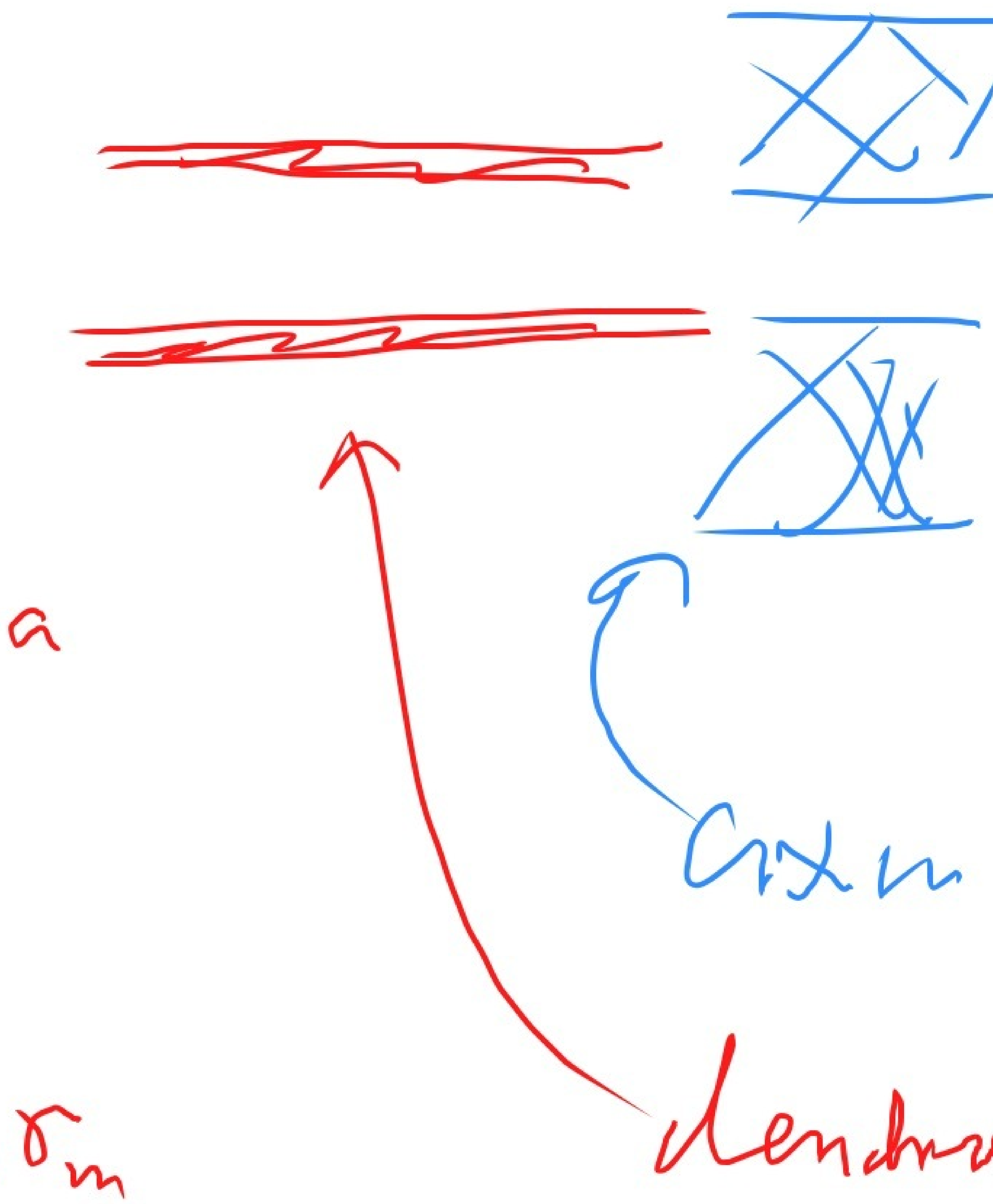


$C\frac{dV}{dt}$ = currents + algebra

cable equation

$$\tau \frac{\partial V}{\partial t} = \lambda^2 \frac{\partial^2 V}{\partial x^2} - (V - E_L) + \underbrace{i_e r_m}_{\delta(x)\delta(t)}$$

