Linear LIF model for gamma distributions of ISIs

Marius Pachitariu[1,2], Joseph K Jun[1,3], Jeffrey C Erlich[1,3], Bingni W Brunton[1,3], Carlos D Brody[1,3]

[1] Princeton University [2] Gatsby Unit, UCL [3] HHMI

INTRODUCTION

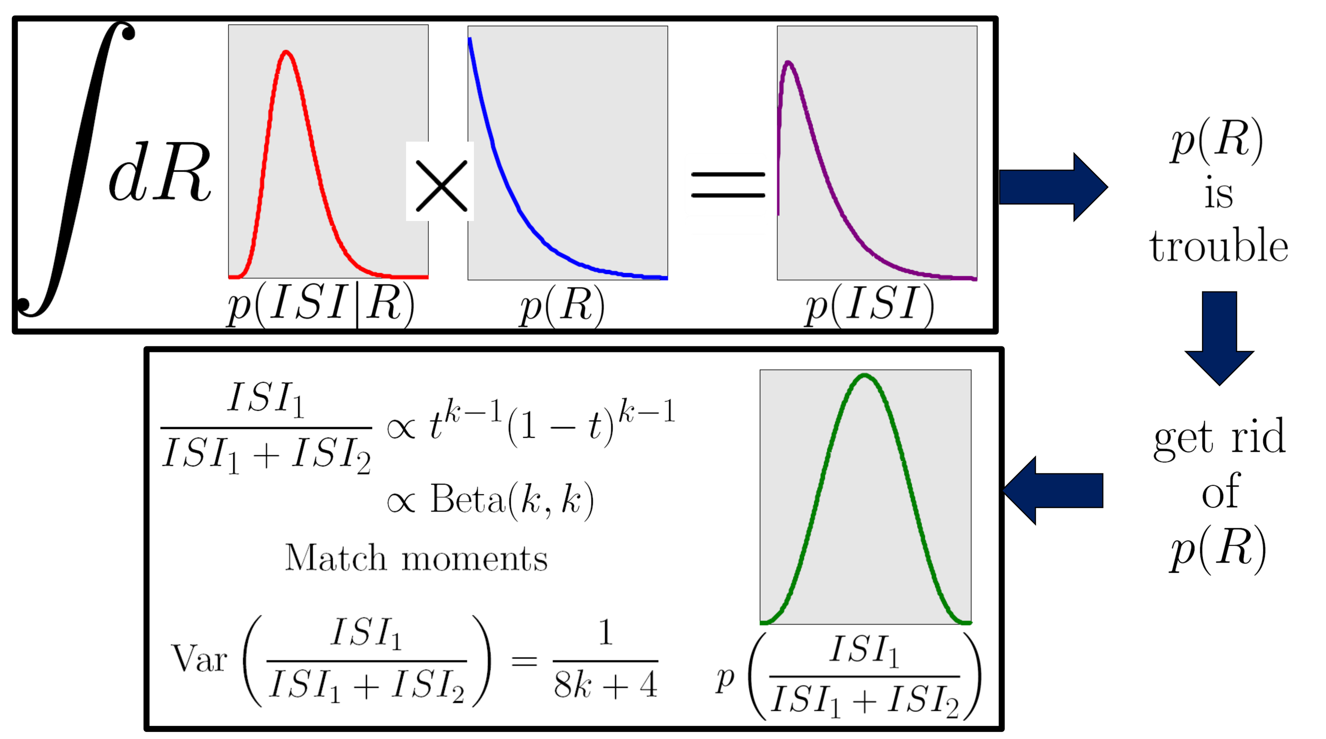
- Evidence has been accumulating for significant departures from the Poisson model in a variety of brain regions, primarily motor-related like M1 and SMA [1] and high-visual, like Area 5 [2]. Neurons in these areas display smaller coefficients of variation (CV) in their interspike interval (ISI) distributions.
- Slice recordings with constant current injection typically show regular spike trains, but a degree of regularity is maintained even under in-vivo like stimulation with noisy inputs[3].
- The Poisson point process might be an appropriate/useful simplification for a variety of brain areas but not in the rat Frontal Orienting Fields (FOF) where our lab has been conducting electrophysiological recordings. The FOF is a premotor area, but neurons also show persistent mnemonic activity.
- Thev can be modeled with renewal point processes with $\mathrm{Gamma}(k,\theta)$ distributions of ISIs.

