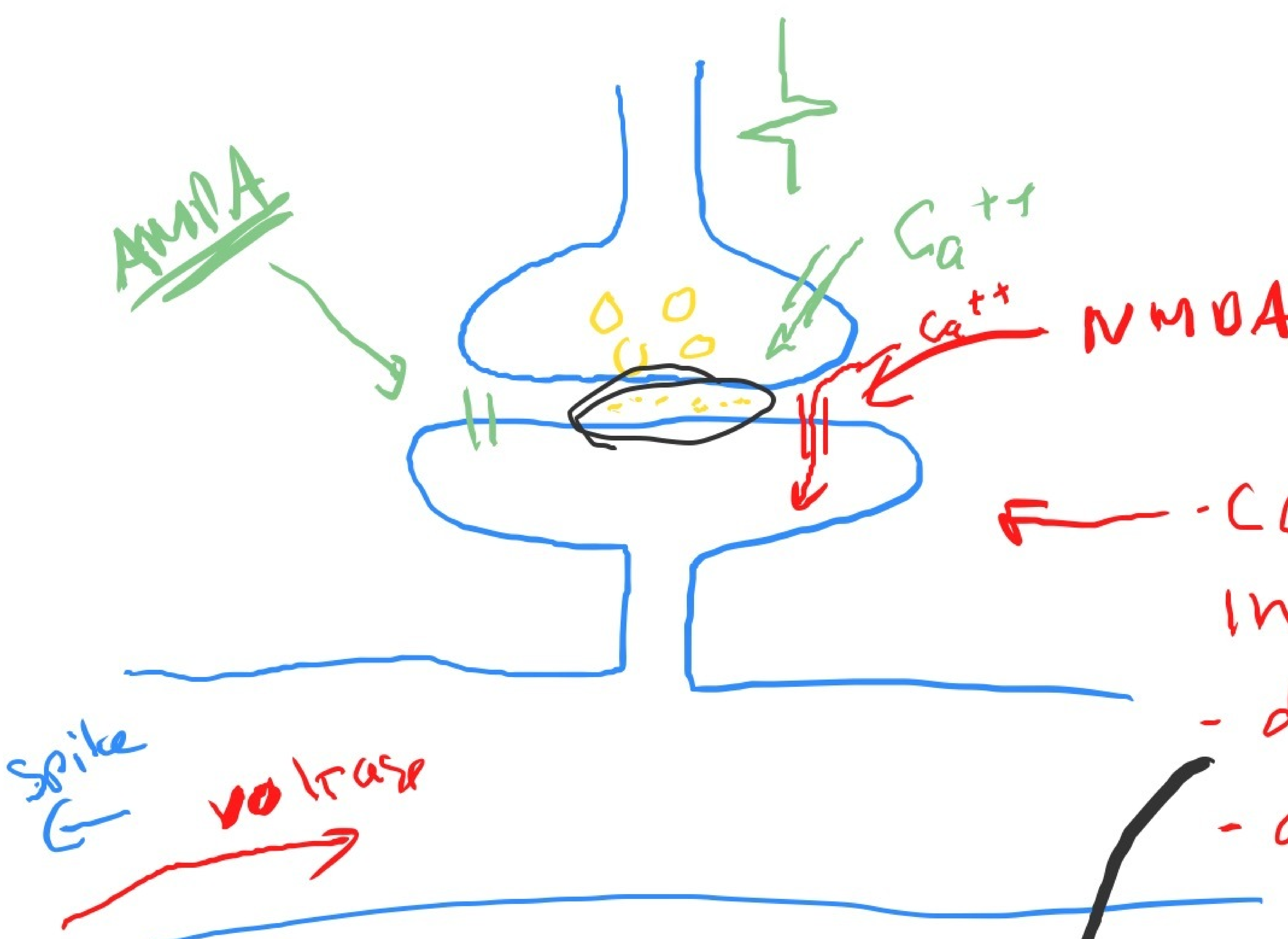
