# Gatsby Computational Neuroscience Unit
# Theoretical Neuroscience

## Final Examination
## 21 Jan 2014

## Part I – short questions

There are five sections with four questions each. Please answer three out of each four, starting the answers for each new section on a new page. Don't forget to write your name at the top of each block of answers.

You have a maximum of 3 hours for this exam.

Good luck!

# 1 Biophysics

1. You do an experiment in which multiple synapses on one branch of a dendritic tree are simultaneously activated. The PSP size as a function of the number of activated synapses is linear + sigmoidal For example, the PSP size, $V_{PSP}$ versus the number of activated synapses, $n$, looks something like

$$V_{PSP}(n) = an + \frac{1}{1 + e^{-\beta(n-n_0)}}$$

with $n_0 \approx 5$. Provide two mechanisms for this nonlinearity.

2. What is short term depression and what causes it?

3. You do an experiment in which you patch a neuron. When you slowly increase the amount of current you inject into it, it doesn't spike, whereas when you rapidly increase the amount of current, it does spike. Explain why.

4. Why do active potassium channels increase the refractory period of a neuron?

## 2 Networks

1. In large networks of excitatory and inhibitory neurons operating in the balance regime (i.e., on the unstable branch of the excitatory nullcline), why are inhibitory to inhibitory connections crucial for stability?

2. Sketch the excitatory and inhibitory nullclines that would turn a randomly connected network of excitatory and inhibitory neurons into a "flip-flop," meaning it could operate in one of two possible states: almost no neurons firing, and all neurons firing at very high rate.

3. Consider a fully connected Hopfield network of the form

$$s_i(t+1) = \text{sign}\left[\frac{1}{N}\sum_{j=1}^{N}\sum_{\mu=1}^{p}\xi_i^\mu \xi_j^\mu s_j(t)\right]$$

where the $\xi_i^\mu$ are binary vectors whose entries are either $+1$ or $-1$. Show that if the $\xi_i^\mu$ are orthogonal, meaning $\sum_i \xi_i^\mu \xi_i^\nu = N\delta_{\mu,\nu}$, then $2N$ patterns can be stored. (The factor of 2 comes from the fact that if $\mathbf{s}$ is a stable pattern, then so is $-\mathbf{s}$.)

4. A neuron receives $K$ excitatory inputs. Whenever presynaptic neuron $i$ fires, it produces a EPSP of the form $V_i e^{-t/\tau_i}\Theta(t)$ where $\Theta$ is the Heaviside step function: $\Theta(t) = 1$ if $t > 0$ and 0 otherwise. Presynaptic neuron $i$ fires at rate $\nu_i$. Write down an exact expression for the time-averaged change in membrane potential due to the EPSPs. Assume that synapses are "current based," meaning the EPSP size does not depend on the membrane potential. Use typical values for $\tau_i$, $V_i$, $\nu_i$ and $K$ to estimate the size of the depolarization.

# 3 Coding

1. Suppose that the instantaneous membrane potential of a cell can be modelled as a Gaussian-shaped function (i.e. tuning curve) of a one-dimensional stimulus plus Gaussian noise. Discuss the differences between the Fisher information and Shannon's average mutual information as measures of the fidelity of the membrane response. What do they each quantify? How do their mathematical forms compare? Which would you prefer to see reported in a paper and why?

2. The (yet-to-be-discovered) $I_Q$ channel is densest on the soma of cortical pyramidal cells. It binds to free intracellular $Ca^{++}$ with the following effects: first an almost instantaneous conformal change in the channel makes it weakly permeable to $Na^+$ ions; this permeability inactivates (like the Hodgkin-Huxley $Na^+$ channel) with a time-constant of 50 ms; then a metabotropic cascade leads to an increase (time-constant of 100 ms) in the efficiency of the $Na^+$ ion pump; the pump returns to normal with a time-constant of 1000 ms.

   You are asked to model a cell that expresses the $I_Q$ channel as well as the usual Hodgin-Huxley channels using a GLM. Sketch the form of spike-history term you would expect to find — show a clearly labelled time axis, explain how each of the features in the term relates to the channel properties, stating any assumptions you need to make.

3. What conditions need to be imposed on the parameters and structure of a GLM point process model so that it describes a (non-trivial) renewal process?

   An inhomogeneous "renewal" process with time-varying average intensity $\overline{\lambda}(t)$ may be constructed by modifying this GLM, or by considering a renewal process in rescaled time. Write the form of the conditional intensity functions for both of these constructions. What assumptions does each approach make when used as a neural model?

4. Consider a generalisation of a "scalar" population code to represent a two-dimensional quantity ($\vec{x} = [x_1, x_2]$), in which the average firing rate of each neuron is a rectified linear function of the input:

$$r_i(\vec{x}) = [w_{i1}x_1 + w_{i2}x_2]^+$$

   for some encoding parameters $\vec{w}_i = [w_{i1}, w_{i2}]$. Assuming that the actual firing rates are Gaussian-distributed with constant variance about these means, and that the noise is independent across cells, what is the form of the optimal decoder? [You may neglect the rectification, provided that you justify doing so.]

   Now suppose that the noise has equal variance in all cells and is positively correlated with all the pairwise correlation coefficients given by $\rho$. As $\rho$ increases do you expect the code to become more or less accurate? How might your answer depend on the the distribution of weights $\vec{w}_i$ in the population?

# 4 Systems

1. What are four main neuromodulators in the mammalian brain? How does their action differ from conventional neurotransmitters?

2. How could patterns of activity across the retina *in utero* influence the wiring of the visual system?

3. Why is synaptic depression useful for detecting changes in pre-synaptic firing rates, but not so useful in determining steady state pre-synaptic firing rates?

4. What is Weber's law? Given that neuronal firing is approximately Poisson, why is it hard to explain?

# 5 Learning

1. Describe, with proof that they achieve these ends, correlational and covariance forms of Hebbian learning.
2. How does the Bienenstock-Cooper-Munro (BCM) learning rule differ from conventional Hebbian learning? What consequence does this have for its stability, and why?
3. What are forwards and backwards blocking in conditioning? How can the latter be explained using a Kalman filter model?
4. Consider a choice between two options $a = \{1, 2\}$ with $Q-$values $Q_a$. What stochastic choice rule maximizes the mean reward subject to the entropy of the choices being $\mathcal{H}$? What values of $\mathcal{H}$ are possible?