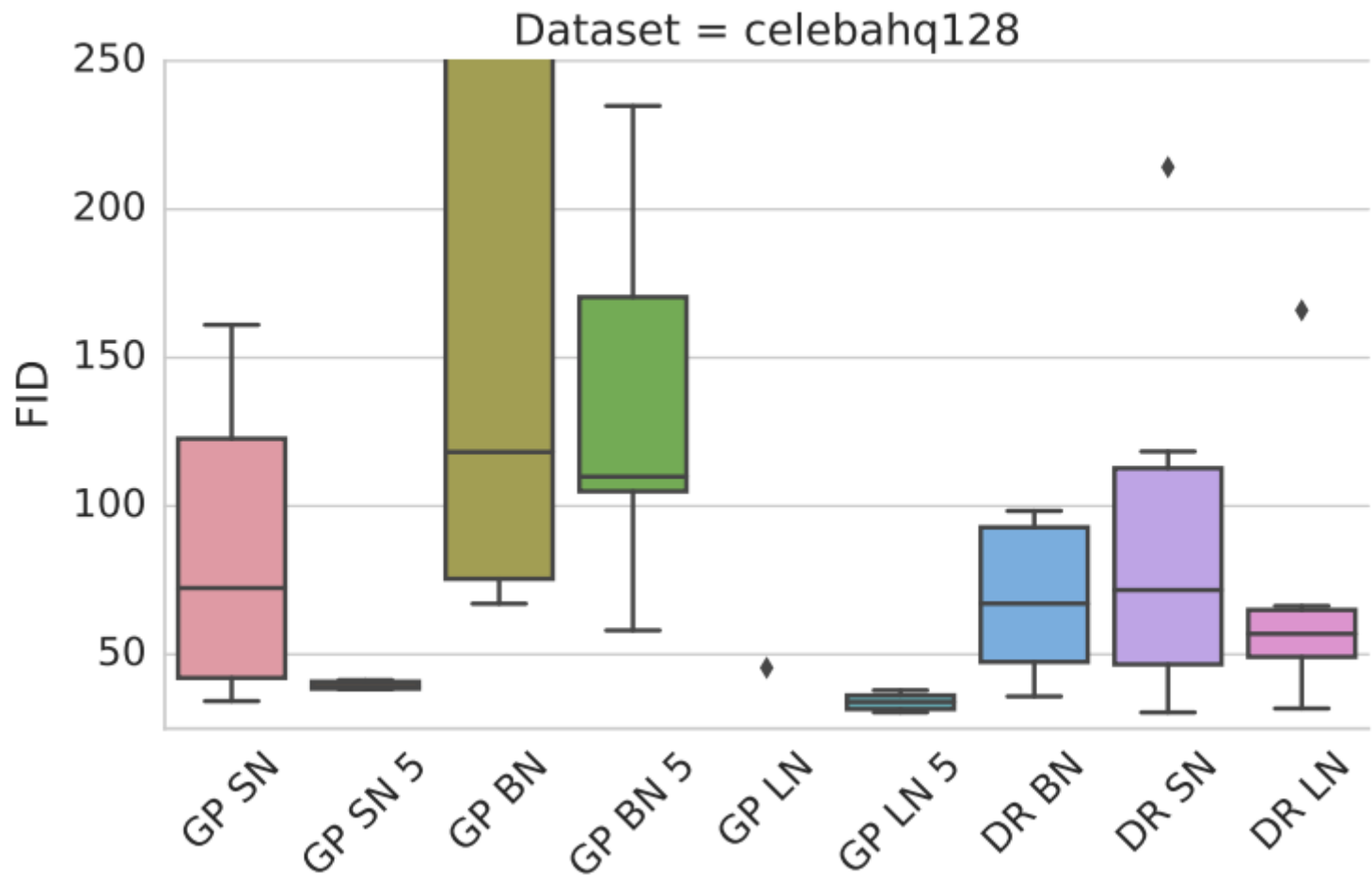
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 - Similar distance with unbiased, ~normal estimator!

Comparing approaches [Kurach+ ICML-19]