$$\mathrm{Gamma}(x;k,\theta) = x^{k-1}\frac{\exp(-x/\theta)}{\Gamma(k)\theta^k}$$

$k = \text{Shape}$
$\theta = \text{Scale}$
$\langle x\rangle|_{\mathrm{Gamma}(k,\theta)} = k\theta$

METHODS

- Estimate Regularity (Gamma Shape)



$$\frac{ISI_1}{ISI_1+ISI_2} \propto t^{k-1}(1-t)^{k-1} \propto \mathrm{Beta}(k,k)$$

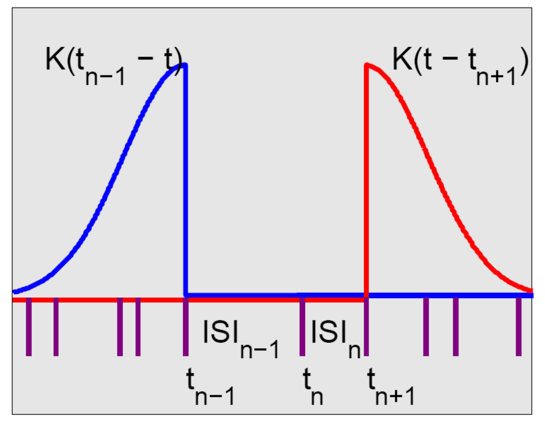
Match moments

$$\mathrm{Var}\left(\frac{ISI_1}{ISI_1+ISI_2}\right) = \frac{1}{8k+4}$$

$p\left(\frac{ISI_1}{ISI_1+ISI_2}\right)$

- Estimate Firing Rate

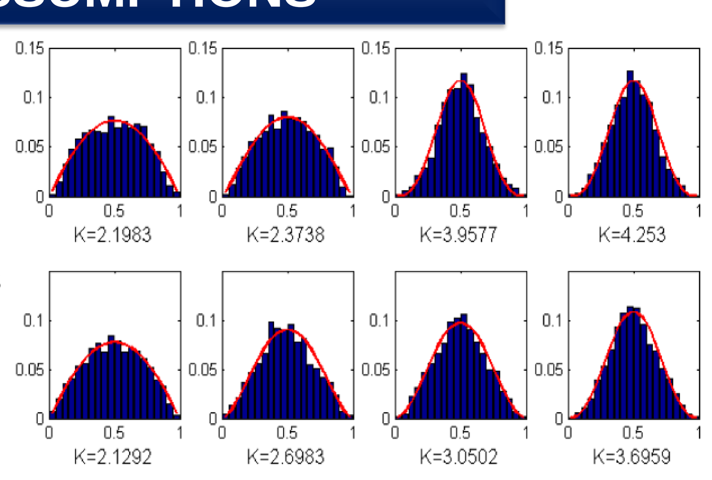
$$R_n = \int dt\ K(t_{n-1}-t)\sum_{k=1}^{n-1}\delta(t_k) + \int dt\ K(t-t_{n+1})\sum_{k=n+1}^{N}\delta(t_k)$$