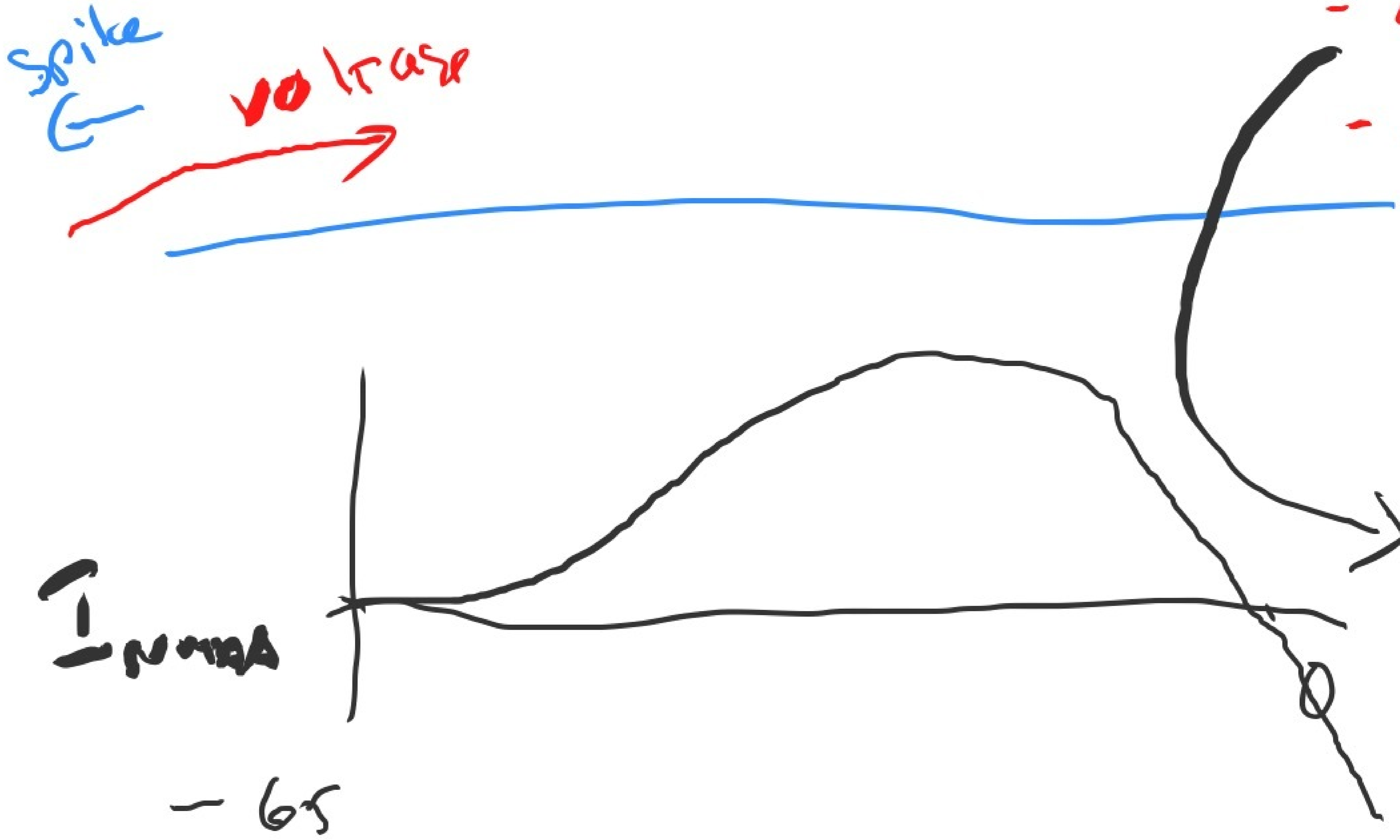
plasticity