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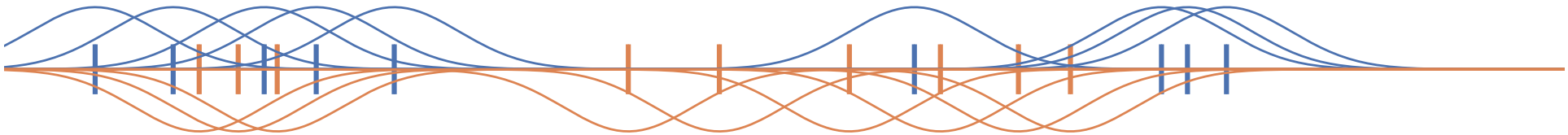
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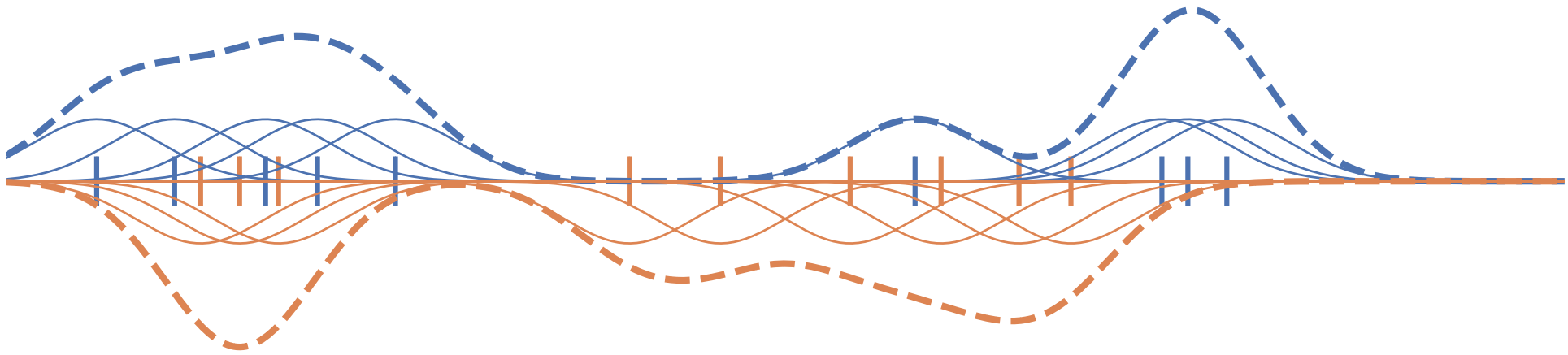
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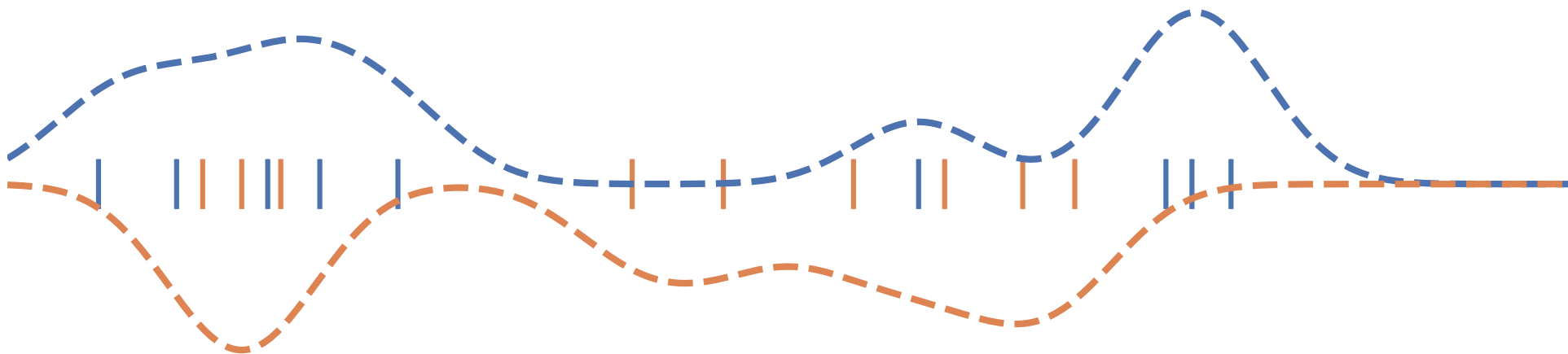
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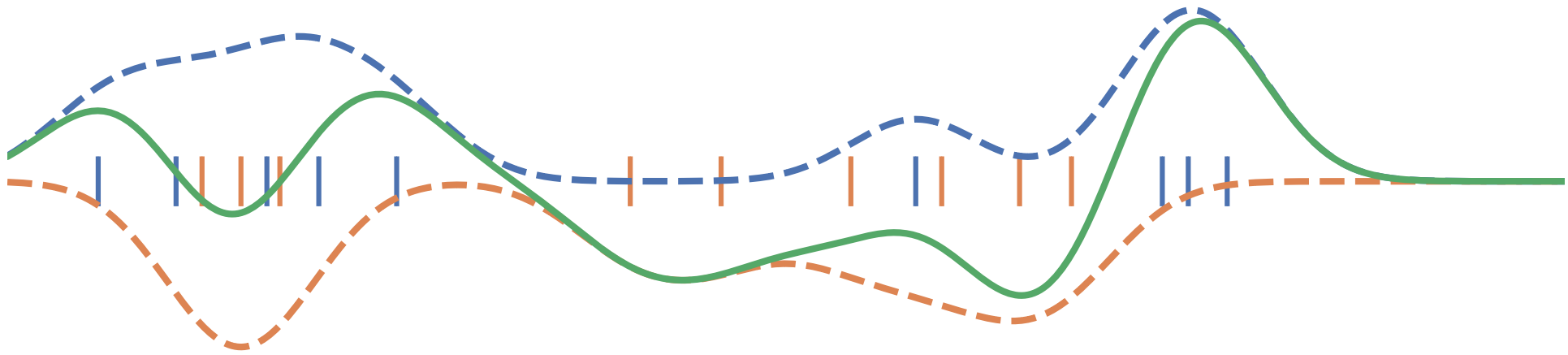
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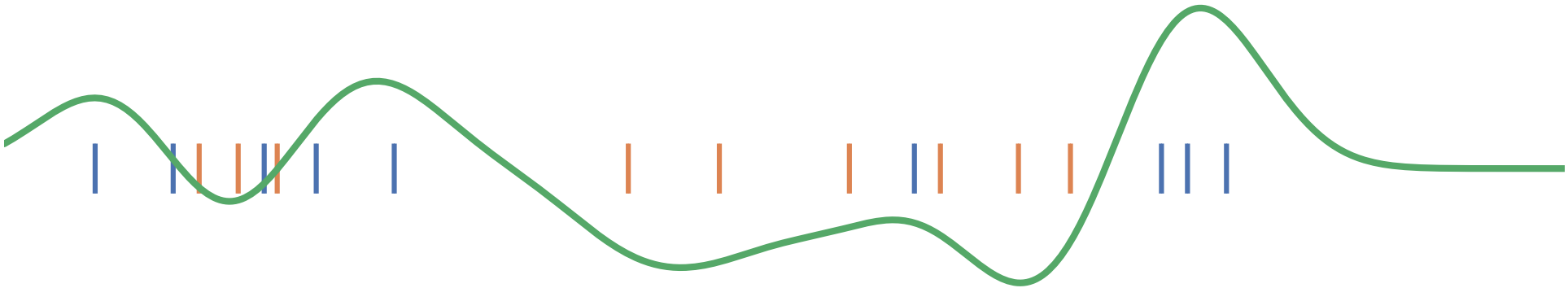
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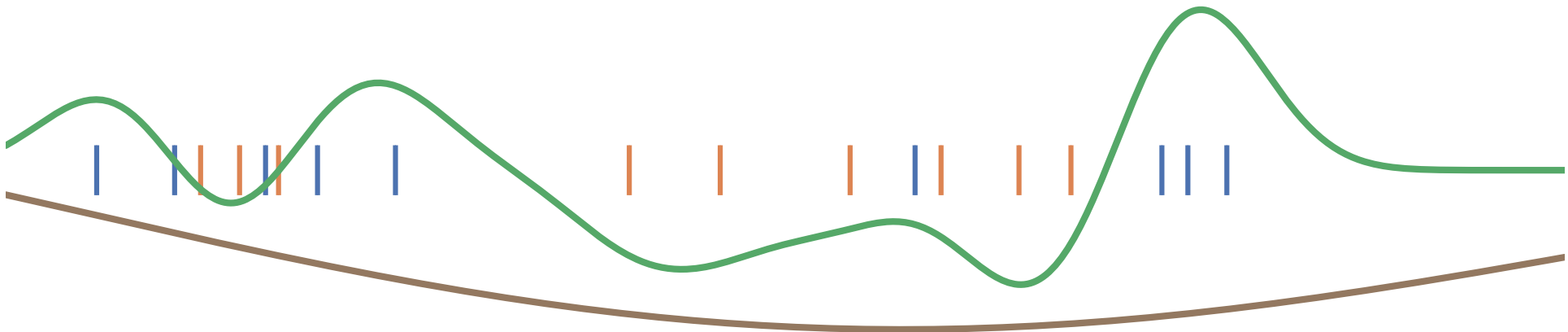
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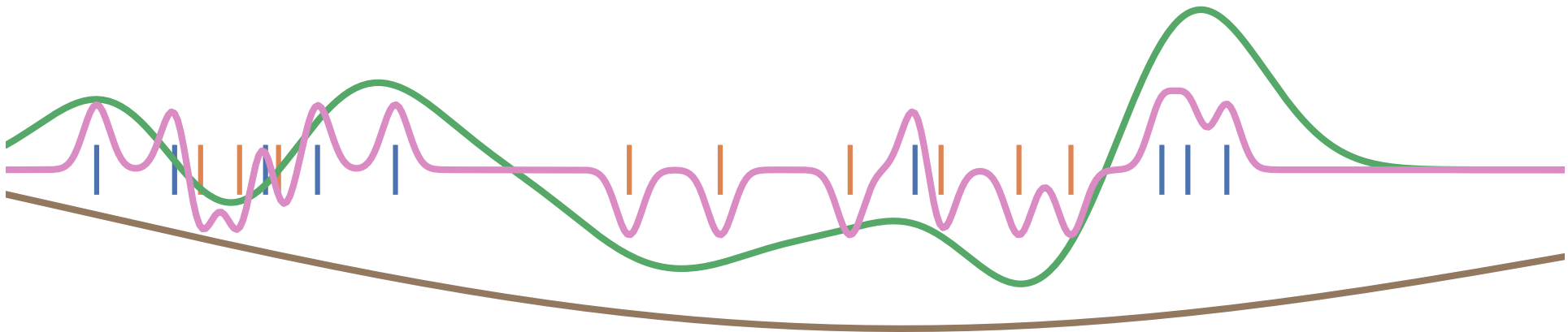
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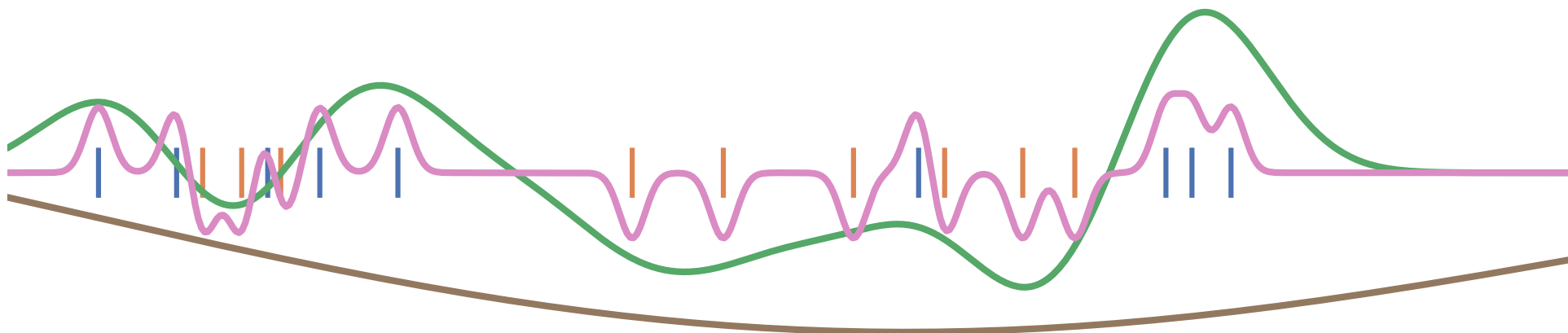


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Optimal f analytically: $f^*(t) \propto \mathbb{E}_{X \sim \mathbb{P}} k(t, X) - \mathbb{E}_{Y \sim \mathbb{Q}} k(t, Y)$