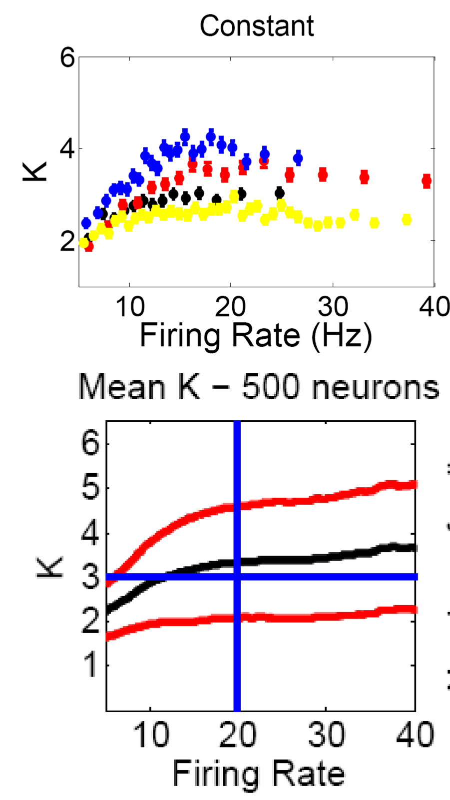
VERIFY ASSUMPTIONS

1) $\frac{ISI_n}{ISI_n+ISI_{n+1}}$ is Beta distributed

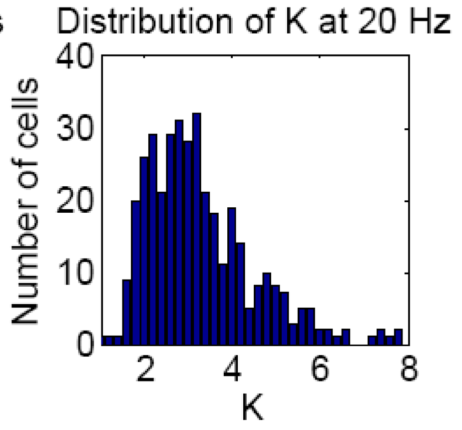
2) Rate changes slowly…
…Autocorrelation of spike trains is high (0.6-0.8 at time lag of 400ms, in 200 ms bins).

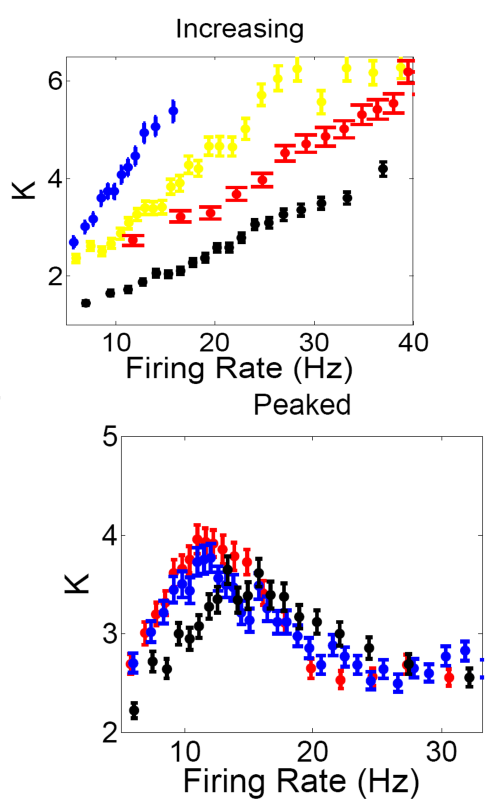


DATA RESULTS



High gamma shapes
Different profiles:
a) Constant (~60%)
b) Increasing (~30%)
c) Peaked (~10%)

