Question 1

You are sitting in a room waiting for an auditory signal. The probability per unit time for the signal to appear is γ, and once it appears it stays there forever.

1. The signal has not appeared at time 0. What is the probability that it will appear at time t? What is the probability it will not appear in the interval $[0, T]$?(2 marks)

2. A neuron with firing rate $\nu(t)$ and refractory period Δt emits spikes as follows: the probability that the neuron fires in a small interval dt around t is $\nu(t)dt$ if the neuron fired no previous spikes in the interval $[t - \Delta t, t]$ and zero otherwise. If $\nu(t) = \nu =$constant, what is the mean firing rate of this neuron? (3 marks)

3. A neuron in auditory cortex fires at a background rate of ν_0 before the signal in part 1 appears and jumps to rate ν_1 (and stays there) after it appears. As in part 2, the neuron has an absolute refractory period of Δt. You observe spikes at times t_1, t_2, ..., t_n. Write down the probability distribution that the signal came on at time t_s given the spikes, denoted $p(t_s|\text{spikes})$. Include the probability that the signal never came on. The normalization that appears in this problem will involve an integral – don't do it! (5 marks)

4. You observe spikes in the interval $[0, T]$. Assume the signal came on at time t_0. Sketch $\langle \log p(t_s|\text{spikes}) \rangle$ versus t_s where the average is over ensembles of spike trains. (10 marks)

5. Based on this plot, give a back of the envelope estimate of the value of of t_0 at which you will say that no signal occurred; that is, estimate the value of t_0 such that $\int dt_s p(t_s|\text{spikes})$ is larger, on average, than $p(\text{no signal}|\text{spikes})$.

Question 2

Consider a network of N analog neurons that obey the time-evolution equations

$$\tau \dot{x}_i = \phi(J\overline{x} - \theta) - x_i \tag{1}$$

where "dot" denotes a time derivative, $\overline{x} \equiv N^{-1}\sum_i x_i$ is the mean firing rate, and $\phi(z)$ is the gain function. We will demand that the gain function be threshold-concave-nondecreasing, meaning

$$\begin{aligned} z < 0: &\quad \phi(z) = 0 \\ z \geq 0: &\quad \phi'(z) \geq 0 \text{ and } \phi''(z) \leq 0\,. \end{aligned}$$

1. Write down the time evolution equation for $\overline{x}$. (1 mark)
2. Find the equilibria graphically in a regime in which there are three. Which of the equilibria are stable and which are unstable? (2 marks)
3. There are two main models for network bursting in the literature: synaptic depression and spike-frequency adaptation.
   (a) What biophysical mechanism is thought to be responsible for synaptic depression? (2 marks)
   (b) What biophysical mechanism is thought to be responsible for spike-frequency adaptation? (2 marks)
4. To model these two mechanisms, one typically lets J or θ have slow, activity dependent dynamics. For synaptic depression,

$$\tau_0 \dot{J} = -(J - J_0(\overline{x}))\,, \tag{2}$$

   while for spike-frequency adaptation,

$$\tau_0 \dot{\theta} = -(\theta - \theta_0(\overline{x}))\,. \tag{3}$$

   In both cases these variables evolve slowly, meaning $\tau_0 \gg \tau$. If Eqs. (2) and (3) are to provide adaptation, how must J_0 and θ_0 depend on $\overline{x}$? (Essentially, what is the slope of $J_0(\overline{x})$ and $\theta(\overline{x})$?) Sketch the two functions. (4 marks)

5. Given our model, including the properties of ϕ, which of these two mechanisms, if either, can lead to network bursting, and why? Assume $\tau_0 \gg \tau$, so that the x_i relax essentially instantaneously to their equilibria. For the model(s) that show bursting, sketch the trajectories of $\overline{x}$ and either J or θ versus time. (8 marks)

6. Let the threshold, θ, be a random variable,

$$\tau \dot{x}_i = \phi(J\overline{x} - \theta_i) - x_i$$

where the θ_i are drawn i.i.d. from a Gaussian distribution with mean θ and variance σ^2.

   (a) Derive a time evolution equation for $\overline{x}$ in the large N limit. (4 marks)

   (b) Sketch the resulting effective gain function. (3 marks)

   (c) With this new model with variable thresholds, which of the mechanisms can produce bursting? (4 marks)