1. learning rules
2. learning



learning rule depends on time of pre- and post-synaptic spikes

- causes a change in synaptic strength
- depends on post-synaptic voltage
- depends on post synaptic spikes



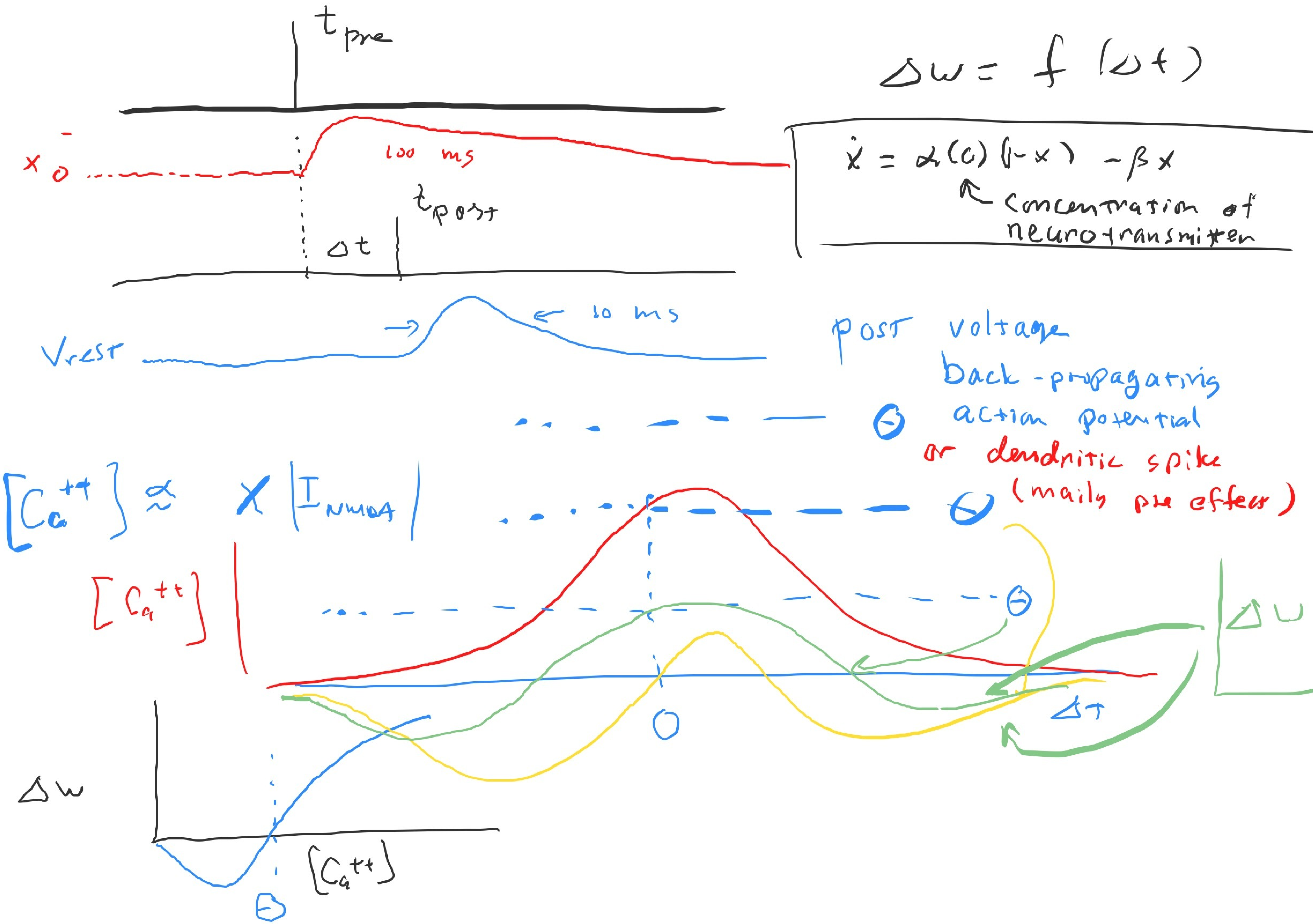
$$I_{NMDA} = \frac{-g_0 X (V - E)}{\frac{[Mg^{++}]}{3.57} e^{-\frac{V}{16.1\,mV}}}$$