10 ms

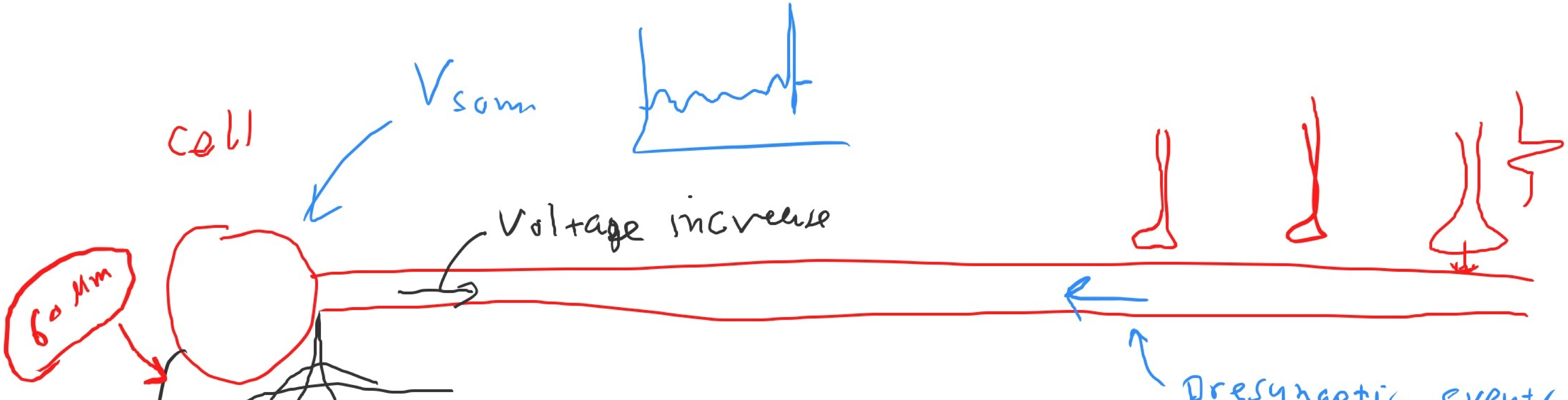
$$\lambda^2 = \frac{r_m a}{2 r_L}$$

- very fast current injection
- from other neurons

C is different

$C = c_m A$

property of membrane



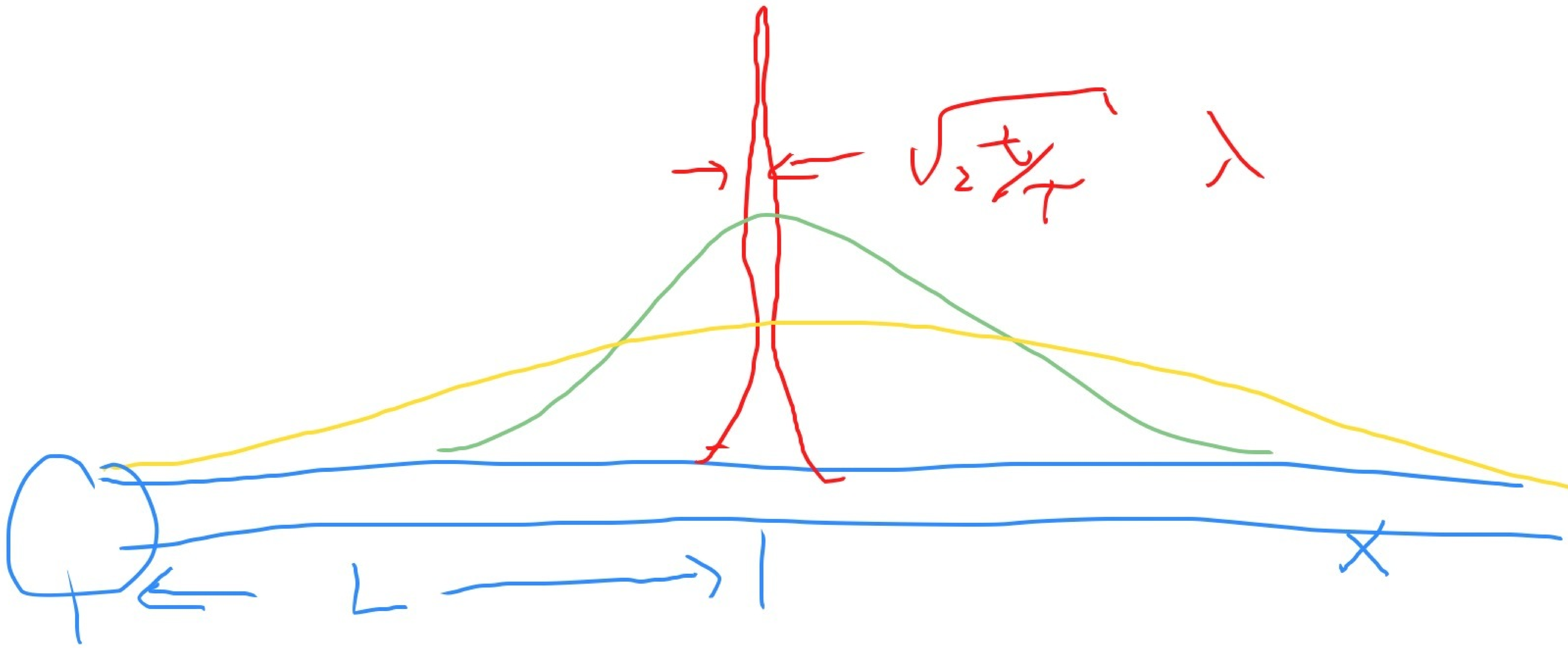
$$u = V - E_L$$

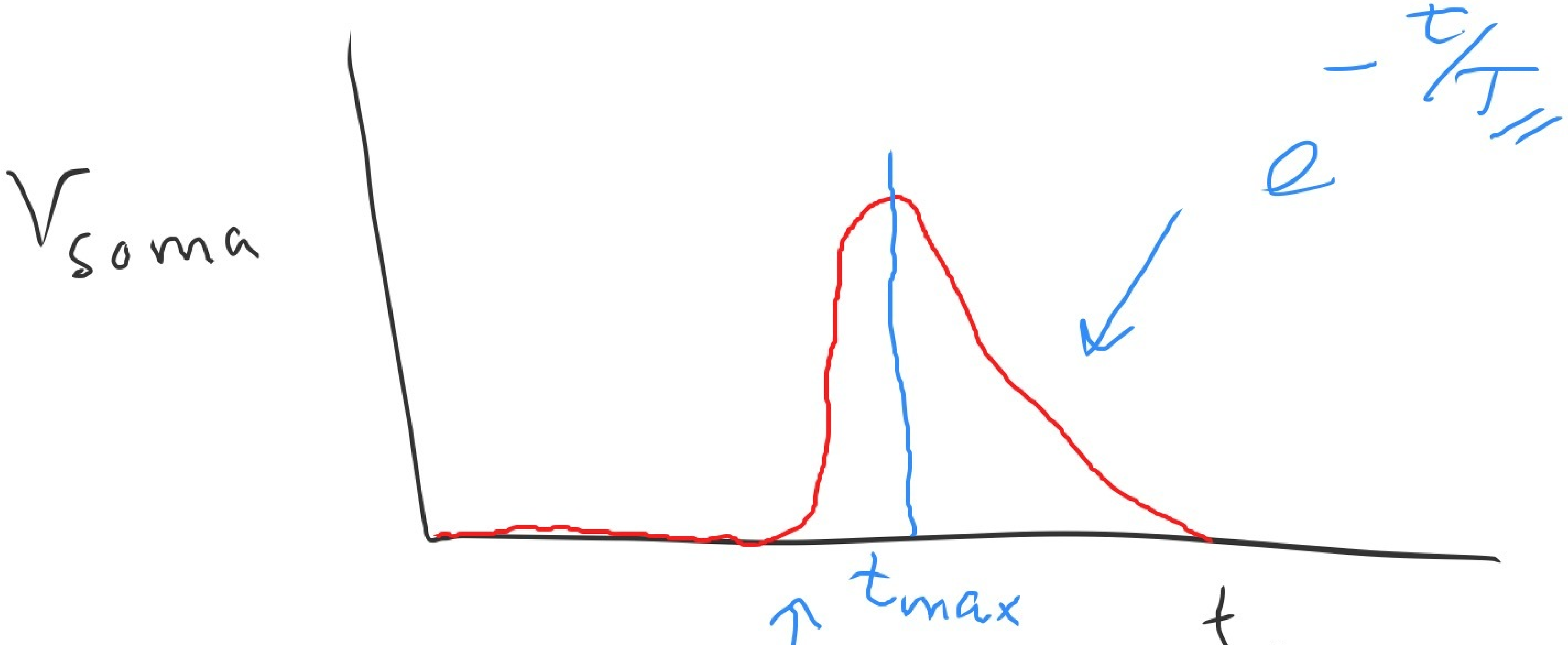
$$\tau \frac{\partial u}{\partial t} = \lambda^2 \frac{\partial^2 u}{\partial x^2} - u + \delta(x)\,\delta(t)$$

$$\boxed{u = e^{-t/\tau} \frac{e^{-\frac{x^2}{4\lambda^2 t/\tau}}}{\sqrt{4\pi \lambda^2 t/\tau}} \quad \Bigg\} \text{ Gaussian with standard dev } \sqrt{2\frac{t}{\tau}}\,\lambda}$$

F.T. w.r.t. x

$$\tau \frac{\partial u}{\partial t} = -\lambda^2 k^2 u - u \qquad u \propto e^{-\frac{\lambda^2 k^2 + 1}{\tau} t}$$





$$\sqrt{\frac{t}{\tau}}\,\lambda \approx L \qquad t \simeq \frac{L^2}{\lambda^2}\tau$$

$$u \propto \exp\left[\underbrace{\frac{-x^2}{4\lambda^2 t/\tau} - \frac{t}{\tau} - \frac{1}{2}\log\frac{t}{\tau}}\right]$$

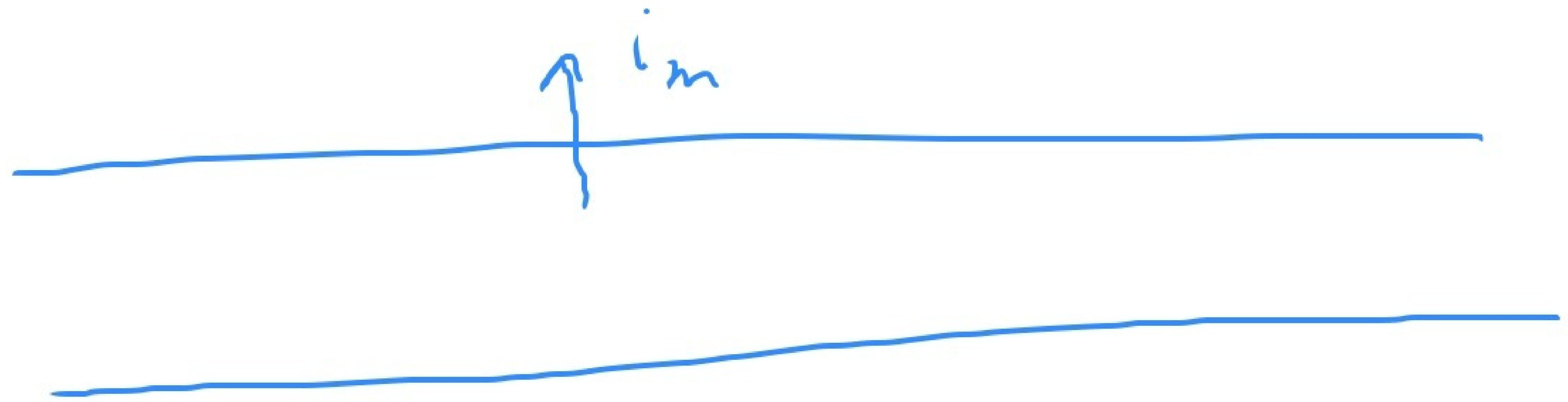
$$\frac{d}{d(t/\tau)}\left[\quad\right] = \frac{x^2}{4\lambda^2 (t/\tau)^2} - 1 - \frac{1}{2(t/\tau)} = 0$$

L

$$\left(\frac{t}{\tau}\right)^2 + \frac{1}{2}\frac{t}{\tau} - \frac{x^2}{4\lambda^2} = 0$$

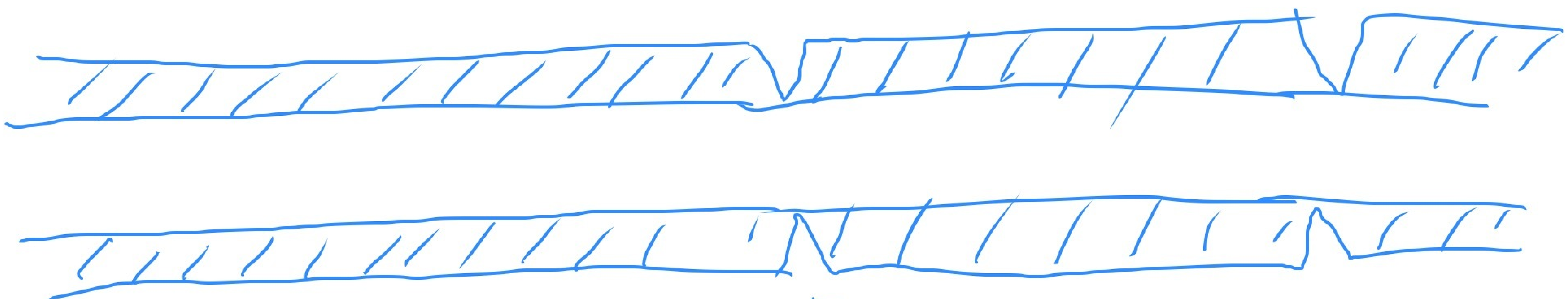
$$\frac{t}{\tau} = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{x^2}{\lambda^2}}}{2} = \frac{1}{4}\left[\sqrt{1+\frac{L^2}{\lambda^2}} - 1\right]$$

$\frac{L^2}{\lambda^2} \gg 1$ $\quad \frac{t}{\tau} \simeq \frac{1}{4}\frac{L}{\lambda}$ $\quad$ "speed": $\frac{L}{t} = \frac{4\lambda}{\tau} \sim \sqrt{a}$



