Short questions.

1. What happens to a cell's effective time constant near threshold in the following three models:
   (a) Hodgkin Huxley.
   (b) Linear integrate and fire.
   (c) Quadratic integrate and fire.
2. Why is the magnesium block of NMDA receptors important for learning?
3. Provide two ways to distinguish experimentally between type I and type II neurons (all-or-none versus graded action potentials, respectively).
4. A neuron is firing regularly at frequency f ("regularly" = all interspike intervals are 1/f). After each action potential the synaptic strength at a postsynaptic neuron decreases by a factor of γ ($\gamma < 1$). Between action potentials the strength recovers with time constant τ. If the synaptic strength in the absence of firing is 1, show that the steady state strength right before a spike is $(1 - e^{-1/f\tau})/(1 - \gamma e^{-1/f\tau})$.
5. In the classic model of high connectivity (and therefore high gain) randomly connected networks, operation is on the unstable branch of the excitatory nullcline. Explain why inhibitory-inhibitory feedback is necessary for a stable equilibrium on that branch.

Long question 1.

Consider a network of N neurons whose firing rates, r_i, evolve according to

$$\tau \frac{dr_i}{dt} = h\left(\sum_j W_{i-j} r_j\right) - r_i \tag{1}$$

where h is a nonlinear function and W is symmetric ($W_{i-j} = W_{j-i}$). Assume $r_i = f(\theta_i)$ is the globally stable equilibrium of Eq. (1), where $\theta_i = 2\pi i/N$, $i = 1, ..., N$ and f is a 2π periodic function, $f(\theta) = f(\theta + 2\pi)\, \forall\, \theta$.

1. Show that $r_i = f(\theta_k - \theta_i)$ is a solution to Eq. (1) for any integer k.

2. Consider the augmented equation

$$\tau \frac{dr_i}{dt} = h\left(\sum_j W_{i-j} r_j\right) + \lambda \sum_j J_{i-j} r_j - r_i \tag{2}$$

   where J is chosen so that $\sum_j J_{i-j} f(\theta - \theta_j) = f'(\theta - \theta_i)$. Assume $r_i = f(\theta - \theta_i)$ is a solution to Eq. (1) for *any* θ (not just $\theta = \theta_k$). Show that the solution to Eq. (2) is a moving bump. What is its speed?

3. Suppose we let the firing rates in Eq. (1) evolve for a long time, and then suddenly perturb Eq. (1) by letting $W_{i-j} \rightarrow W_{i-j} + \delta W_{ij}$. Assume that the elements of δW_{ij} (which is *not* Töplitz) are small. Qualitatively, what is the subsequent behavior of the firing rates?

4. Same question, except for Eq. (2): Suppose we let the firing rates in Eq. (2) evolve for a long time, and then suddenly perturb Eq. (2) by letting $W_{i-j} \rightarrow W_{i-j} + \delta W_{ij}$. Assume that the elements of δW_{ij} are small. Qualitatively, what is the subsequent behavior of the firing rates?

Long question 2.

Consider a Hopfield network in the low temperature limit,

$$x_i(t+1) = \text{sign}\left[\sum_j W_{ij} x_j(t) - \theta_i\right] \tag{3}$$

where

$$W_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu \, .$$

As usual, the ξ_i^μ can take on two values, ± 1, both with probability 1/2. For the first two parts below, $\theta_i = 0$.

For all parts assume N is large!!!

1. Let the ξ_i^μ be uncorrelated, meaning $p(\xi_i^\mu, \xi_i^\nu) = p(\xi_i^\mu) p(\xi_i^\nu)$, $\mu \neq \nu$, and $p(\xi_i^\mu, \xi_j^\mu) = p(\xi_i^\mu) p(\xi_j^\mu)$, $i \neq j$. Compute the mean and variance of $N^{-1} \sum_j \xi_j^\mu \xi_j^\nu$.

2. Show that the capacity (the number of attracting fixed points) is proportional to N. Hint: show that if p/N is a small constant, then, if $x_i = \xi_i^\mu$ on the right hand side of Eq. (3), $\text{sign}\left[\sum_j W_{ij} x_j(t)\right]$ is equal to ξ_i^μ for most i.

3. Let ξ_i^μ and ξ_i^ν be correlated: $\text{prob}(\xi_i^\mu = \xi_i^\nu) = (1+\epsilon)/2$, $\mu \neq \nu$, $\epsilon > 0$. (But we still have $p(\xi_i^\mu, \xi_j^\mu) = p(\xi_i^\mu) p(\xi_j^\mu)$, $i \neq j$.) Compute the mean of $N^{-1} \sum_j \xi_j^\mu \xi_j^\nu$.

4. Using the correlational structure in part 3, show that the capacity of the network scales as $1/\epsilon^{3/2}$, independent of N. Hint: use the same method as in part 2.

5. Again use the correlational structure in part 3. Show that by adjusting the thresholds, θ_i, one can increase the the fidelity of recall – that is, one can increase the probability that $x_i = \xi_i^\mu$ for all the memories.