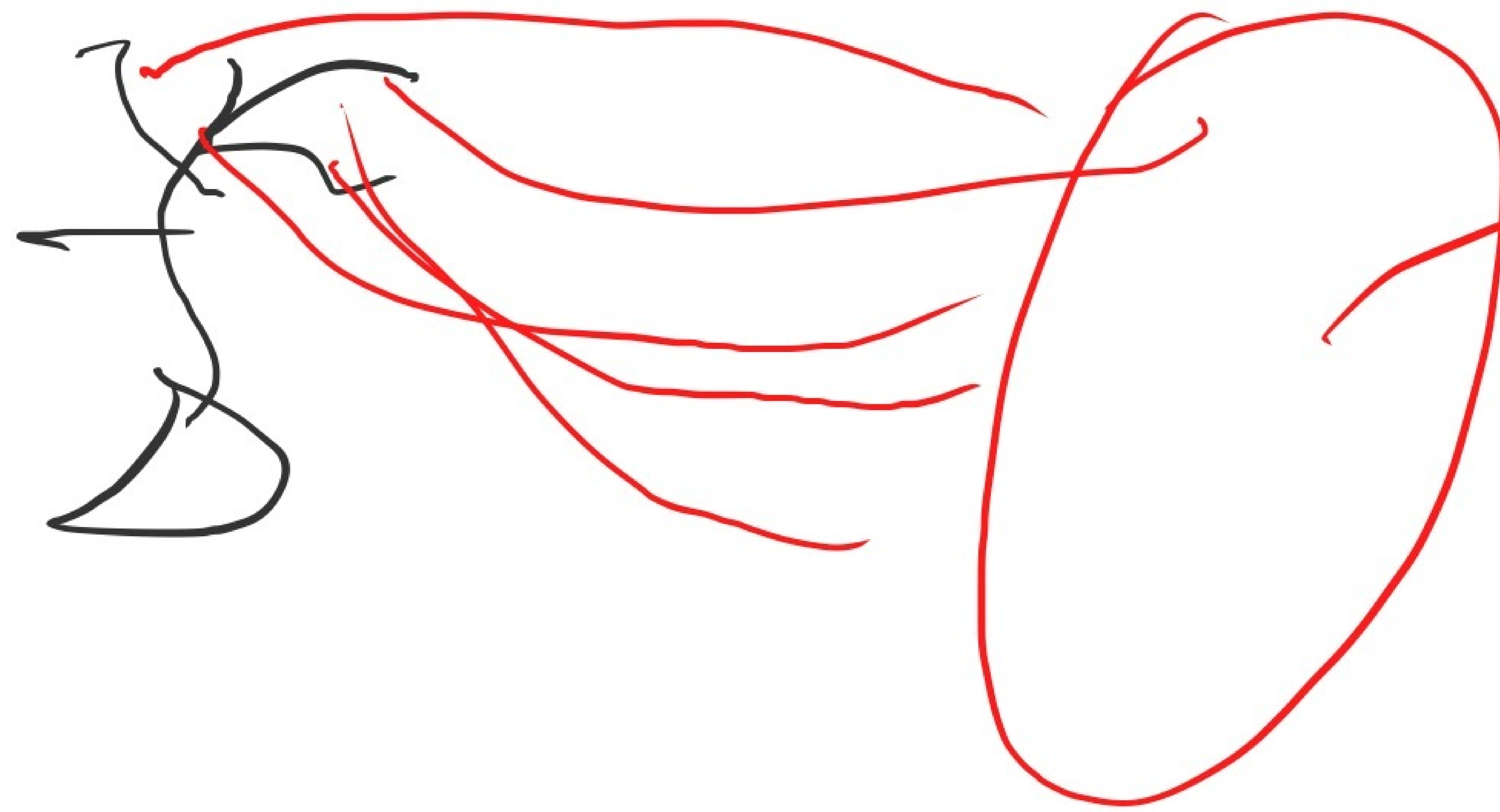
0 mV

$$\Delta w = f(\Delta t)$$

$$\dot{x} = \alpha(c)(1-x) - \beta x$$

↖ concentration of neurotransmitter