dendrites

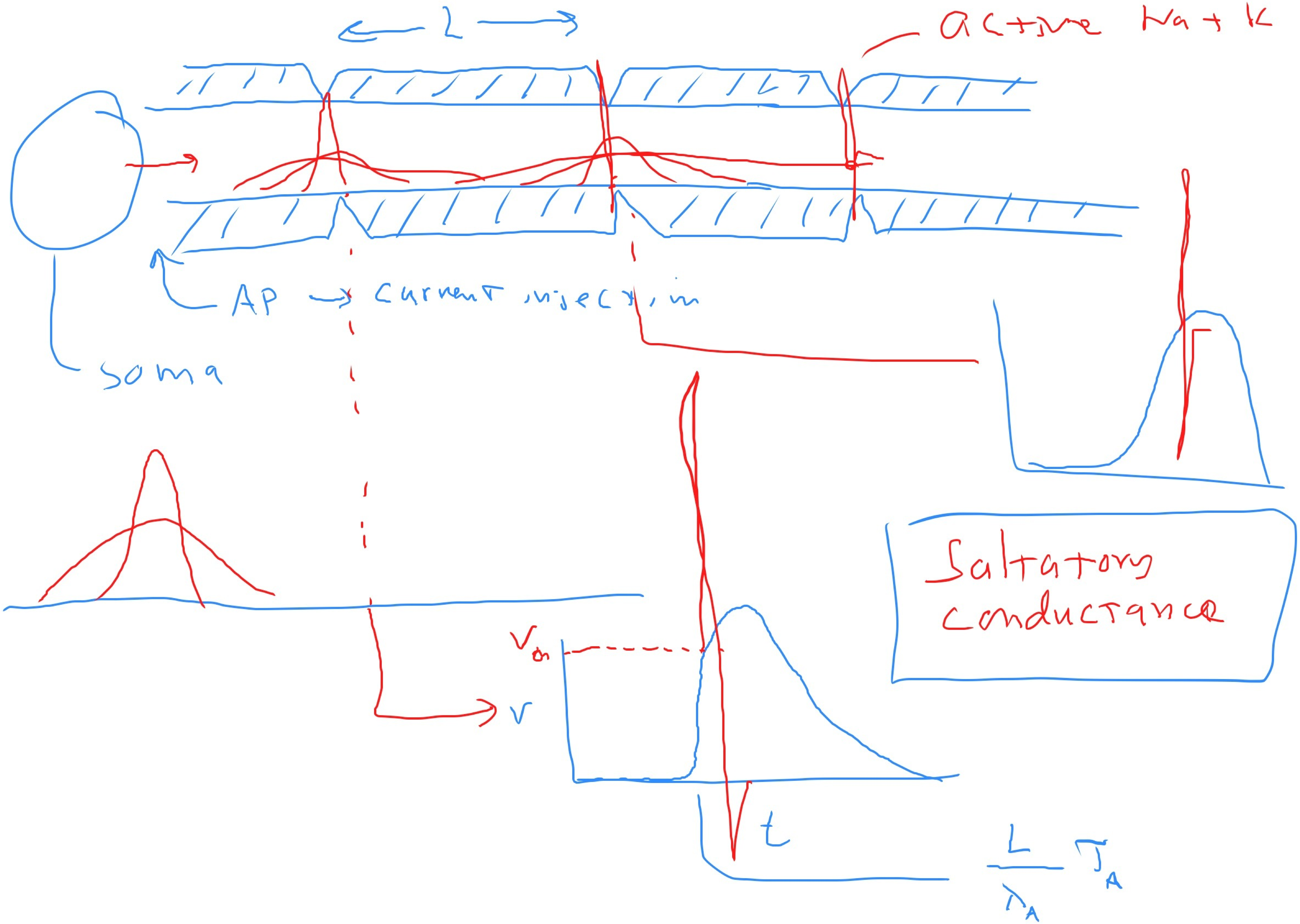
- characteristic length scale $\lambda \sim \underline{1\ \text{mm}}$
- " time " $\tau \sim 10$ ms

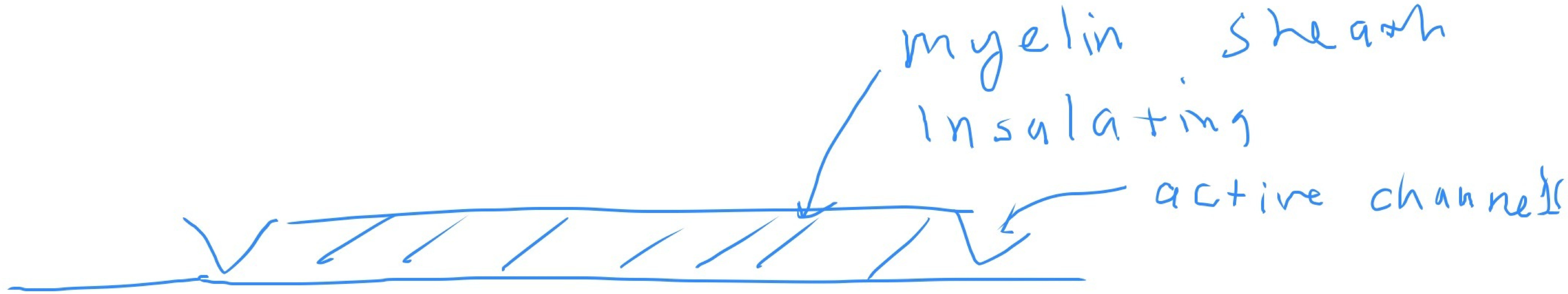


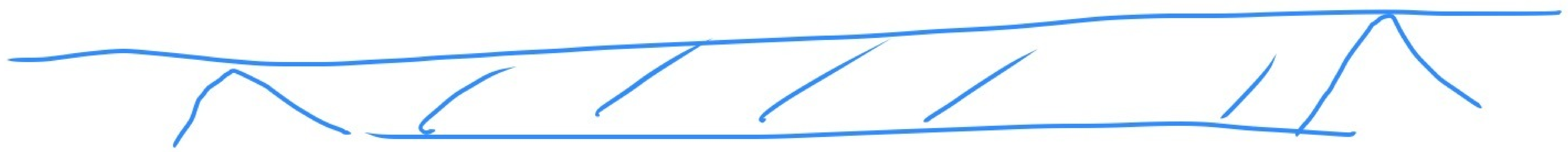
nodes of Ranvier

active Na + K channels

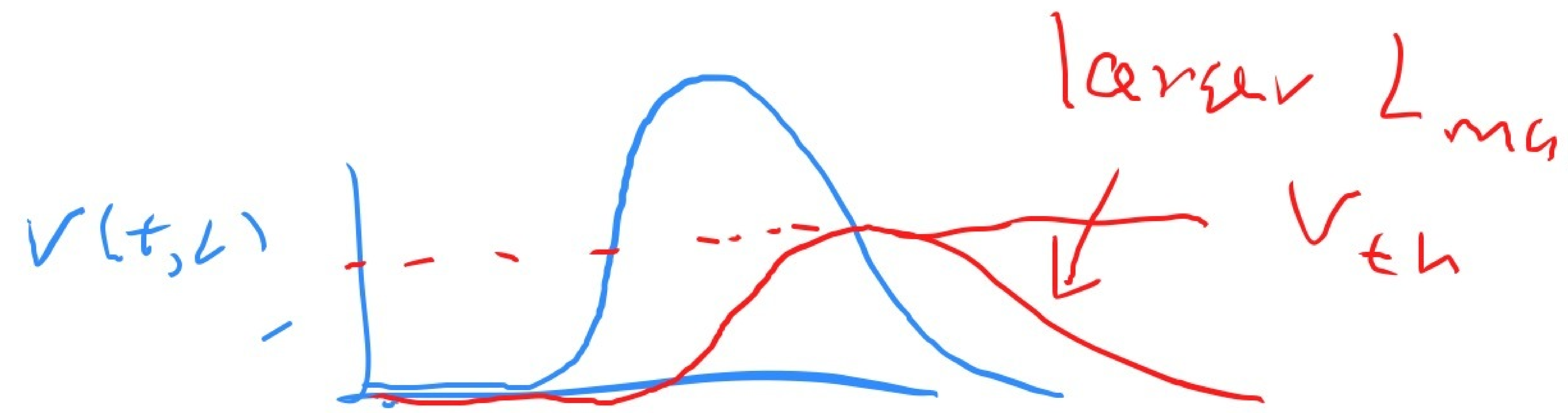
- HH eqns

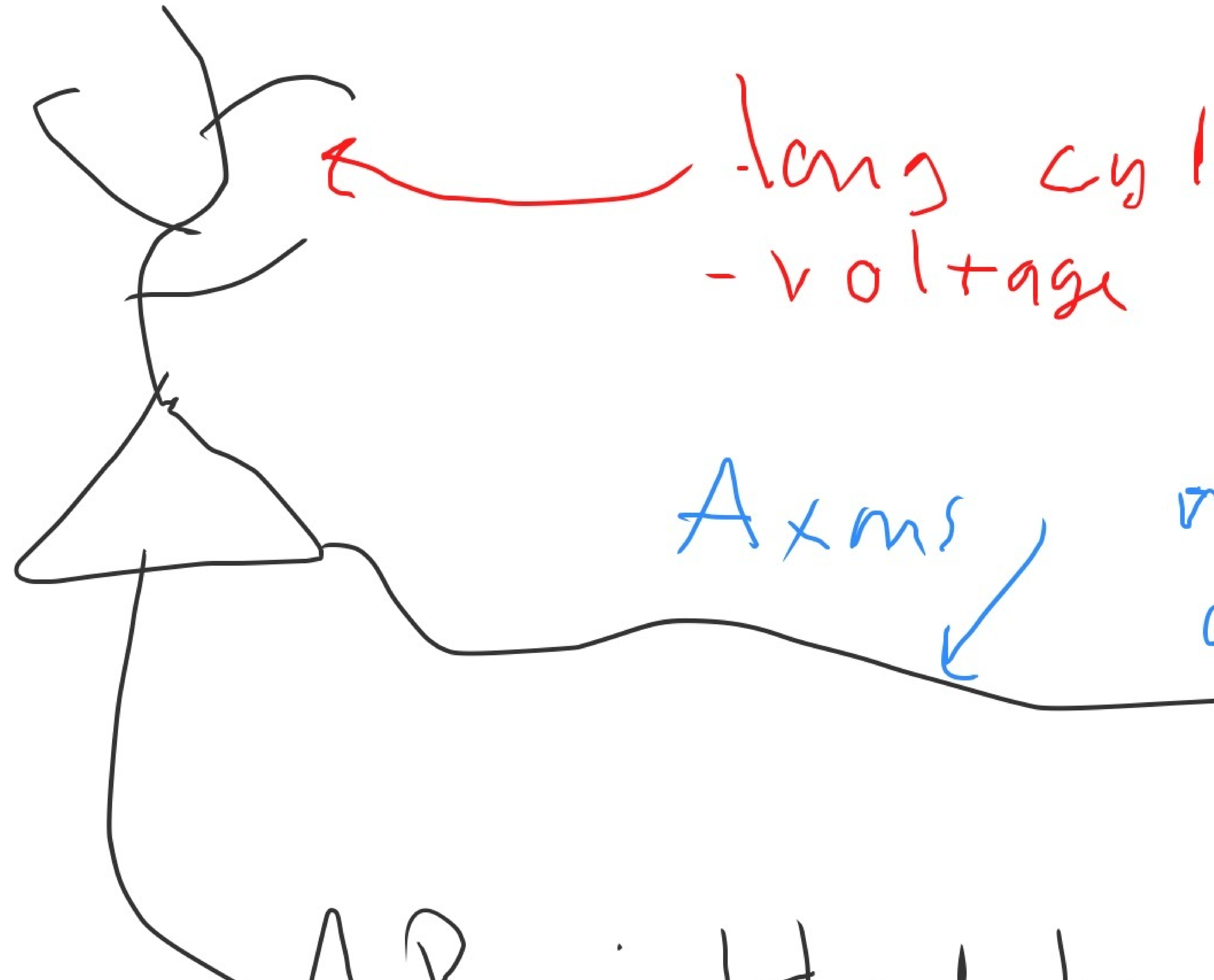

L
active Na + K
AP → current injection
soma
V_{th}
V
t
$\frac{L}{\lambda_A}\tau_A$
Saltatory conductance