Estimating MMD

$$\text{MMD}_k^2(\mathbb{P}, \mathbb{Q}) = \mathbb{E}_{X, X' \sim \mathbb{P}} [k(X, X')] + \mathbb{E}_{Y, Y' \sim \mathbb{Q}} [k(Y, Y')] - 2 \mathbb{E}_{\substack{X \sim \mathbb{P} \\ Y \sim \mathbb{Q}}} [k(X, Y)]$$

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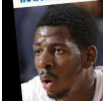




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K_{XX}


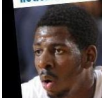










			
	1.0	0.2	0.6
	0.2	1.0	0.5
	0.6	0.5	1.0

Estimating MMD








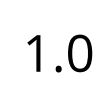




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K_{YY}







		
 1.0	 0.8	 0.7
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K_{XY}

			
	0.3	0.1	0.2
	0.2	0.3	0.3
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MMD models [Li+ ICML-15, Dziugaite+ UAI-15]

- No need for a discriminator – just minimize $\widehat{\text{MMD}}_k$!
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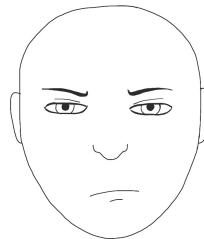
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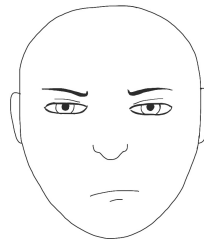
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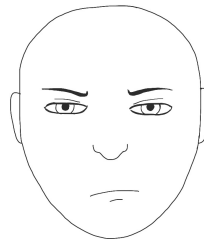


Target (\mathbb{P})



Not great! $\widehat{\text{MMD}}(Q_\theta, \mathbb{P}) = 0.75$

How are these?



MMD models [Li+ ICML-15, Dziugaite+ UAI-15]

- No need for a discriminator – just minimize $\widehat{\text{MMD}}_k$!
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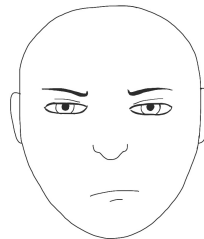


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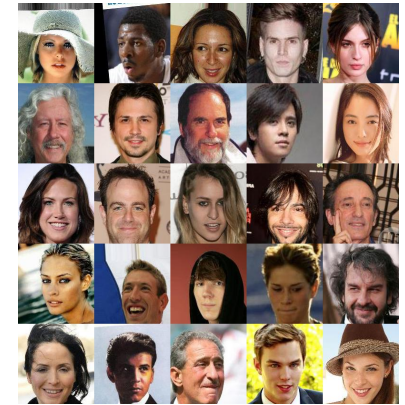
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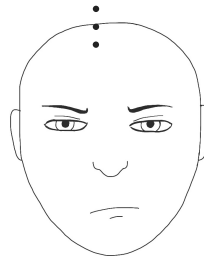


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\mathcal{P}

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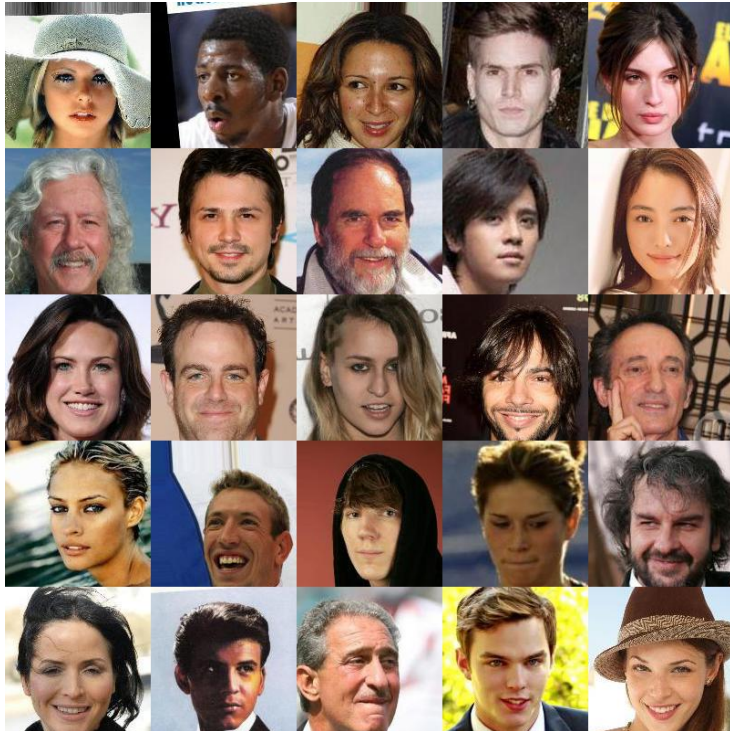