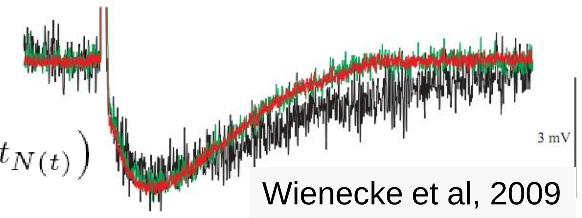


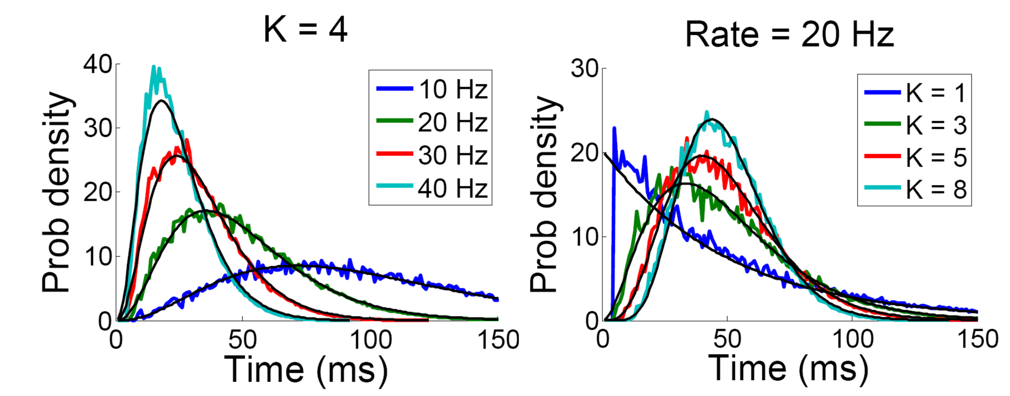


MODEL RESULTS

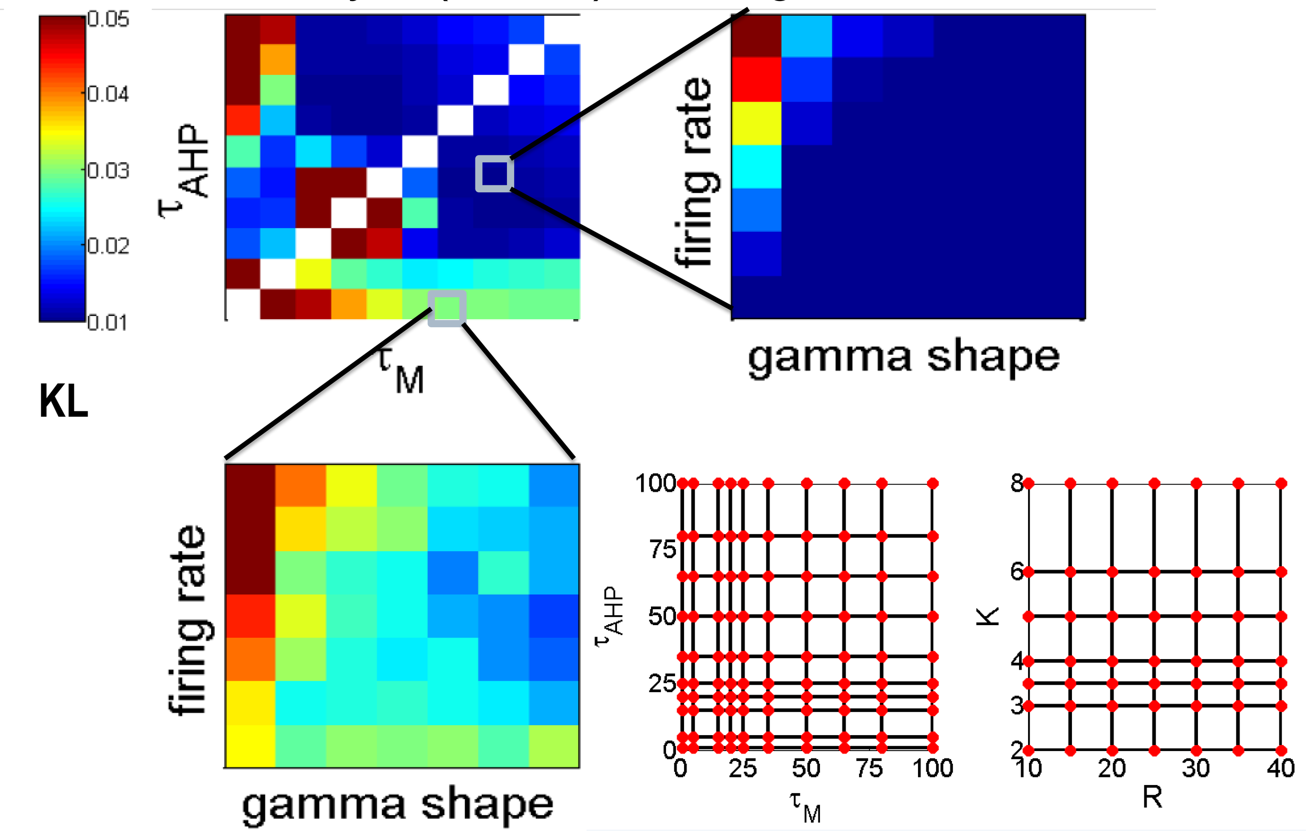
- Probabilistic models of spiking are epiphenomenal.
- What is the minimal structure we must add to an integrate and fire model in order to get Gamma-like distributions of ISIs, of gamma shapes above 3?
  - Answer: AHP-like currents.