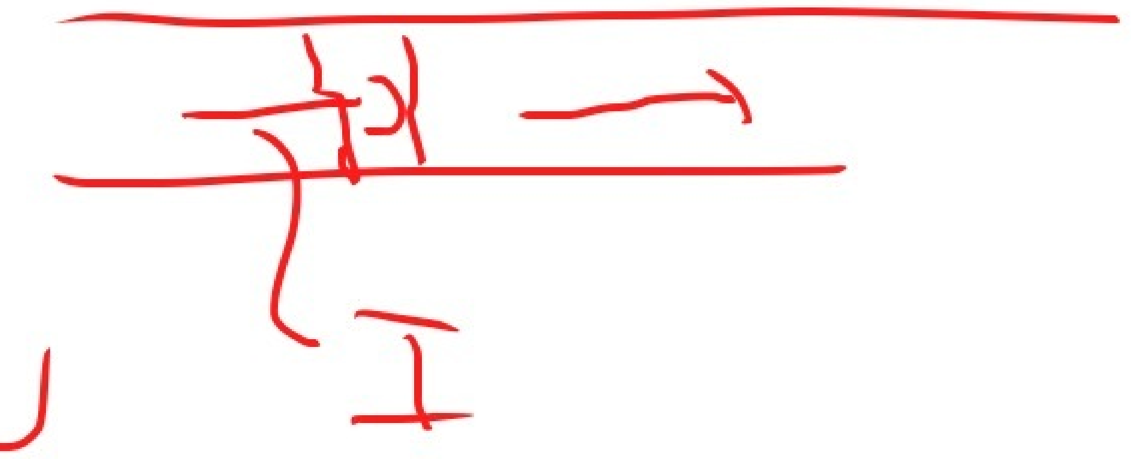


$$V(t) \propto e^{-\frac{x^2}{4\lambda^2 T/T_A} - \frac{1}{2}\log\frac{T}{T_A}}$$





long cylinders
- voltage propagated



Axons

mylenated dendrites
active channels

A.P. : Hodgkin Huxley

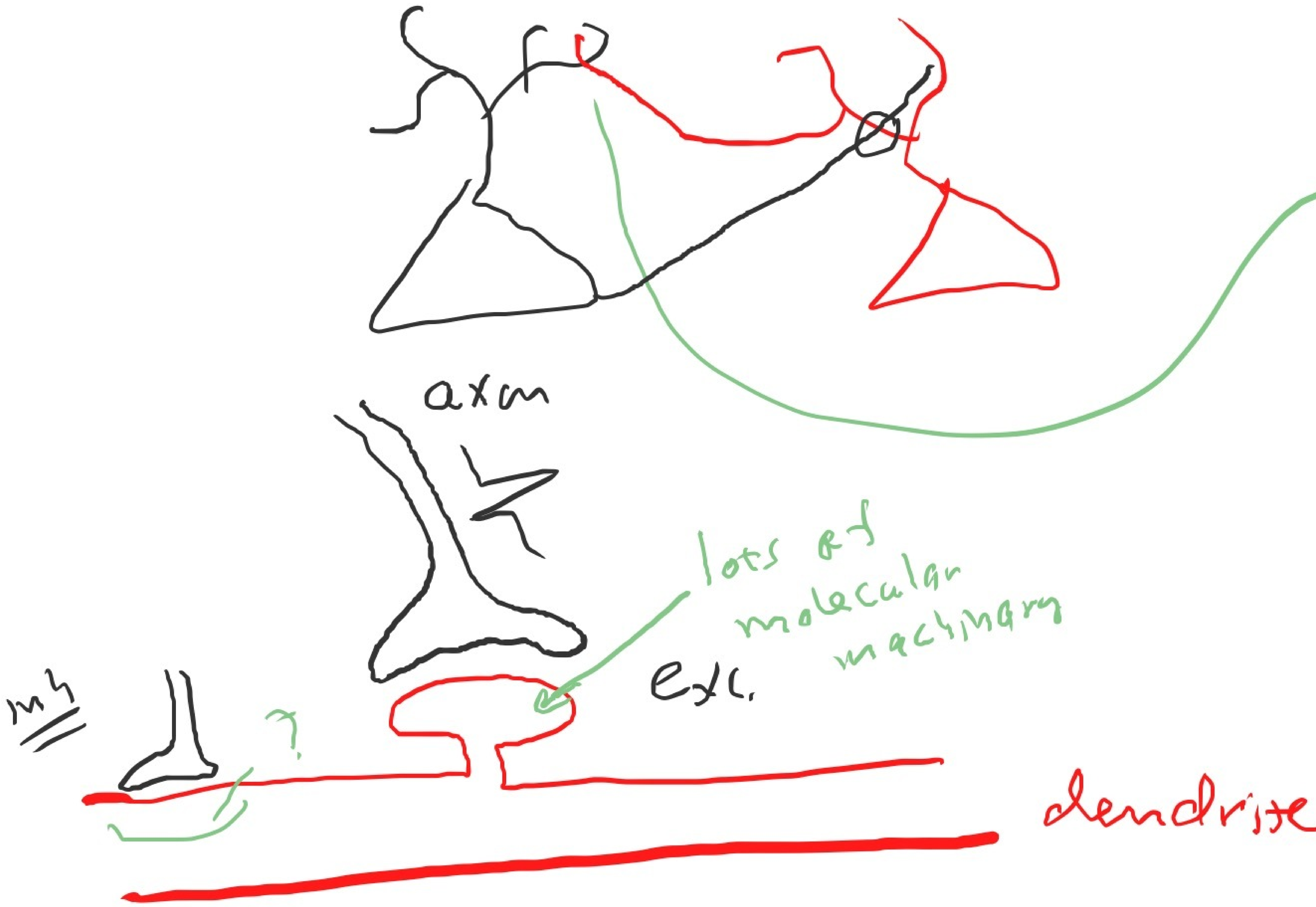
action potentials

- voltage-dependent channels that allow massive influx of Na^+

[Ca^{++} spikes]

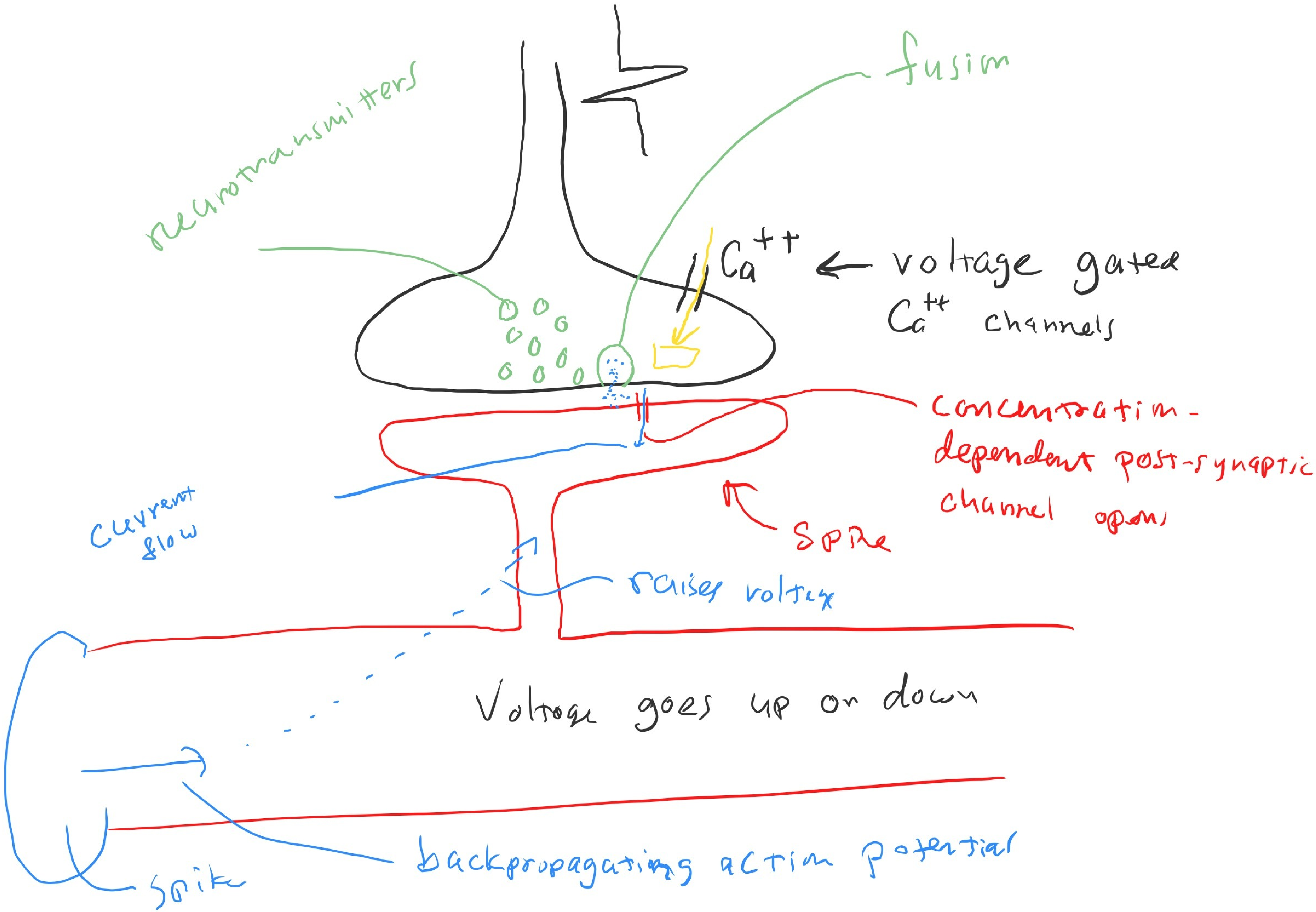
$$\boxed{C\frac{dV}{dt} = \text{currents}}$$