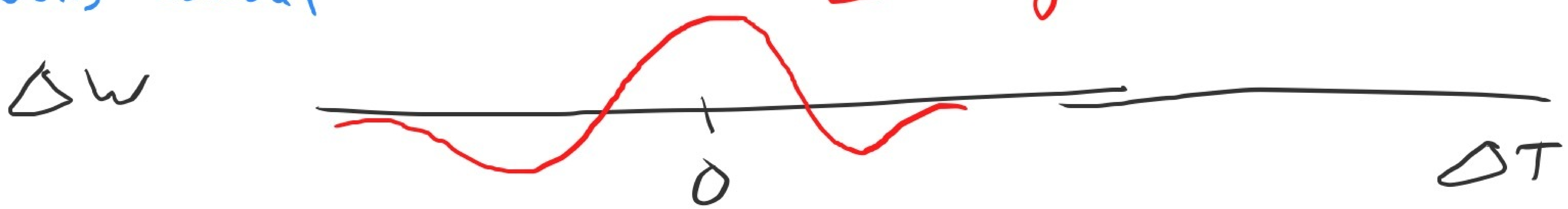




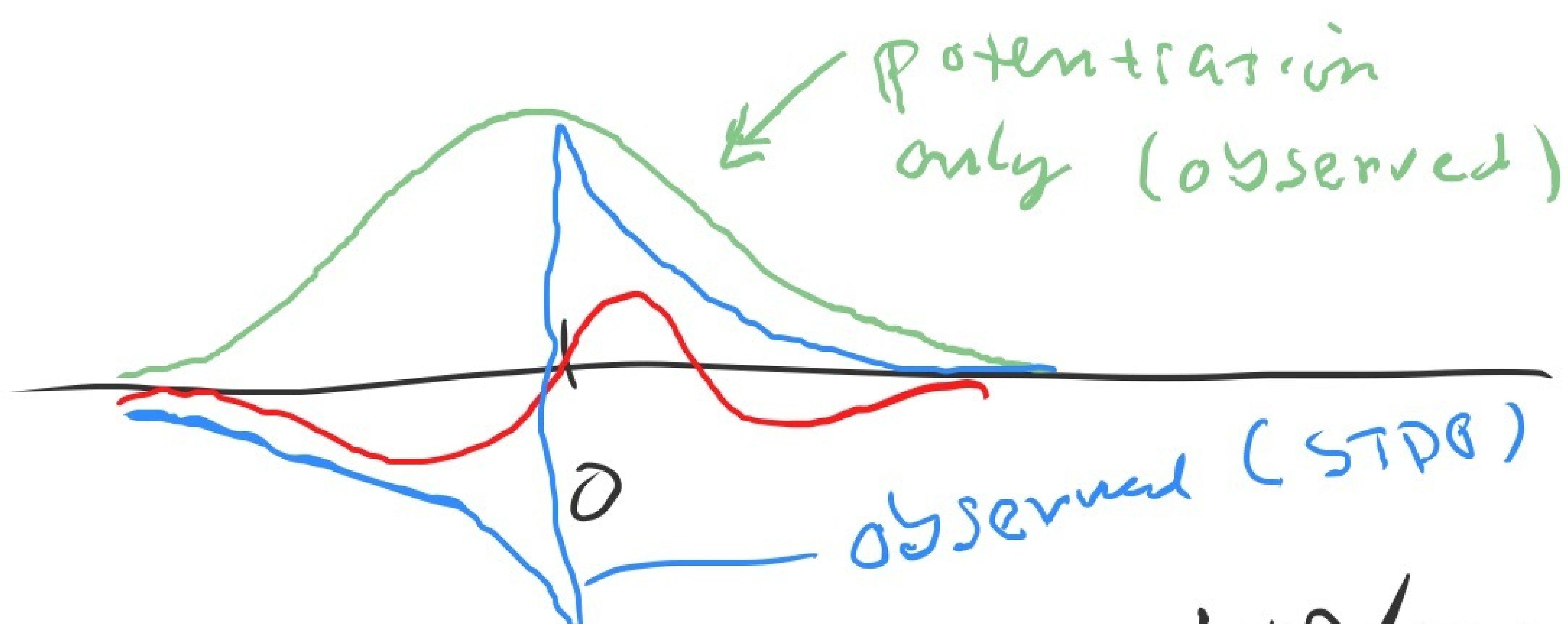
pre-neurons
fire close together,
→ dendritic spike

