

Assignment 1

Theoretical Neuroscience

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1. The Hodgkin-Huxley neuron

Numerically integrate the Hodgkin-Huxley equations with matlab. Best idea is to use the Matlab ode45 function. The equations are:

$$C \frac{dV}{dt} = -\bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_K n^4 (V - E_K) - \bar{g}_L (V - E_L) + I_{stim} \quad (1)$$

$$\frac{dx}{dt} = \alpha_x (1 - x) - \beta_x x \quad \text{where } x \text{ is } m, n \text{ or } h \quad (2)$$

$$\alpha_n(V) = 0.01(V + 55) / [1 - \exp(-(V + 55)/10)] \quad (3)$$

$$\beta_n(V) = 0.125 \exp(-(V + 65)/80) \quad (4)$$

$$\alpha_m(V) = 0.1(V + 40) / [1 - \exp(-(V + 40)/10)] \quad (5)$$

$$\beta_m(V) = 4 \exp(-(V + 65)/18) \quad (6)$$

$$\alpha_h(V) = 0.07 \exp(-(V + 65)/20) \quad (7)$$

$$\beta_h(V) = 1 / [\exp(-(V + 35)/10) + 1] \quad (8)$$

Let $C = 10 \text{ nF/mm}^2$, $\bar{g}_L = .003 \text{ mS/mm}^2$, $\bar{g}_K = 0.36 \text{ mS/mm}^2$, $\bar{g}_{Na} = 1.2 \text{ mS/mm}^2$, $E_K = -77 \text{ mV}$, $E_L = -54.387 \text{ mV}$, and $E_{Na} = 50 \text{ mV}$. Use an integration time step of 0.1 ms.

Remember, $F/S = \text{Farad/Siemens} = 1 \text{ second}$.

- (a) Run the simulations with $I_{stim} = 200 \text{ nA/mm}$. Plot the membrane potential (V) and gating variables (m , h , and n) versus time.
- (b) Write down expressions for the equilibrium values of the gating variables (m_∞ , h_∞ , and n_∞), and plot them versus voltage.
- (c) Plot the firing rate versus I_{stim} , up to a firing rate of 50 Hz. The firing rate should jump suddenly from zero to a non-zero value. This is called a type II behaviour. Type I behaviour is when the firing rate begins at zero and increases continuously without any jumps.
- (d) What happens to the plot of firing rate versus I_{stim} as you decrease \bar{g}_K ?
- (e) Spikes are initiated at the axon hillock, where the axon meets the soma. This is because \bar{g}_{Na} is very high there. What happens to the plot of firing rate versus I_{stim} as you increase \bar{g}_{Na} ?

2. The linear integrate and fire neuron

An approximate treatment of spiking neurons is to think of them as passively integrating input and, when the voltage crosses threshold, emitting a spike. This leads to the linear integrate and fire neuron (sometimes called the leaky integrate and fire neuron, and often abbreviated LIF), which obeys the equation

$$C \frac{dV}{dt} = -g_L(V - \mathcal{E}_L) + I_0.$$

This is just the “linear integrate” part. To incorporate spikes, when the voltage gets to threshold (V_t), the neuron emits a spike and the voltage is reset to rest (V_r).

- (a) Compute the firing rate of the neuron as a function of I_0 . This firing rate will be parameterized by three numbers: \mathcal{E}_L , V_t , and V_r .

Hint #1: The firing rate is the inverse of the time it takes to go from V_r to V_t .

Hint #2: Changing variables, and defining new quantities, almost always makes life easier. For example, you might let $v = V - \mathcal{E}_L$ and define $V_0 \equiv I_0/g_L$ and $\tau \equiv C/g_L$.

- (b) Let $I(t) = g_L V_0 \sin(\omega t)$, $V_r = \mathcal{E}_L$, $V_t = \mathcal{E}_L + \Delta V$, and define $C/g_L \equiv \tau$. Start with $V_0 = 0$ and integrate for a long enough time that the neuron equilibrates. Then increase V_0 very slowly compared to the time constant, τ . Show that the neuron will start spiking repetitively when $V_0 > (1 + \tau^2 \omega^2)^{1/2} \Delta V$.

3. Nullclines. Consider a simplified Hodgkin-Huxley type model,

$$\begin{aligned} \tau \frac{dV}{dt} &= -(V - \mathcal{E}_L) - hm(V)V \\ \tau_h \frac{dh}{dt} &= h_\infty(V) - h \\ m(V) &= \frac{1}{1 + \exp(-(V - V_t)/\epsilon_m)} \\ h_\infty(V) &= \frac{1}{1 + \exp(+(V - V_h)/\epsilon_h)} \end{aligned}$$

with parameters

$$\begin{aligned} \mathcal{E}_L &= -65 \text{ mV} \\ V_t &= -50 \text{ mV} \\ \epsilon_h &= 10 \text{ mV} \\ \epsilon_m &\ll 1 \text{ mV}. \end{aligned}$$

The remaining parameter, V_h , will be specified as needed (it will take on a range of values).

- (a) Sketch the nullclines in V - h space for $V_h = -60, -50$ and -40 mV. Put voltage on the x -axis and h on the y -axis. For each equilibrium, tell us whether it is stable or unstable, or hard to tell without a detailed stability analysis.

- (b) Find the condition on V_h that guarantees more than one equilibrium.
- (c) For a value of V_h such that there is more than one equilibrium, sketch the trajectories starting at V slightly greater than V_t and $h = 1$.
- (d) Show (graphically) that the amplitude of the spike is an increasing function of τ_h .