How do neurons communicate?



gap junctions

- very common during development
- less common in adults
- almost universally ignored by theorists



neurotransmitters
fusion
Ca^{++} ← voltage gated Ca^{++} channels
concentration-dependent post-synaptic channel opens
spike
current flow
raises voltage
Voltage goes up or down
backpropagating action potential
spike

$$I_{post} = -g_0 x (V - \Sigma)$$

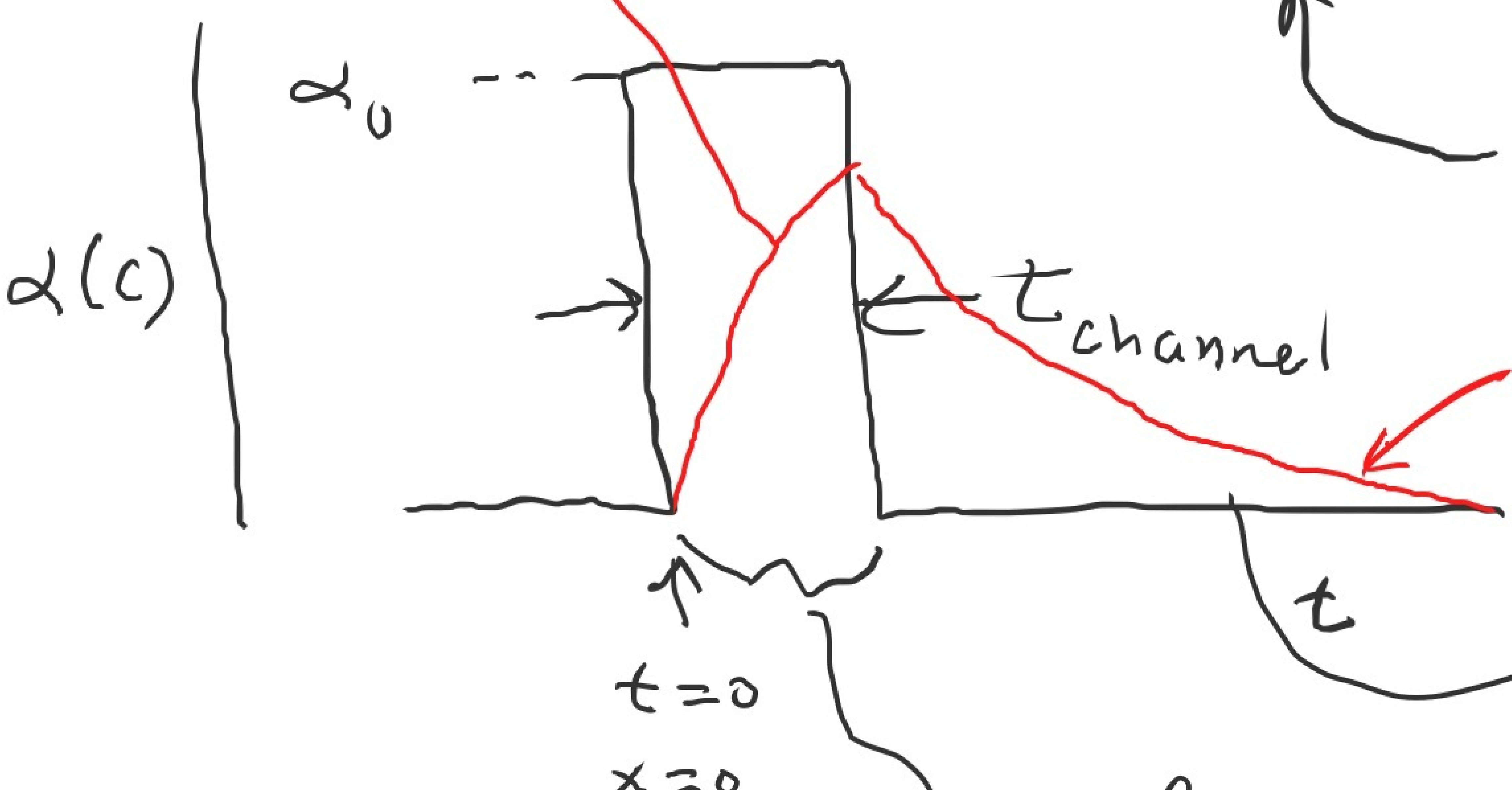
concentration dependent

time constant $\frac{1}{\alpha_0 + \beta}$

$$\frac{dx}{dt} = \alpha(c)(1-x) - \beta x$$

channels opening

channels closing



$x(t)$, time constant $\frac{1}{\beta}$

$$x(t) \propto e^{-\beta t}$$

$$\frac{dx}{dt} = \alpha_0(1-x) - \beta x = \alpha_0 - (\alpha_0 + \beta)x$$

$$x(t) = \frac{-\alpha_0 e^{-(\alpha_0+\beta)t}}{\alpha_0 + \beta} + \frac{\alpha_0}{\alpha_0 + \beta}$$

		E	τ_{rise}	τ_{decay}
E	AMPA	0	~0.1 ms	5
	NMDA	0	1-5	150-1000
	$GABA_A$	-70	0.3	5
	$GABA_B$	-100	10	200

$\frac{1}{\alpha_0+\beta}$

$\frac{1}{\beta}$

NMDA
$$I = \frac{-g_0\, x\,(V - E_{NMDA})}{1 + \frac{[Mg^{++}]}{3.7}\exp\left[-\frac{V}{16.1\text{ mV}}\right]}$$

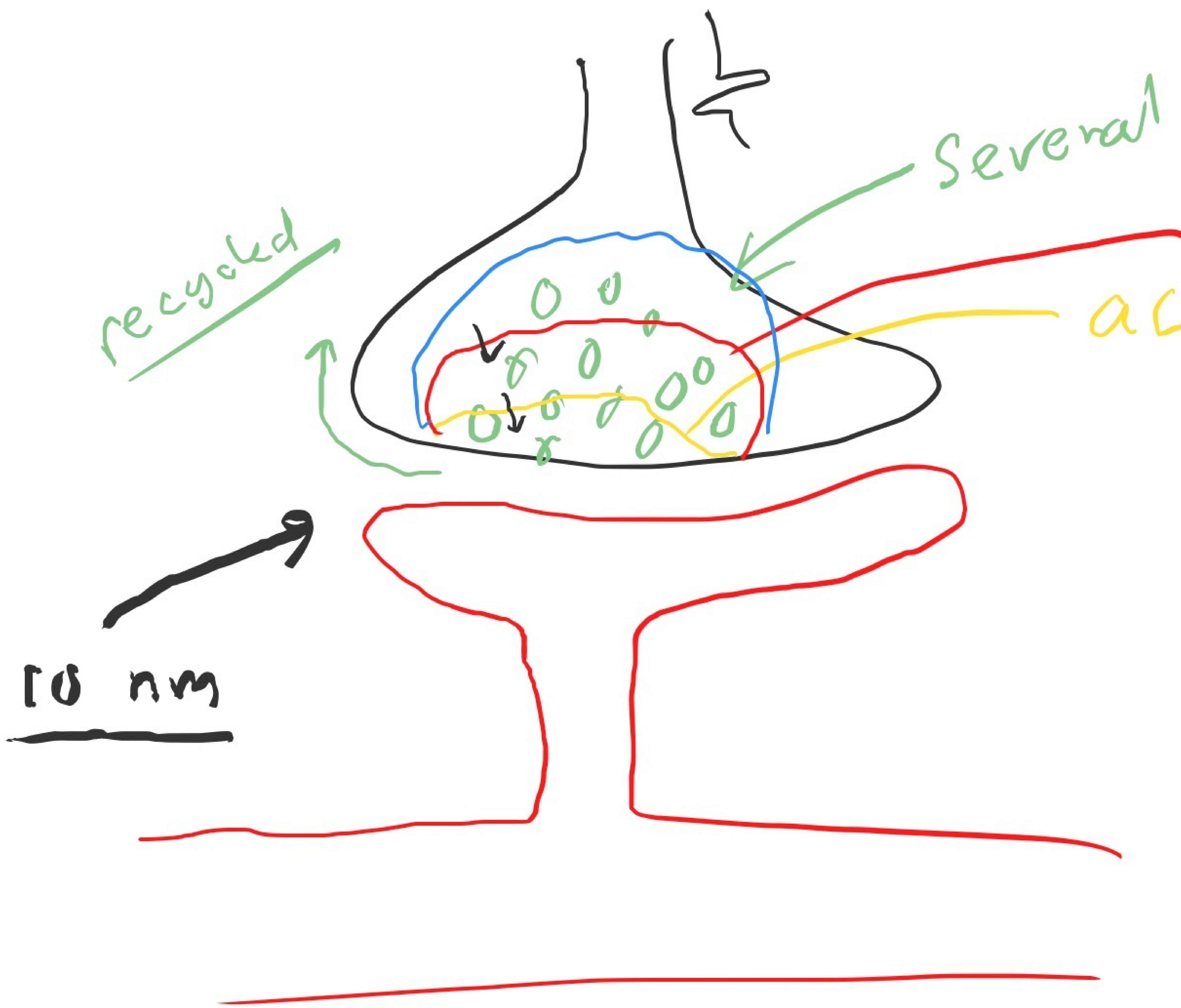
$V = -65$

e^{+4}

1. a little bit about synaptic release

2. plasticity (short term)