$$\tau_m\frac{dV}{dt} = -V + dW + I + Fe^{-(t-t_{N(t)})}$$



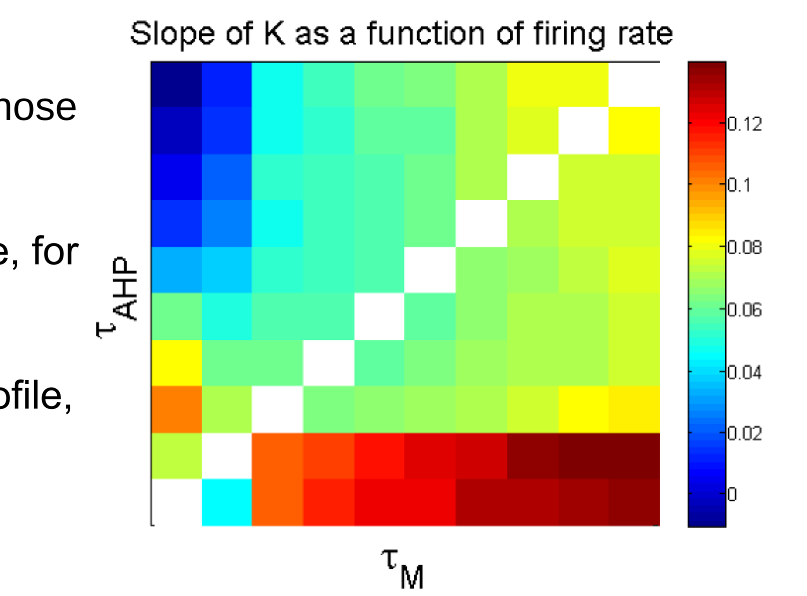


KL divergence between fitted LLIF and gamma models shows necessity of (at least) one long time constant.



Fitted AHP-LLIF models whose input is varied:

1) have a 'constant' profile, for long AHP,
2) have an 'increasing' profile, for short AHP.



LINEAR LIF MODEL

$$\tau_m\frac{dV}{dt} = -V + dW + I + f(t-t_{N(t)})$$

Integrate out after a spike

dW Gaussian noise (synaptic)
I mean input (synaptic)
$f(t-t_{N(t)})$ to be determined
$-D<0$ reset potential
$0 =$ leak potential
$T>0$ threshold for spiking

$$V(t) = -De^{-\frac{t}{\tau_m}} + \frac{1}{\tau_m}\int_0^t dW e^{(t'-t)/\tau_m}\,dt' + \frac{1}{\tau_m}\int_0^t (I+f(t))\,e^{(t'-t)/\tau_m}\,dt'$$

- We need the distribution of crossing times but O-U model with absorbing boundary is impossible to integrate in general.
- Collect the deterministic part in a time-dependent threshold

$$Th(t) = T + De^{-t/\tau_m} - \frac{1}{\tau_m}\int_0^t (I+f(t))\,e^{(t'-t)/\tau_m}\,dt'$$

- A spike is produced when

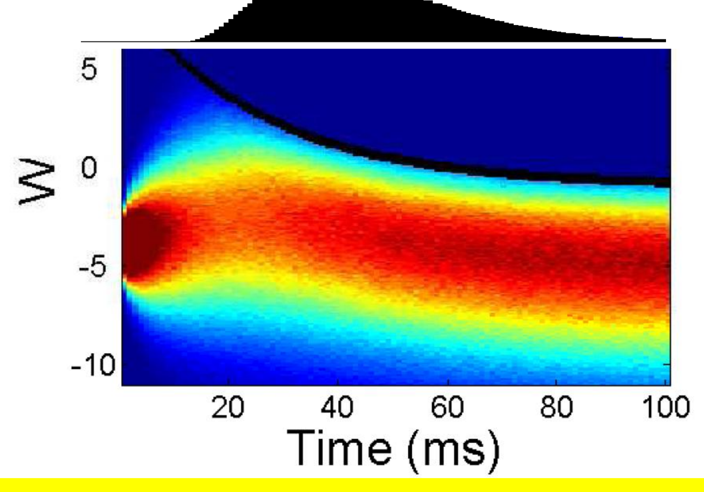
$$Th(t) = \frac{1}{\tau_m}\int_0^t dW e^{(t'-t)/\tau_m}\,dt' \overset{\text{not}}{=} W(t)$$