\mathbb{P}



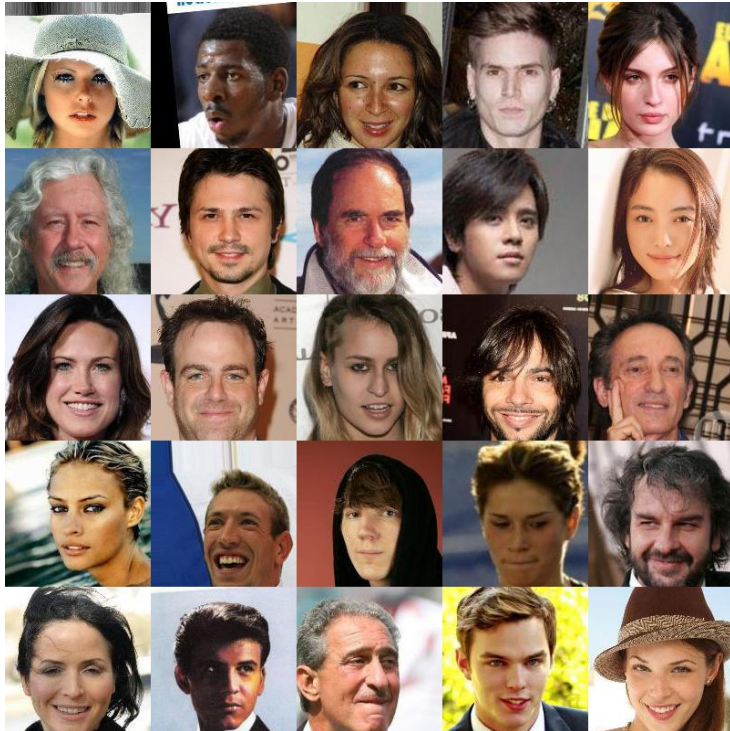
\mathbb{Q}_θ

Celeb-A, mix of rational quadratic + linear kernels

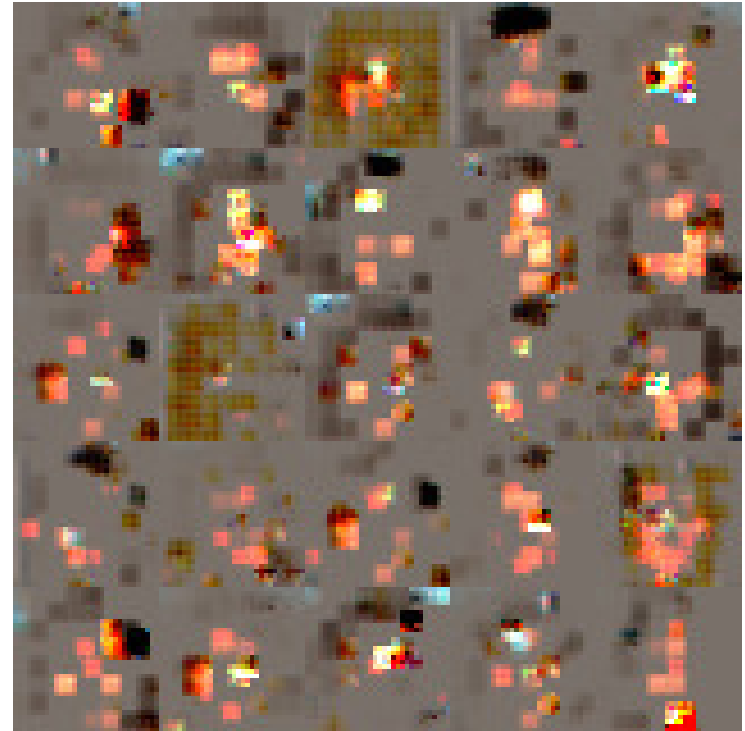


P

Celeb-A, mix of rational quadratic + linear kernels



\mathcal{P}



\mathcal{Q}_θ

MMD loss with a smarter kernel

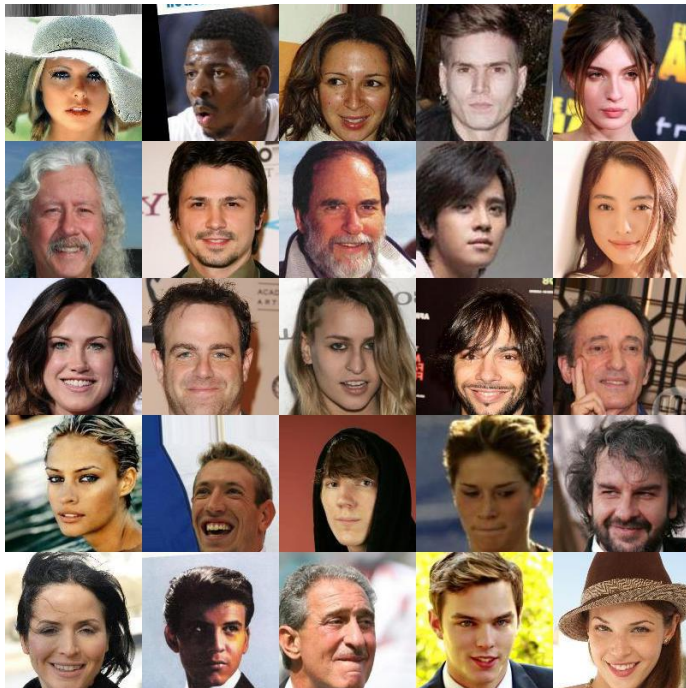
$$k(x, y) = k_{\text{top}}(\phi(x), \phi(y))$$

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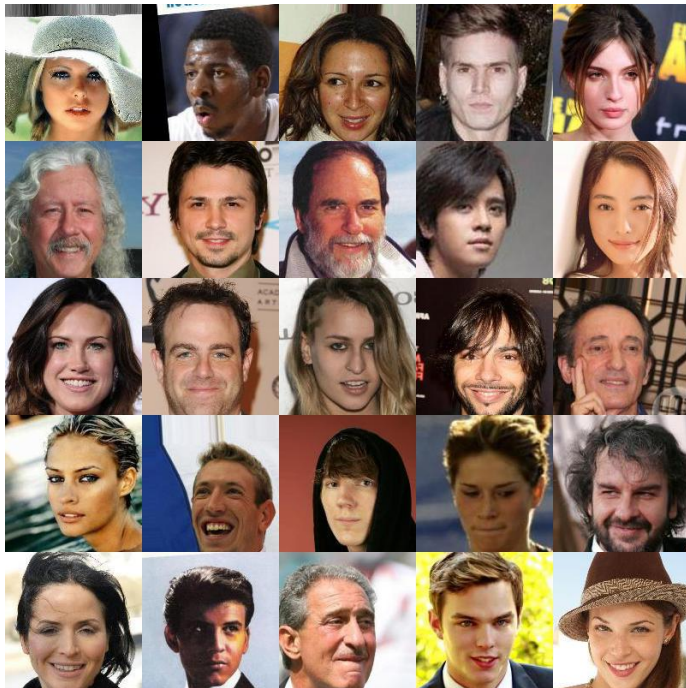
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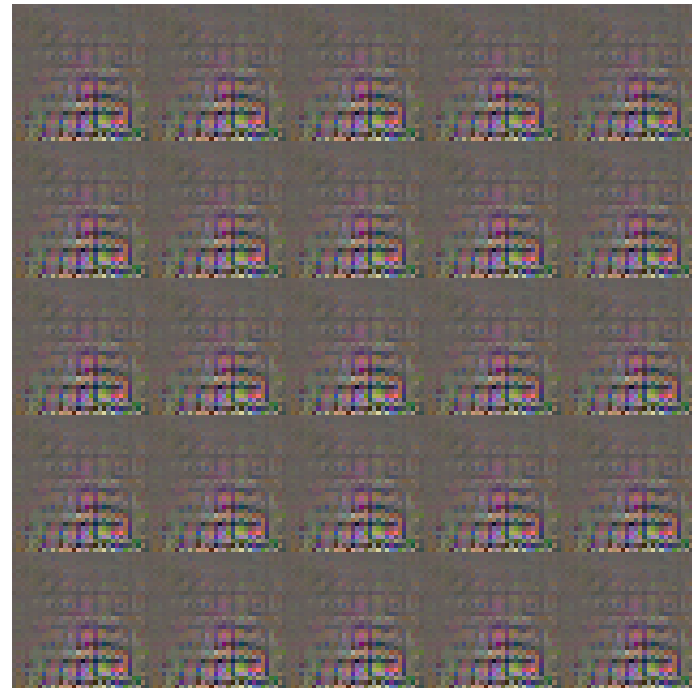
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We just got adversarial examples!



88% **tabby cat**

adversarial
perturbation

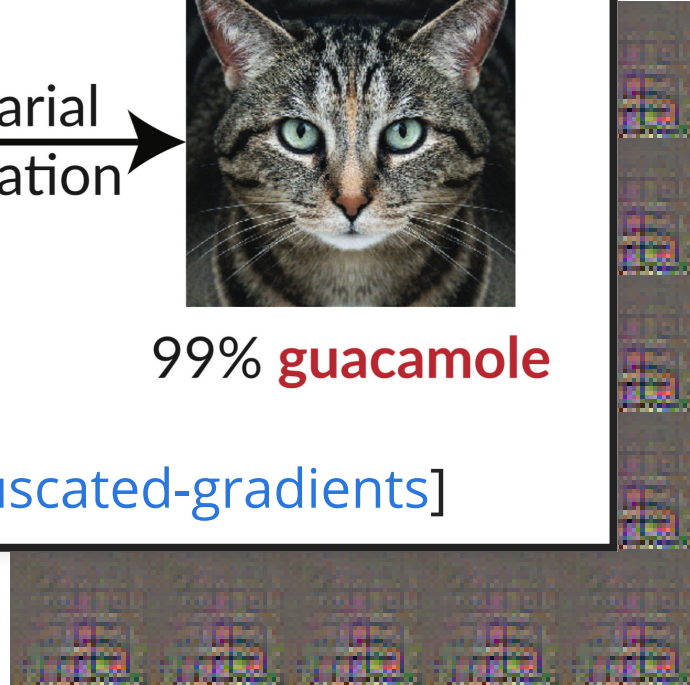


99% **guacamole**

[[anishathalye/obfuscated-gradients](#)]



ominal



\mathbb{P}

Q_{θ}

Optimized MMD: MMD GANs [Li+ NeurIPS-17]

- Don't just use one kernel, use a *class* parameterized by ψ :