3. " (long term)



Action potential arrives

Prob release

$= P_r \simeq 0.5$

$(0.2 - 0.8)$

if there's release, $P_r \downarrow$

if " no " , $P_r \uparrow$

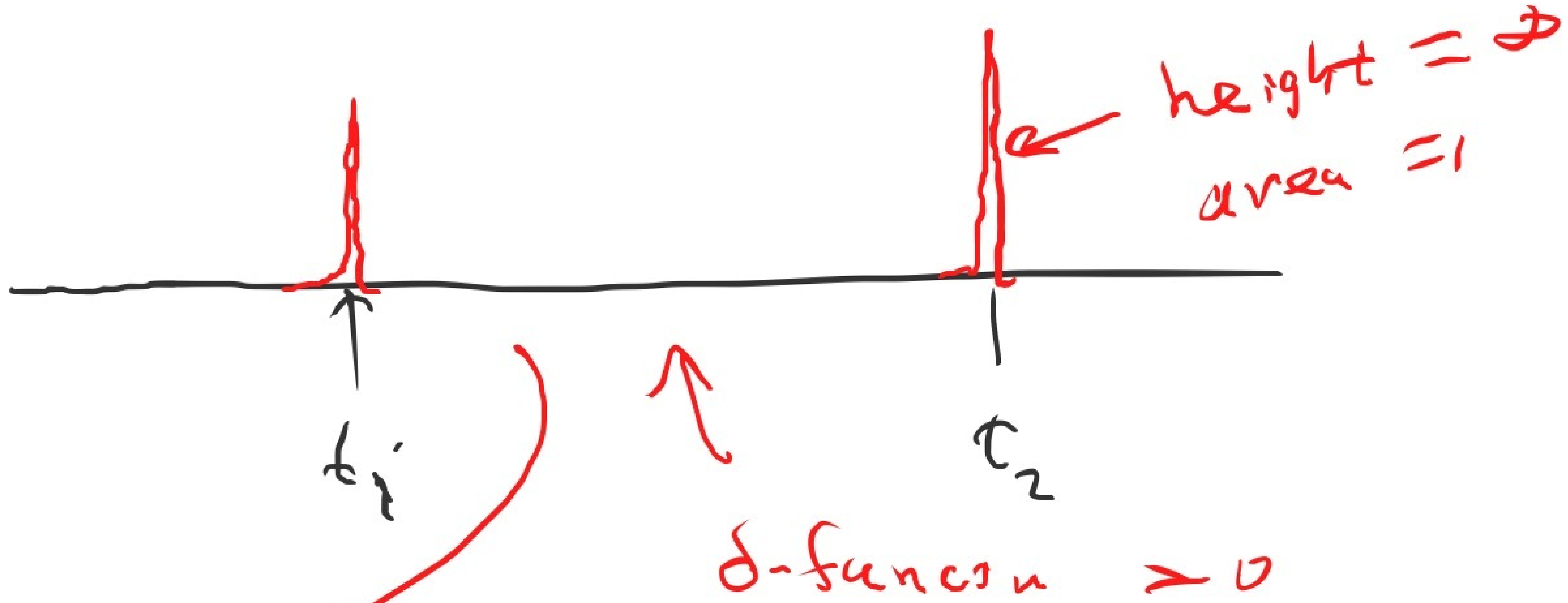
Ca^{++} buildup

$$\tau \frac{dP_r}{dt} = P_0 - P_r + \left[\sum_i \delta(t - t_i) \right] \begin{cases} -\tau(1-f_D) P_r & \text{depression} \\ \text{or} & \\ +\tau f_F (1-P_r) & \text{facilitation} \end{cases}$$

t_i: time of pre-synaptic action potential

$$\tau \frac{dP}{dt} = P_0 - P_r - \tau \sum_i \delta(t - t_i)(1-f_D) P_r$$

$$\frac{dP_r}{dt} = \frac{P_0 - P_r}{\tau} - (1 - f_D) \sum_i \delta(t - t_i)\, P_r$$



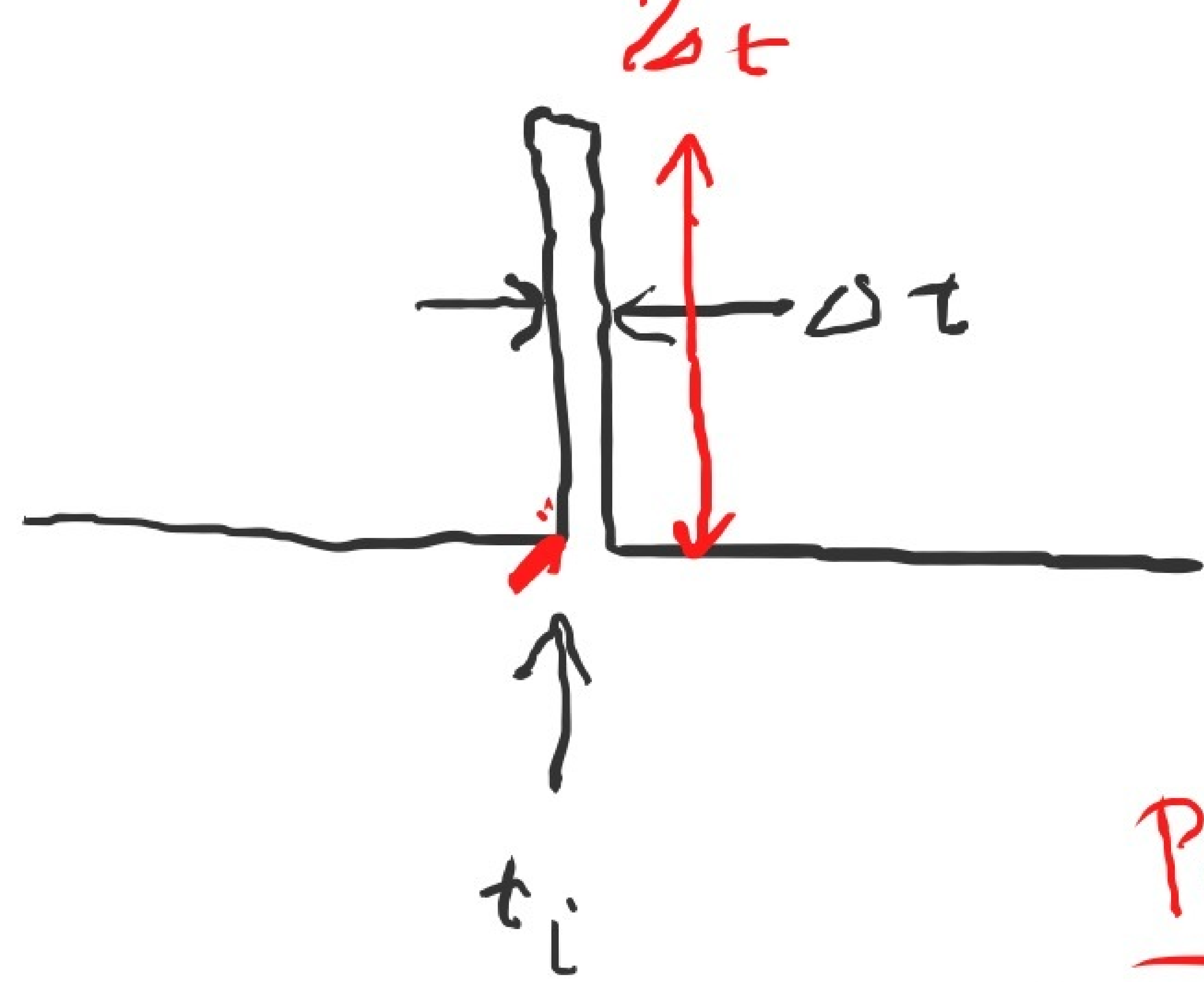
$$\int dt\, \delta(t - t_i) = 1$$

$$\frac{dP_r}{dt} = \frac{P_0 - P_r}{\tau} \qquad \Big\| \qquad \frac{d(P_r - P_0)}{dt} = -\frac{(P_r - P_0)}{\tau}$$

$$P_r - P_0 = \left(P_r(0) - P_0\right) e^{-\frac{t}{\tau}}$$

$$P_r = P_0 + \left(P_r(0) - P_0\right) e^{-t/\tau}$$

$$\frac{dP_r}{d\tau} = \frac{P_r - P_0}{T} - (1-f_D)\,\delta(t-t_i)\,P_r(t)$$



$\Delta t \to 0$

$t \to t_i$

$\frac{P_r - P_0}{T} << \frac{1-f_D}{\Delta T}$

looks like exponential decay towards 0

$t_i < t < t_i + \Delta T$

$$\frac{dP_r}{d\tau} = \cancel{\frac{P_r - P_0}{T}} - (1-f_0)\frac{P_r(t_i)}{\Delta t}$$

$$P_r(t) = P_r(t_i) - P_r(t_i)\left[\frac{1-f_D}{\cancel{\Delta t}}\right]\frac{t-\cancel{t_i}}{\Delta T}$$