STDP:
spike time dependent plasticity
- looks causal

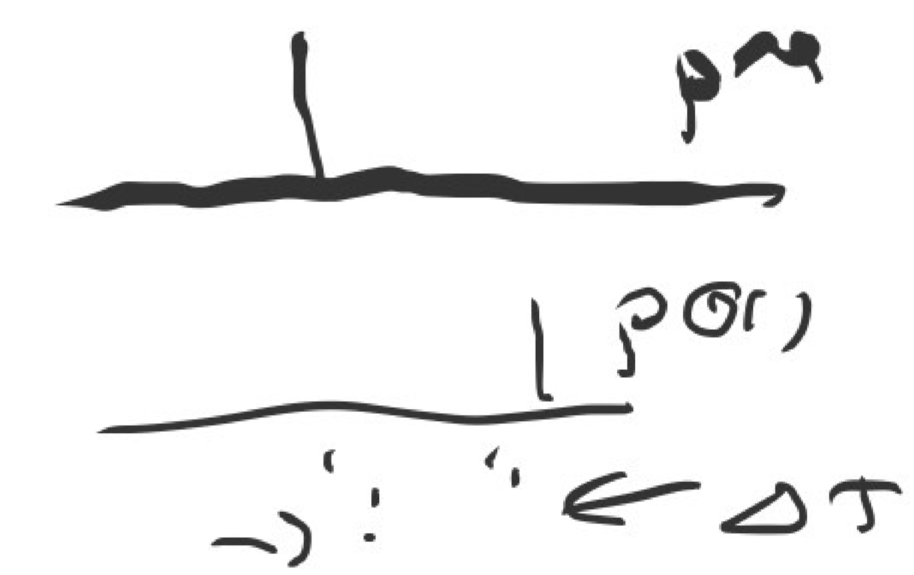
Simple analysis gives a generic shape