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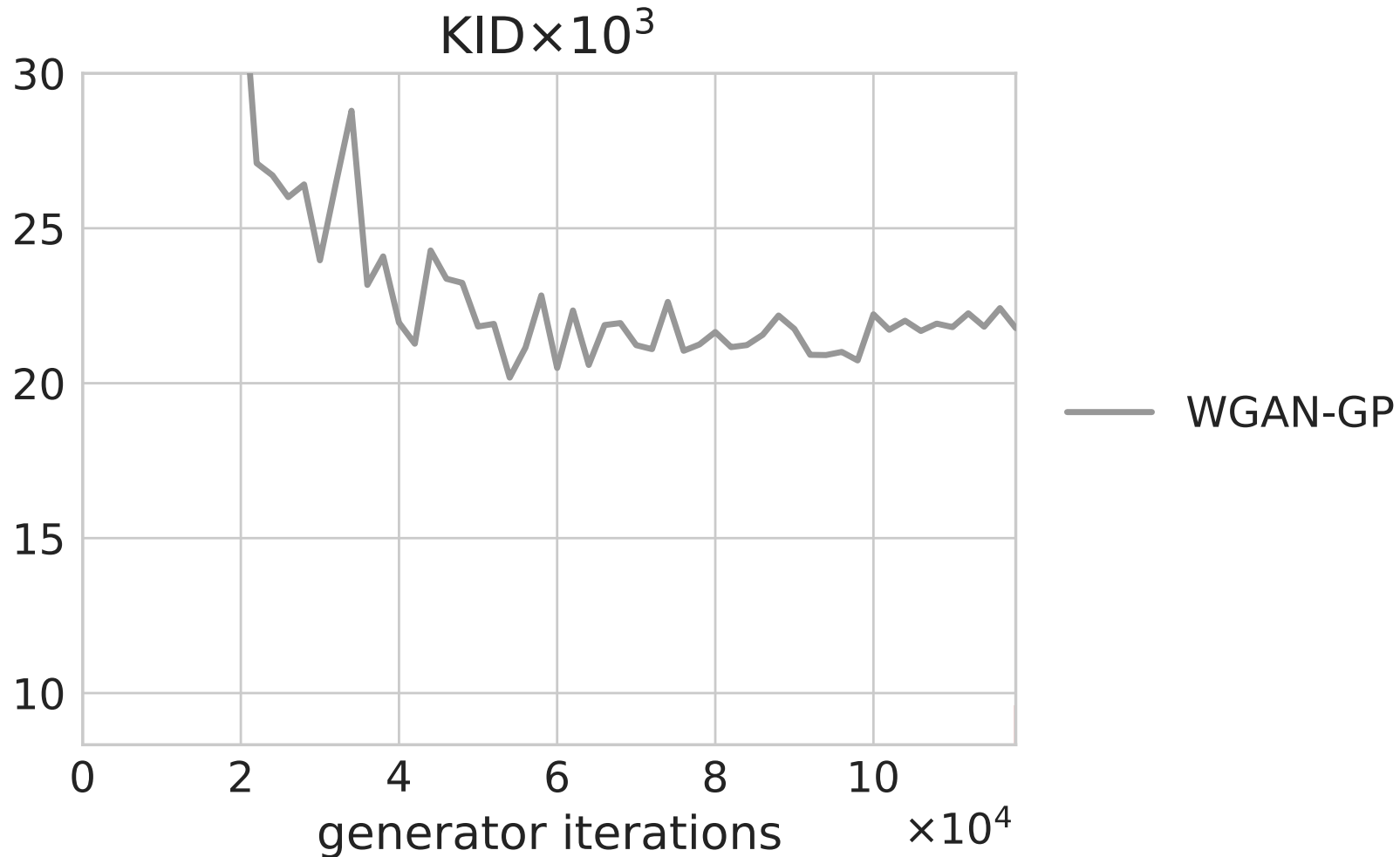
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- Scaled MMD GANs [Arbel+ NeurIPS-18]
correct \mathcal{D}_{MMD} with a gradient penalty to make it continuous

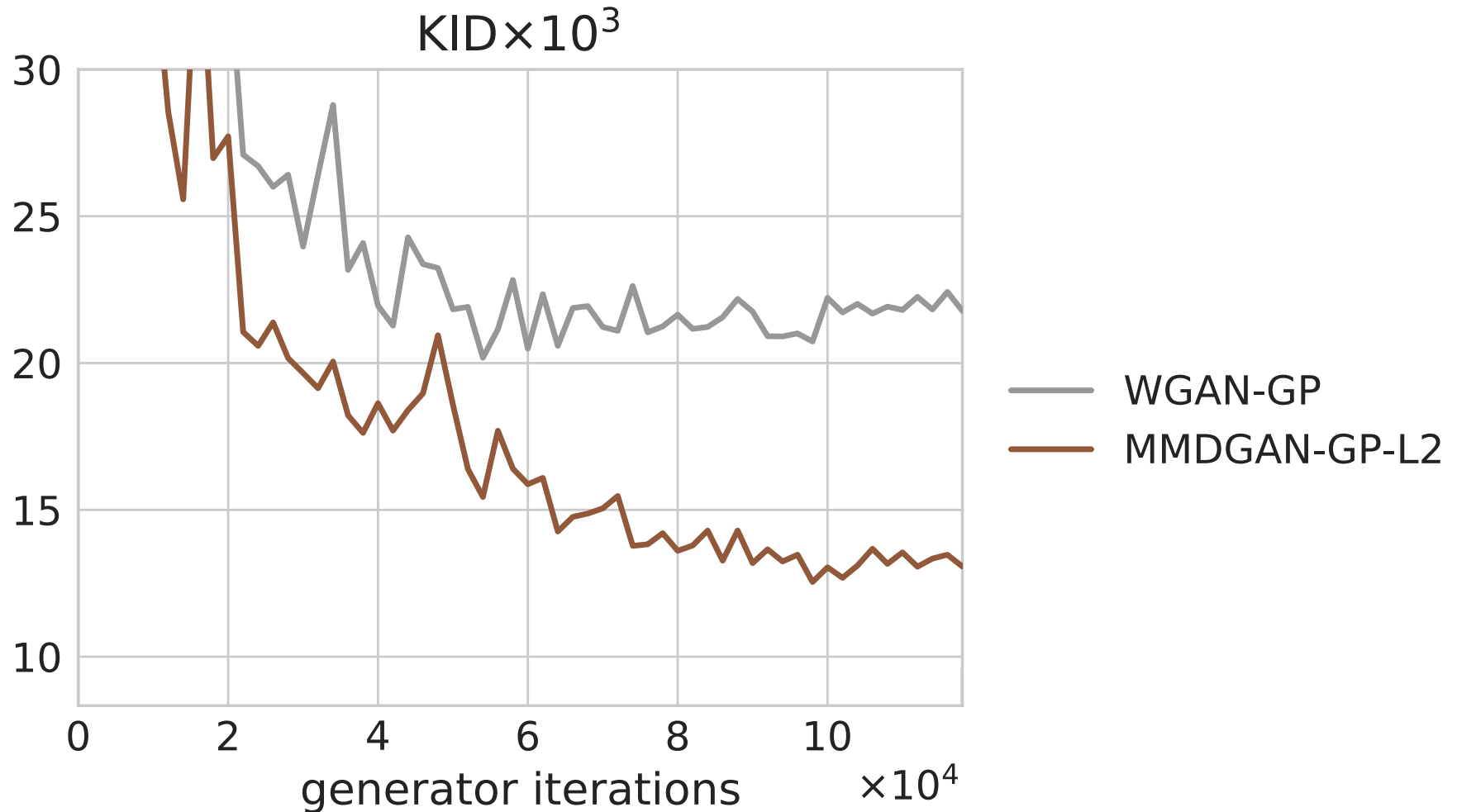
Why MMD GANs?