THE MOVING THRESHOLD FITTING PROCEDURE

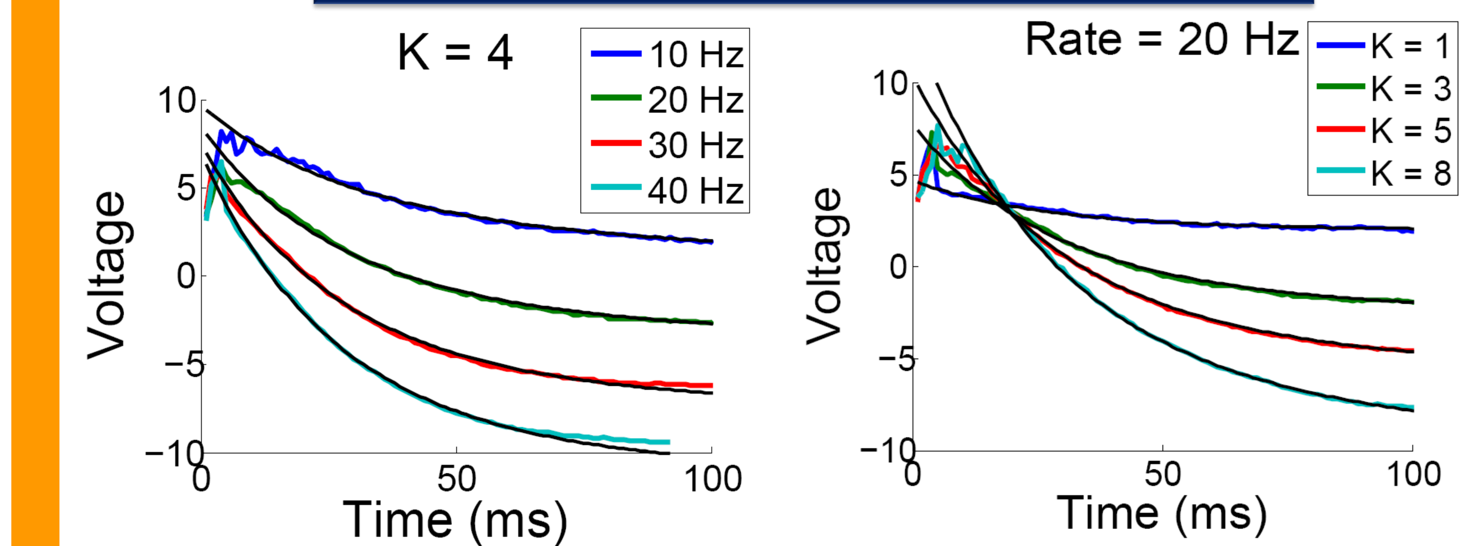
- We ask, for a given distribution p(ISI), what Th(t) would generate it?
- Use sequential Monte Carlo methods in discretized time (dt = 1ms).
- Repeat the following procedure:
  - Approximate $P(W(t)\,|\,W(t') < Th(t'),\ t' < t)$ with a set of (at least) 10000 particles: $\{W_n^t\}|_{1\le n\le 10000}$
  - Choose Th(t) to match empirical and theoretical hazard rates

$$\lambda(t)dt = \frac{p(\mathrm{ISI}=t)dt}{1-\int_0^t p(t')dt'} \approx \frac{|\{W_n^t > Th(t)\}|}{|\{W_n^t\}|}$$

- Eliminate absorbed particles, and choose randomly with replacement 10000 new particles from the ones left.
- Propagate particles to the next time step.



MOVING THRESHOLD RESULTS



Fitted thresholds corresponding to the distributions shown in the central box. Colored lines are AHP model fits. Black lines are AHP-LLIF model fits. Here we used a time constant of 65 ms for AHP and 25 ms for the membrane.

Observation: our moving threshold method can be used for fitting parameters of (generalized) O-U processes in general, with one or two absorbing boundaries.

DISCUSSION

- Neurons in FOF have spike trains three times more regular than Poisson on average (as measured by Fisher information).
- AHP-like adaptation currents predict these regular point processes under a very simple integrate and fire model. Depending on the timescale of AHP we put in our model, we obtain different profiles of regularity as a function of firing rate, which profiles we see in the FOF data.
- The 'constant' profile requires long AHP currents.
- The 'increasing' profile does not.
- The 'peaking' profile might be indicative of 15 Hz oscillations.
- mAHP currents are present in rat pyramidal cells, with time constants around 50ms [4].
- mAHP currents are present in spinal cord motor neurons [5]. These neurons can be classified as either fast or slow, depending on the timescale of their respective AHP current (smaller or larger than 25 ms). Spinal cord motor neurons have very regular spike trains.
- Tempting to hypothesize gradual projection of 'brain code' onto a rate code along the motor pathway, because regular neurons favor a rate code[6].
- Further experiments are necessary to establish the existence of the hypothesized AHP current.

References

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