$$= P_r(t_i) - P_r(t_i)(1-f_0) = f_D P_r(t_i)$$

$$P_r(t_i+\Delta t) = \underline{f_D}\, P_r(t_i)$$

$$P_r(t) = P_r(t_i)\, e^{-\frac{(1-f_0)}{\Delta T}(t-t_i)}$$

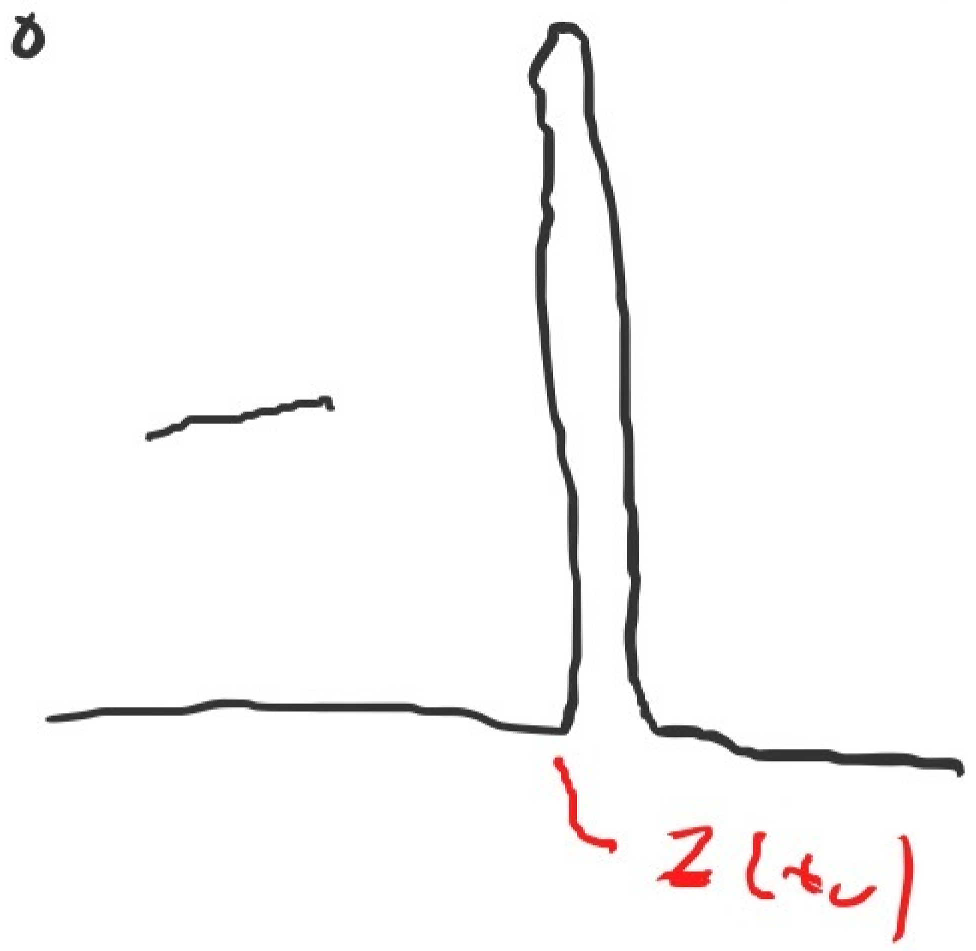
solution

$$P_r(t_i+\Delta t) = P_r(t_i)\, \underline{e^{-(1-f_0)}}$$

$$\simeq P_r(t_i)\left[1-(1-f_0)\right]$$

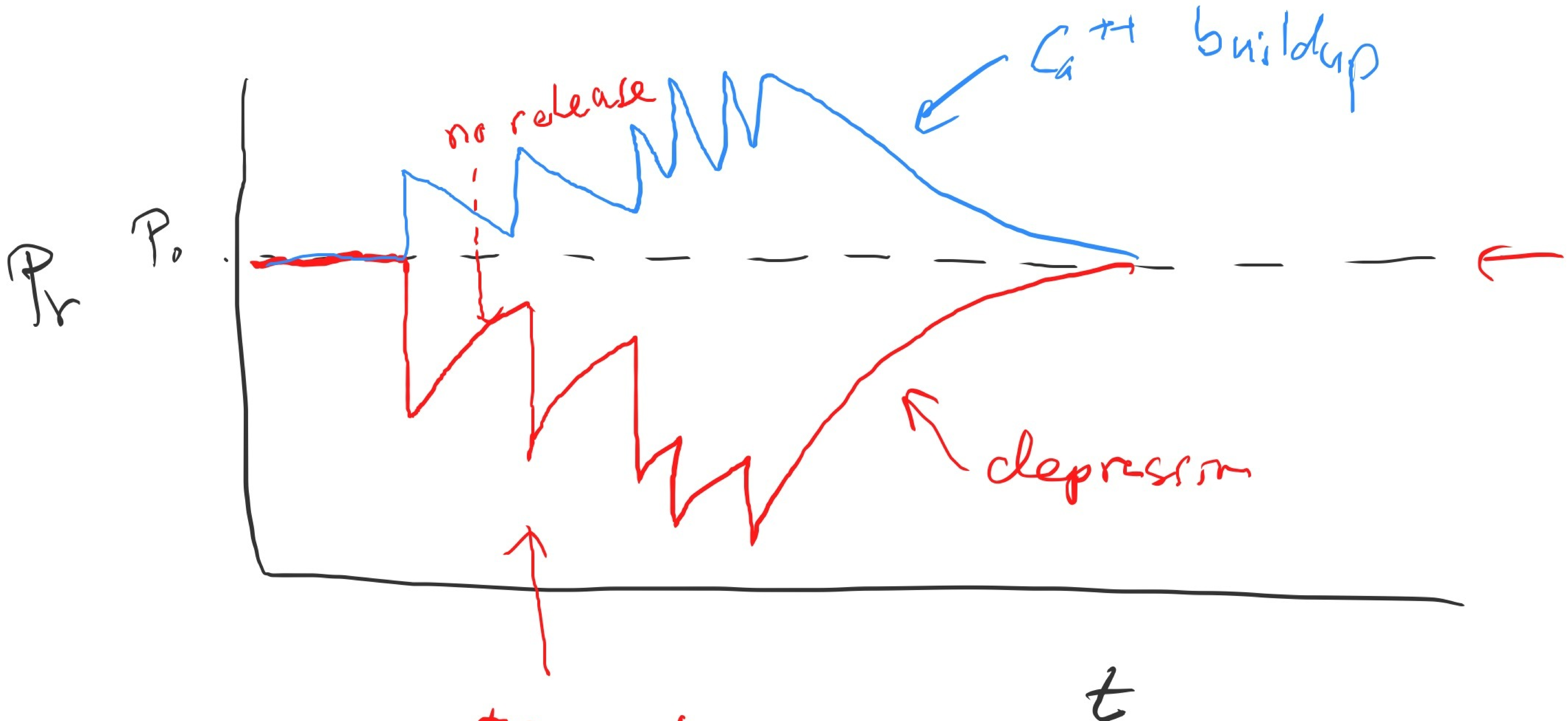
$$\frac{dz}{dt} = \underline{\delta(t)}\, z(t) + g(t)$$

$$z(t) = z(t_0) + \underline{\int_{t_0}^{t} dt' \, \delta(t')\, z(t')} + \underbrace{\int_{t_0}^{t \approx t_0} dt' \, g(t')}_{\approx g(t_0)(t-t_0)}$$



$$\approx \delta(\cancel{t_0})\, z(t_0)\,(t-t_0)$$

$$z(t_0) \int_{t_0}^{t} dt' \, \delta(t') = \underline{z(t_0)}$$



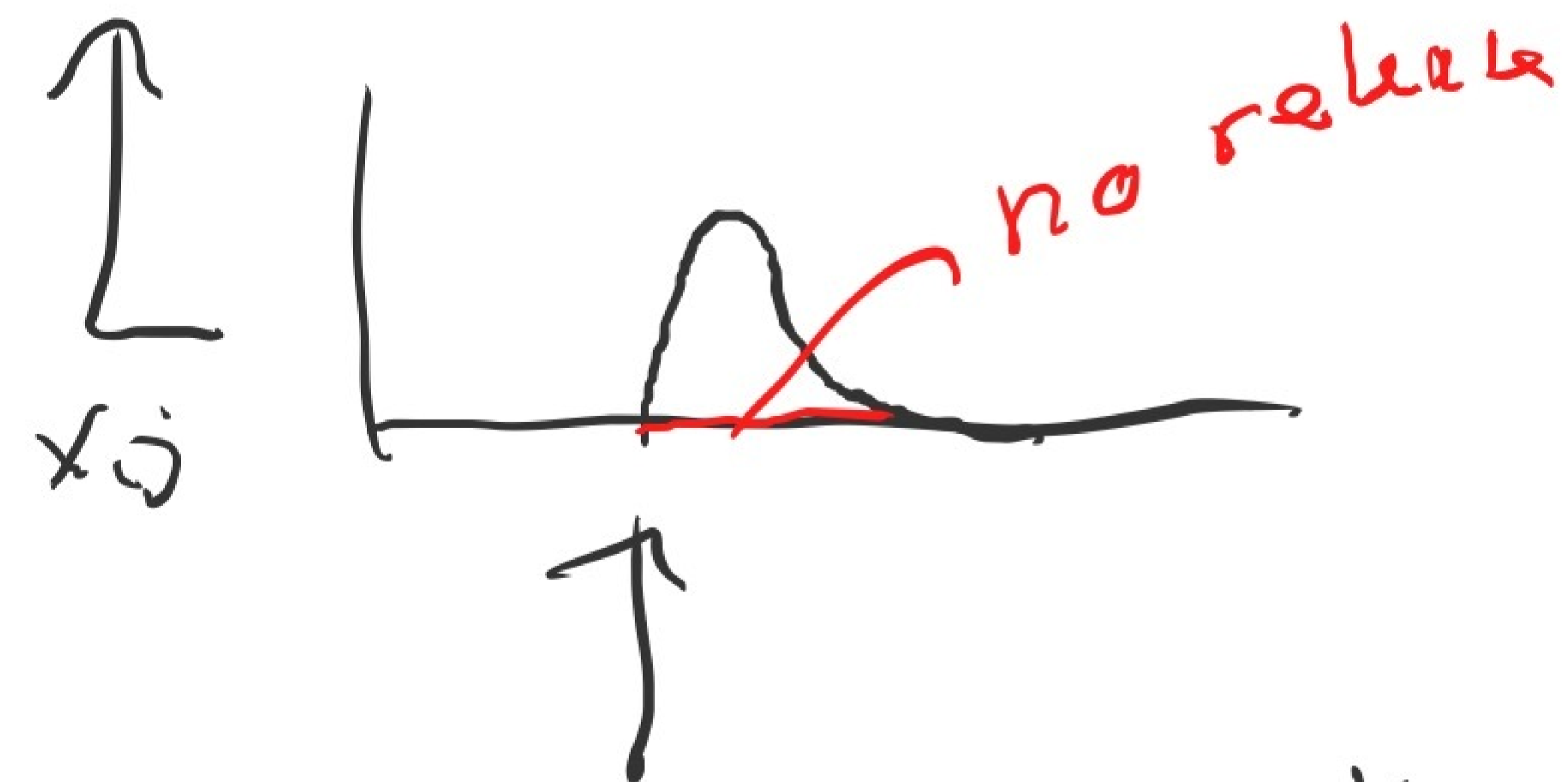
times of presynaptic events
if there was a release