- “Easy parts” of the optimization done *in closed form*



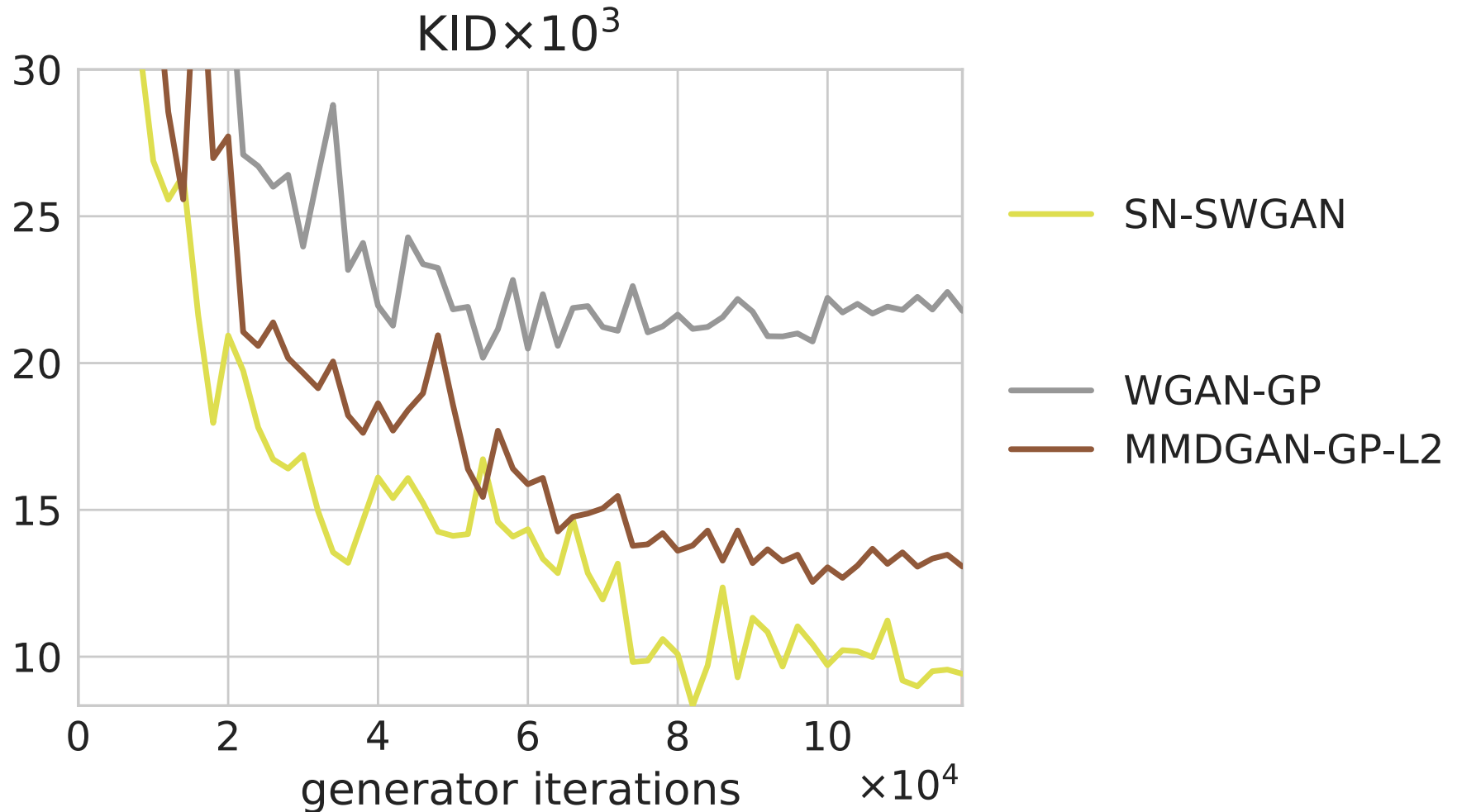
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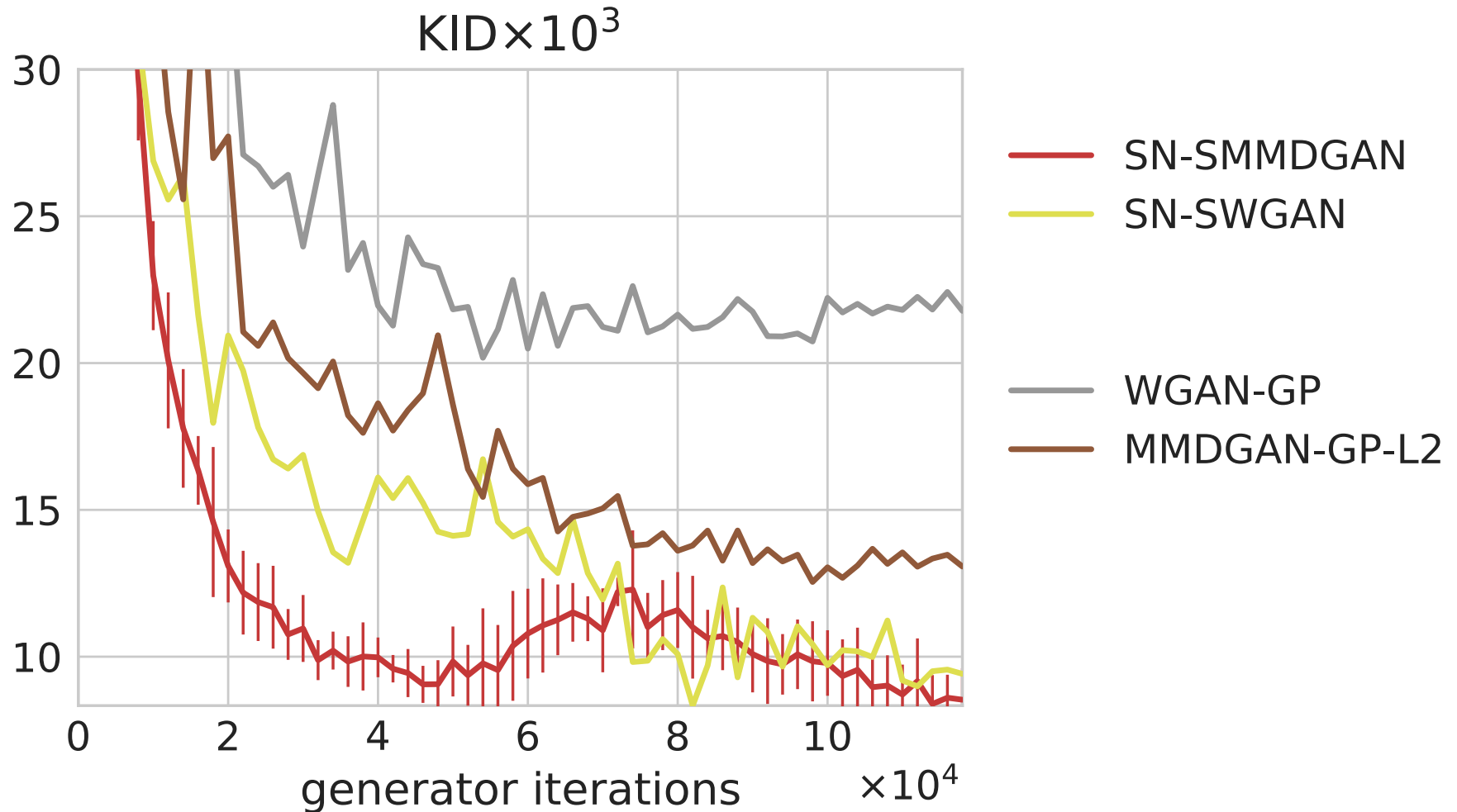
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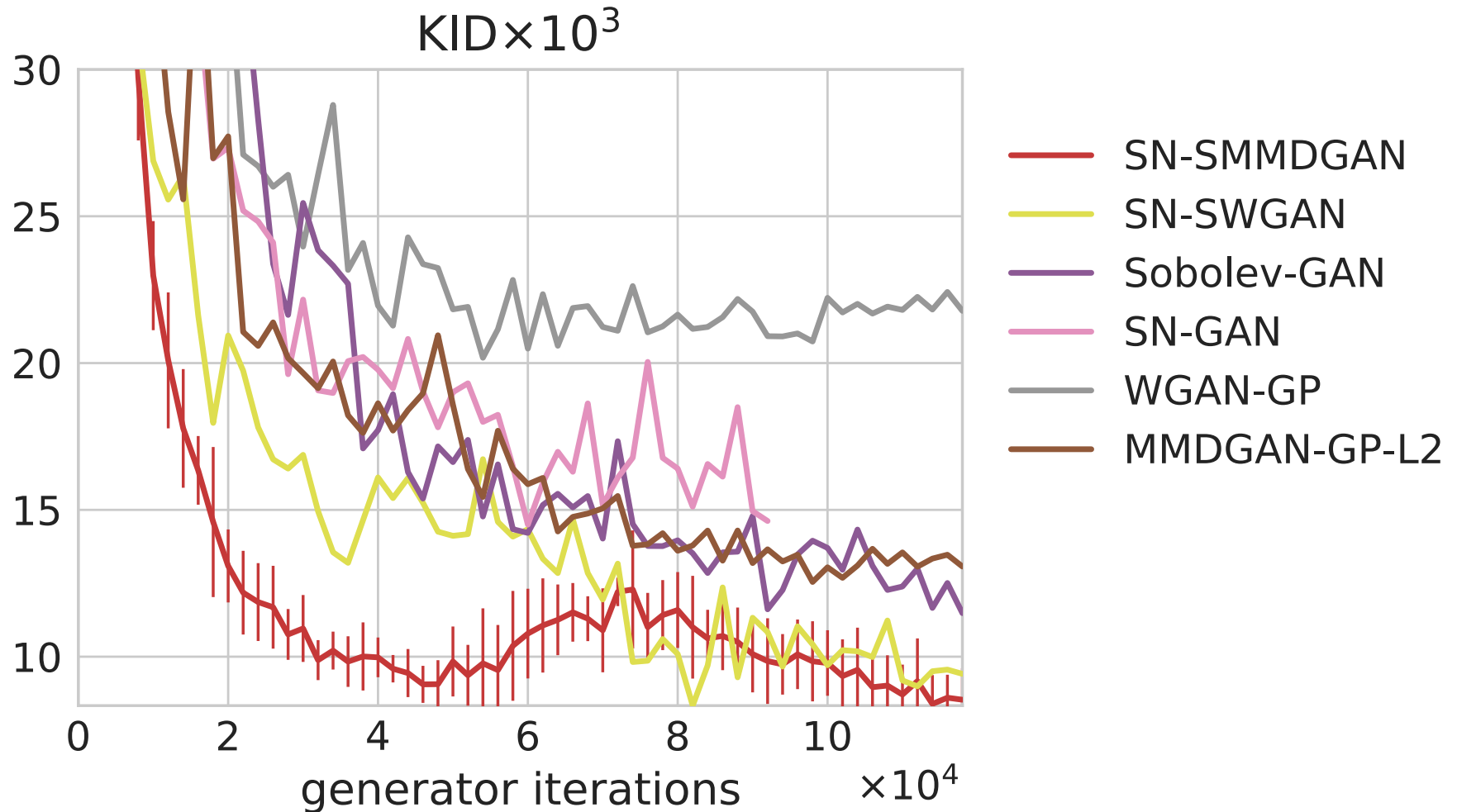
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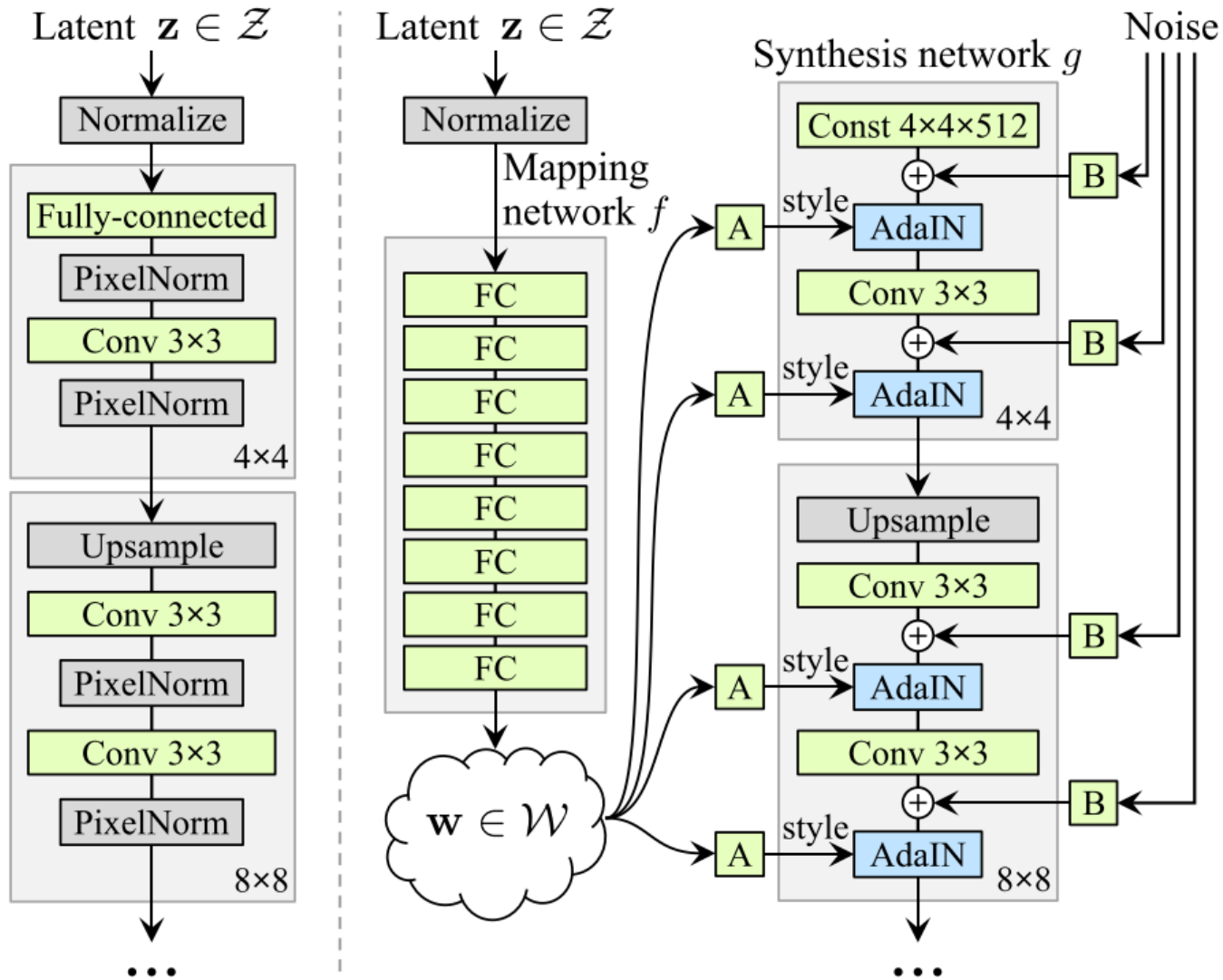