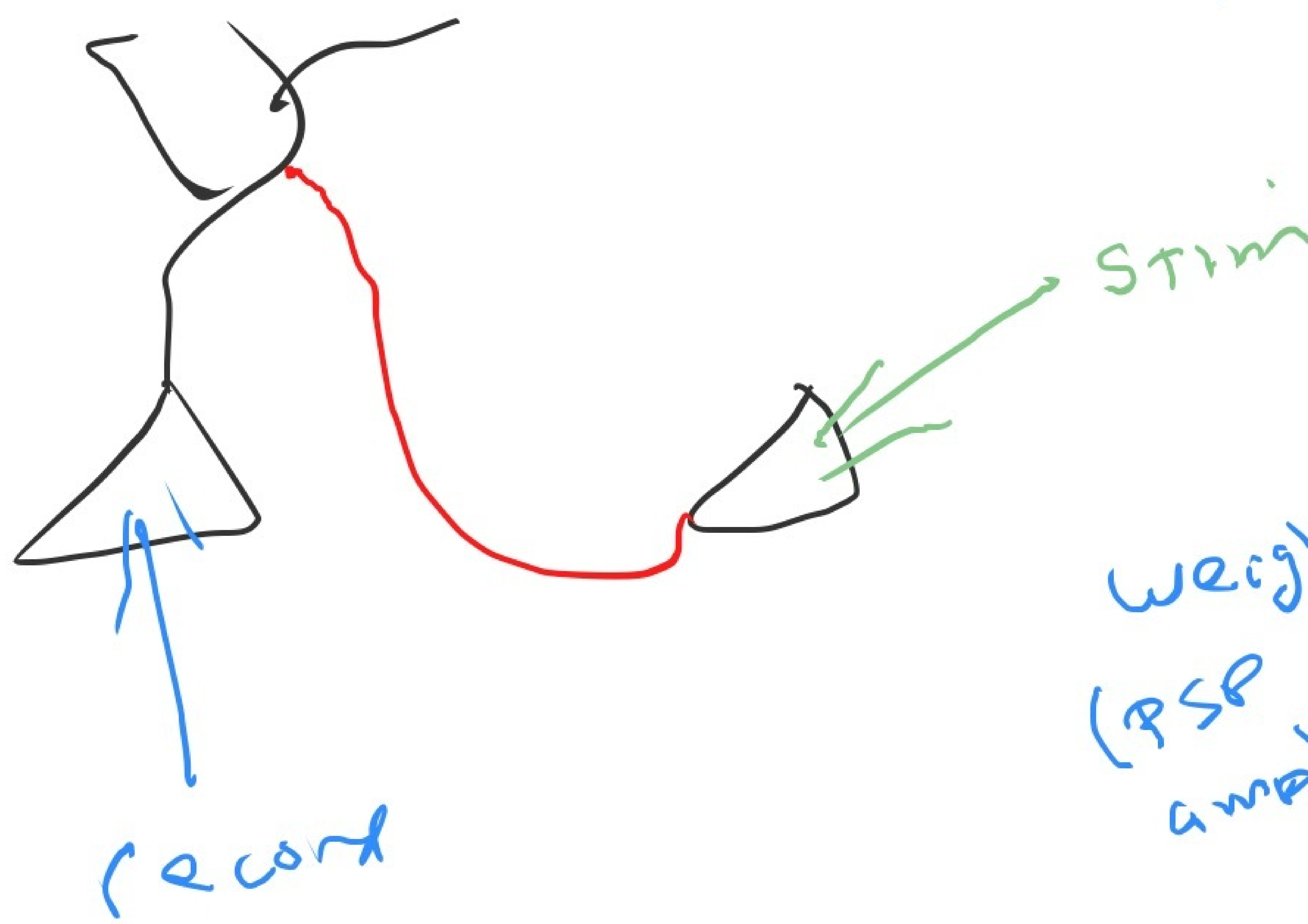
predicted

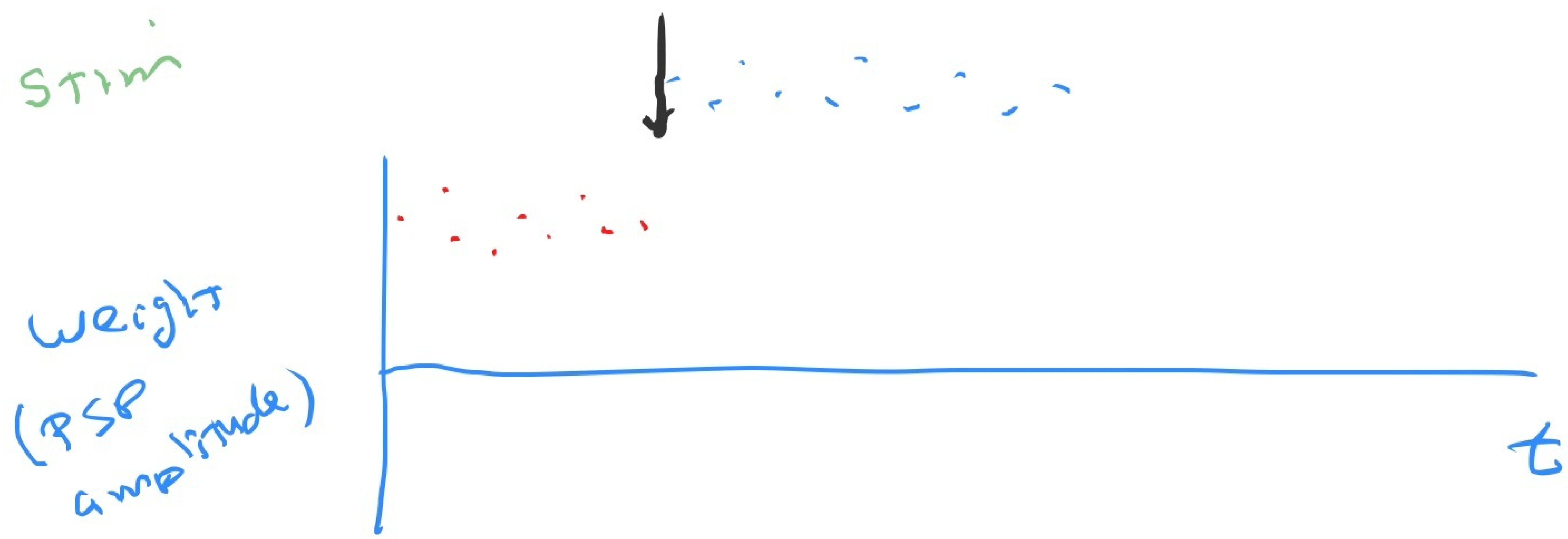