$$\frac{dP_r}{dt} = \frac{P_0 - P_r}{\tau} - (1-f_0)\sum_i \delta(t-\underbrace{t_i}_{\text{random times}})\,\xi_i P_r$$

$$\langle P(\xi_i = 1) \rangle = P_r(t)$$

$$P(\xi_i = 0) = 1 - P_r$$

$$\tau \frac{dV_i}{dt} = [\text{HH} \ldots] - \sum_j W_{ij} x_{ij} (V_i - \mathcal{E}_j)$$



10^{11} equations

missing:

1. dendrites

2a. $\frac{dW_{ij}}{dt}$

b. $\frac{dP_{o,ij}}{dt}$

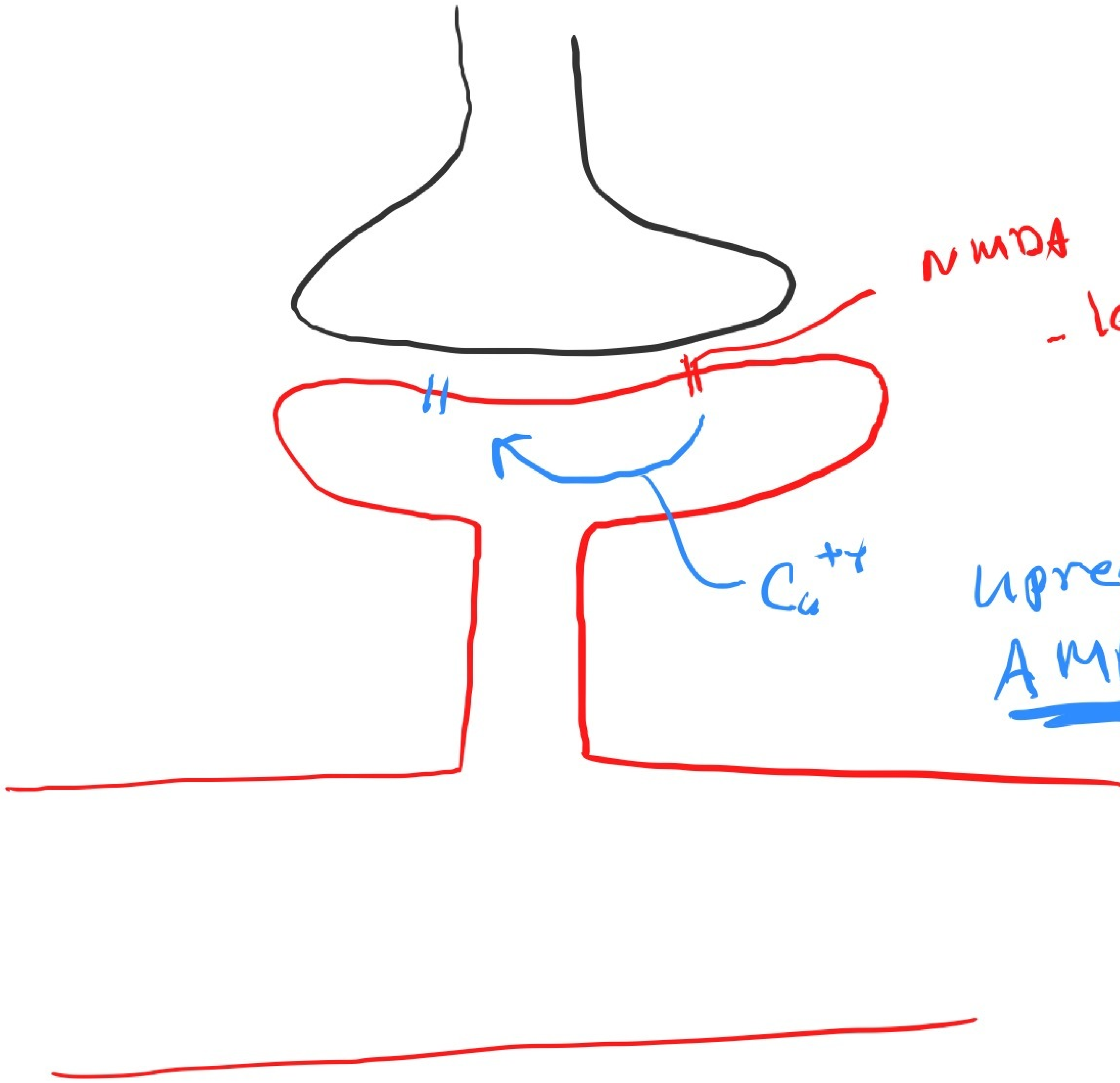
most important missing info about the brain

- experimentally, lots of rules
- don't know which one the brain uses, if any

$$\frac{dP_{ij}}{dt} = \ldots$$

c. initial connectivity ← connectome

NMDA-dependent plasticity



NMDA channel

- lets in a mix of Na^+ + Ca^{++}

upregulates and downregulates **AMPA** channels

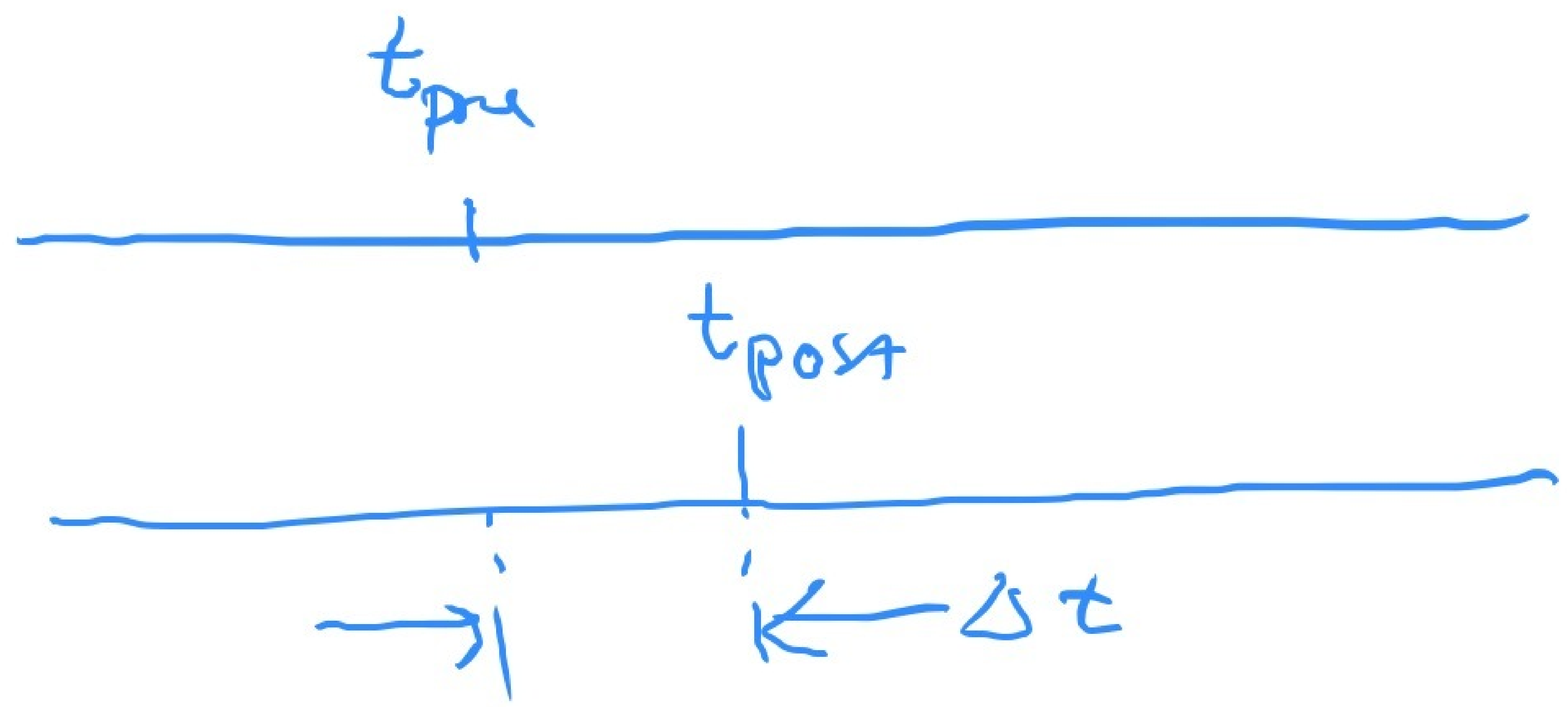
insertion and deletion

- "silent" synapses

NMDA, but no AMPA

W_{ij} $\uparrow$ coincident pre + post

$\downarrow$ (?) Ca^{++} is present but low



time

- what's Ca^{++} concentration as a function of Δt?

NMDA channels open only if V is large