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StyleGANs [Karras+ 2018]



(a) Traditional

(b) Style-based generator

StyleGAN: latent structure

StyleGAN: local noise



(a) Generated image

(b) Stochastic variation

(c) Standard deviation

StyleGANs on a different domain [[@roadrunning01](#)]

▪

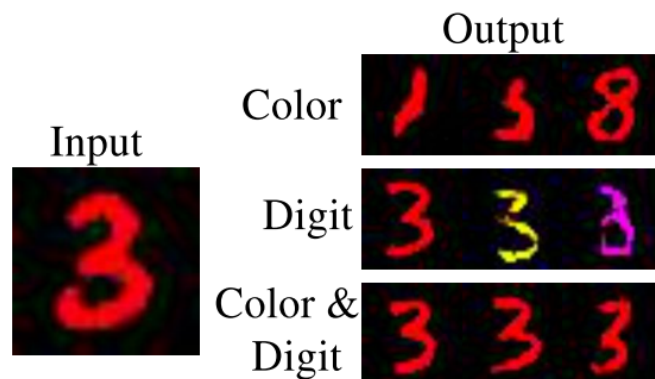
Finding samples you want [Jitkrittum+ ICML-19]

If we want to find “more samples like $\{X\}$ ”:

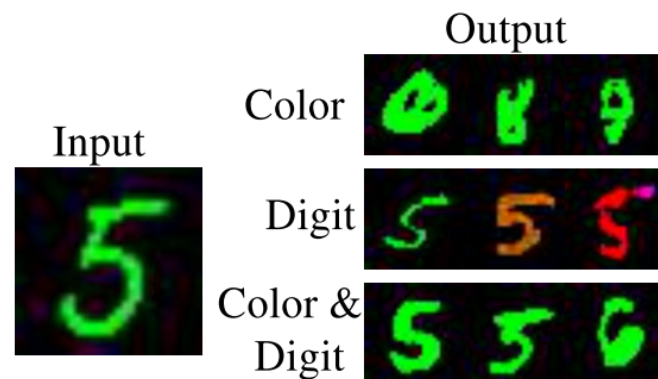
$$\min_{\{Z_1, \dots, Z_n\}} \widehat{\text{MMD}}_k^2 \left(\{X_i\}_{i=1}^m, \{G_\theta(Z_i)\}_{i=1}^n \right)$$



(a) Samples from DCGAN

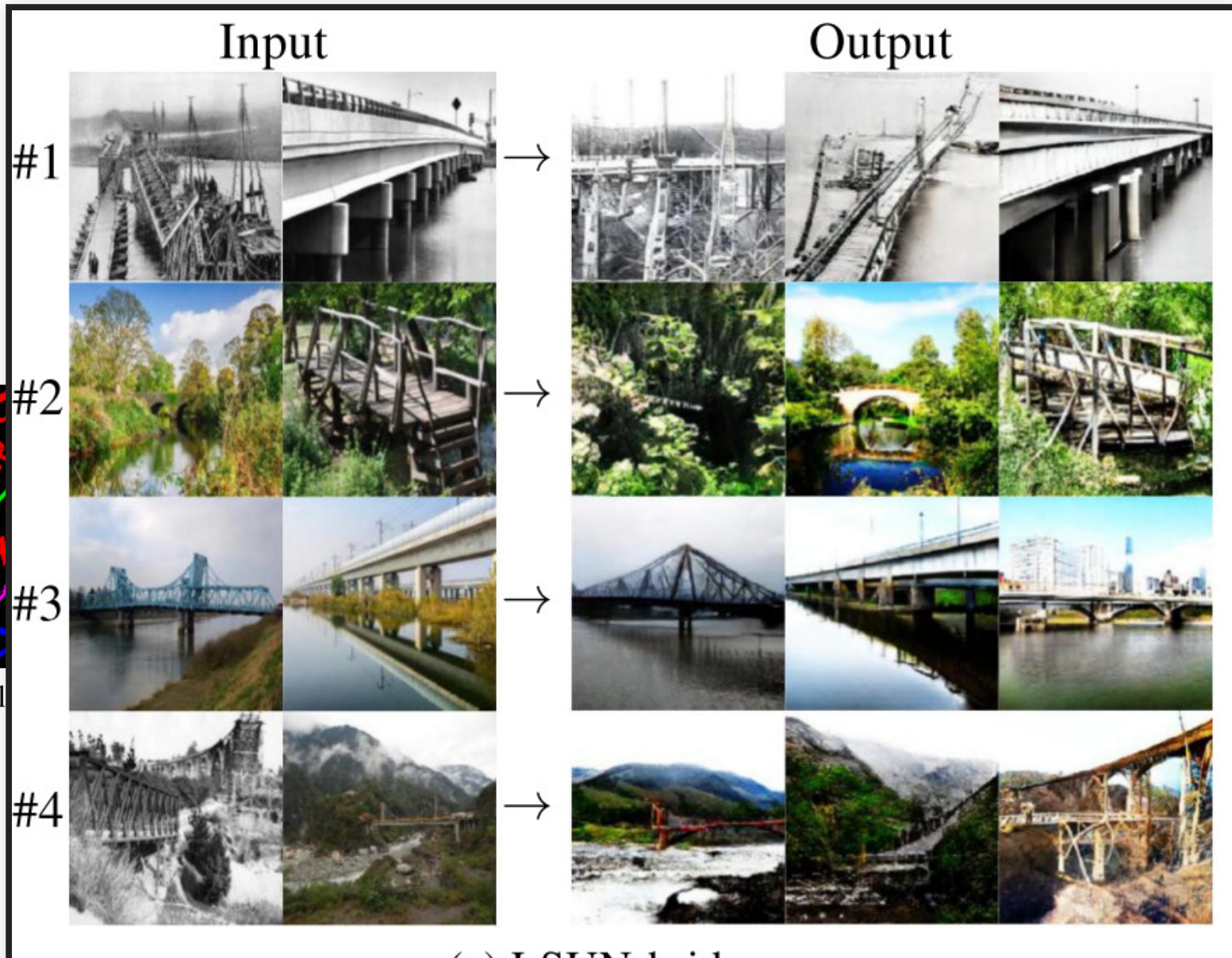


(b) Input: digit 3 in red



(c) Input: digit 5 in green

Finding samples you want [Jitkrittum+ ICML-19]



(a) Sample

Input



Open

Conditional GANs and BigGAN

- Conditional GANs: [Mirza+ 2014]
 - Just add a class label as input to G_{θ} and D_{ψ}
- BigGAN [Brock+ ICLR-19]: a bunch of tricks to make it huge



Image-to-image translation [Isola+ CVPR-17]

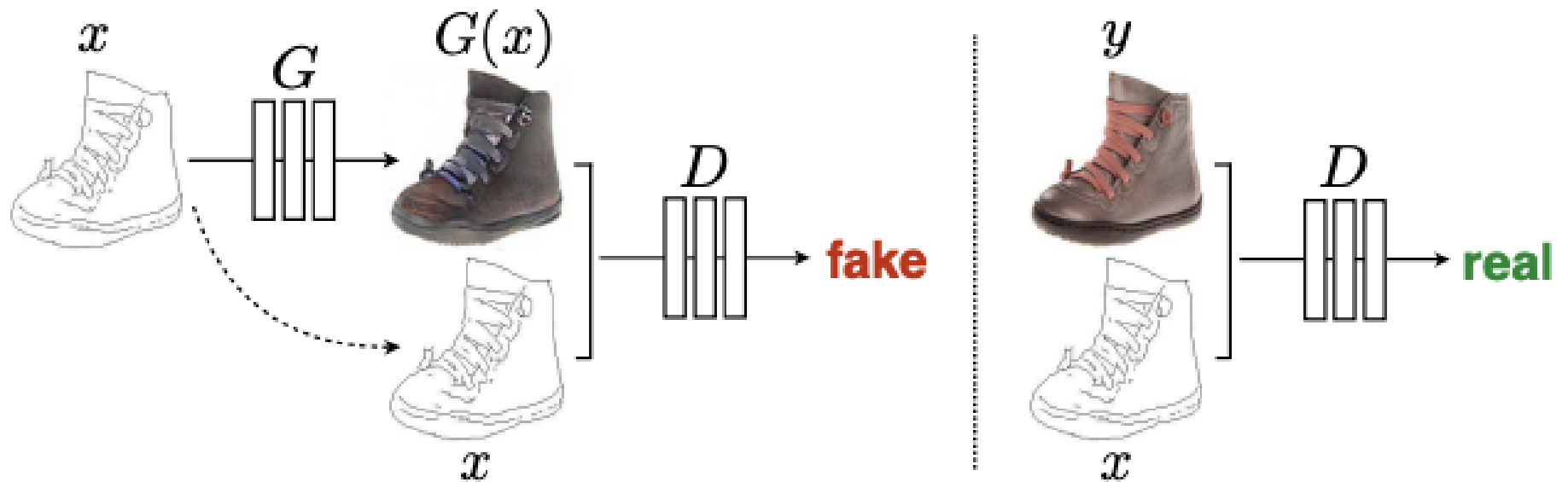


Figure 2: Training a conditional GAN to map edges \rightarrow photo. The discriminator, D , learns to classify between fake (synthesized by the generator) and real {edge, photo} tuples. The generator, G , learns to fool the discriminator. Unlike an unconditional GAN, both the generator and discriminator observe the input edge map.

Image-to-image translation [Isola+ CVPR-17]

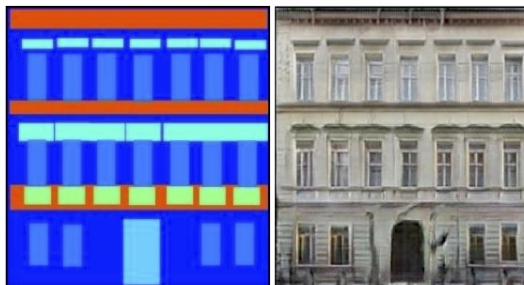
Labels to Street Scene



input

output

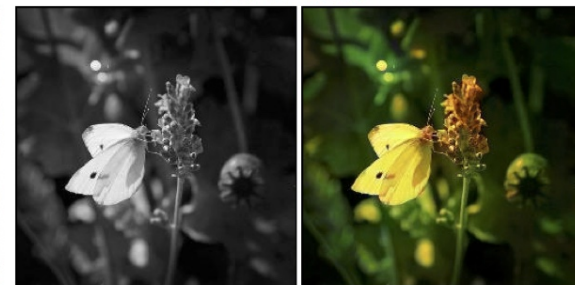
Labels to Facade



input

output

BW to Color



input

output

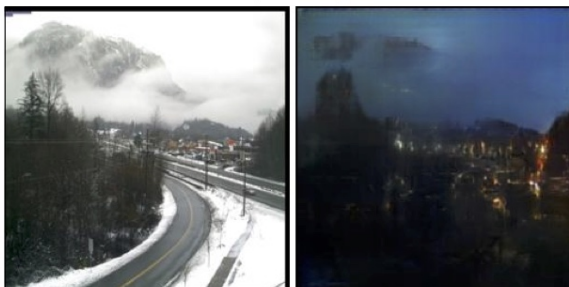
Aerial to Map



input

output

Day to Night



input

output

Edges to Photo



input

output

CycleGAN [Zhu+ ICCV-17]

Monet ↔ Photos



Monet → photo

Zebras ↔ Horses



zebra → horse

Summer ↔ Winter



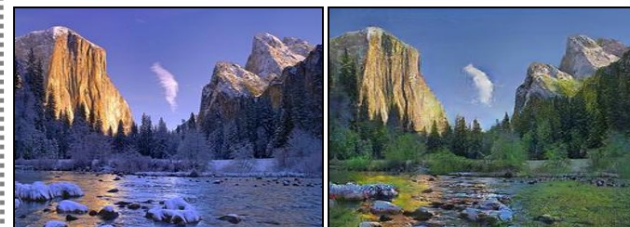
summer → winter



photo → Monet



horse → zebra



winter → summer



Photograph



Monet



Van Gogh



Cezanne



Ukiyo-e

Pose-to-image translation [Chan+ 2018]

-

DeepFakes

Use your new knowledge for good!

Slides (including links to papers) are online:

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