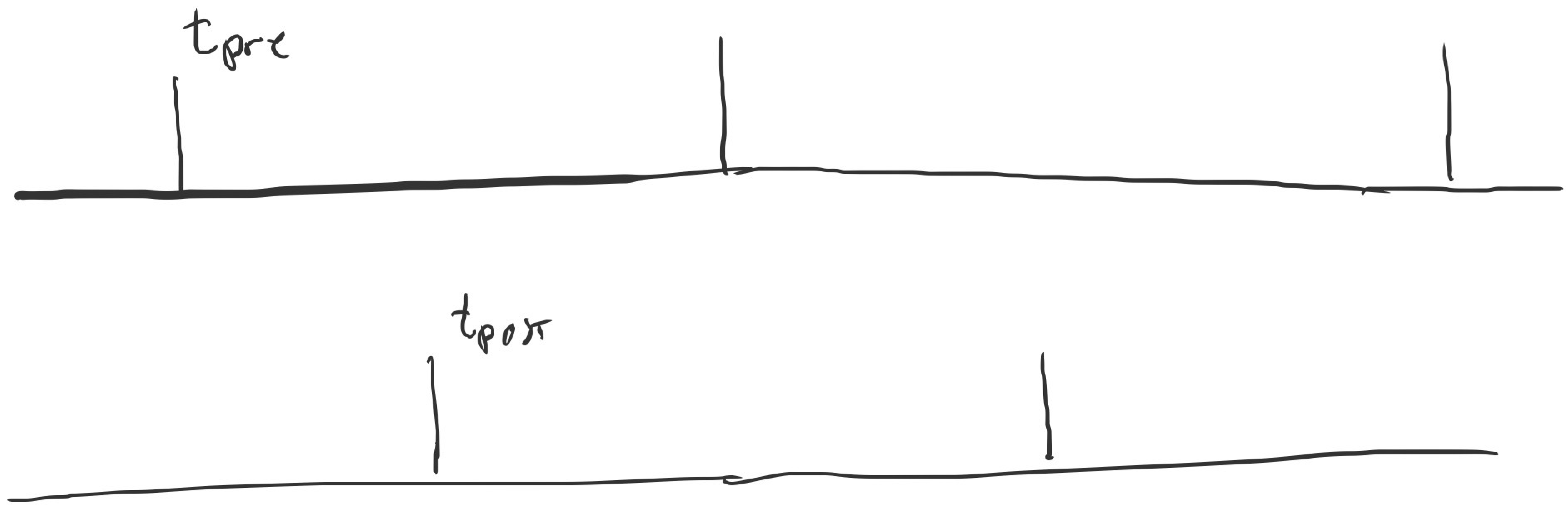
Hebb rule



LTP/LTD protocol







- want a theory that handles this

- Graupner + Brunel

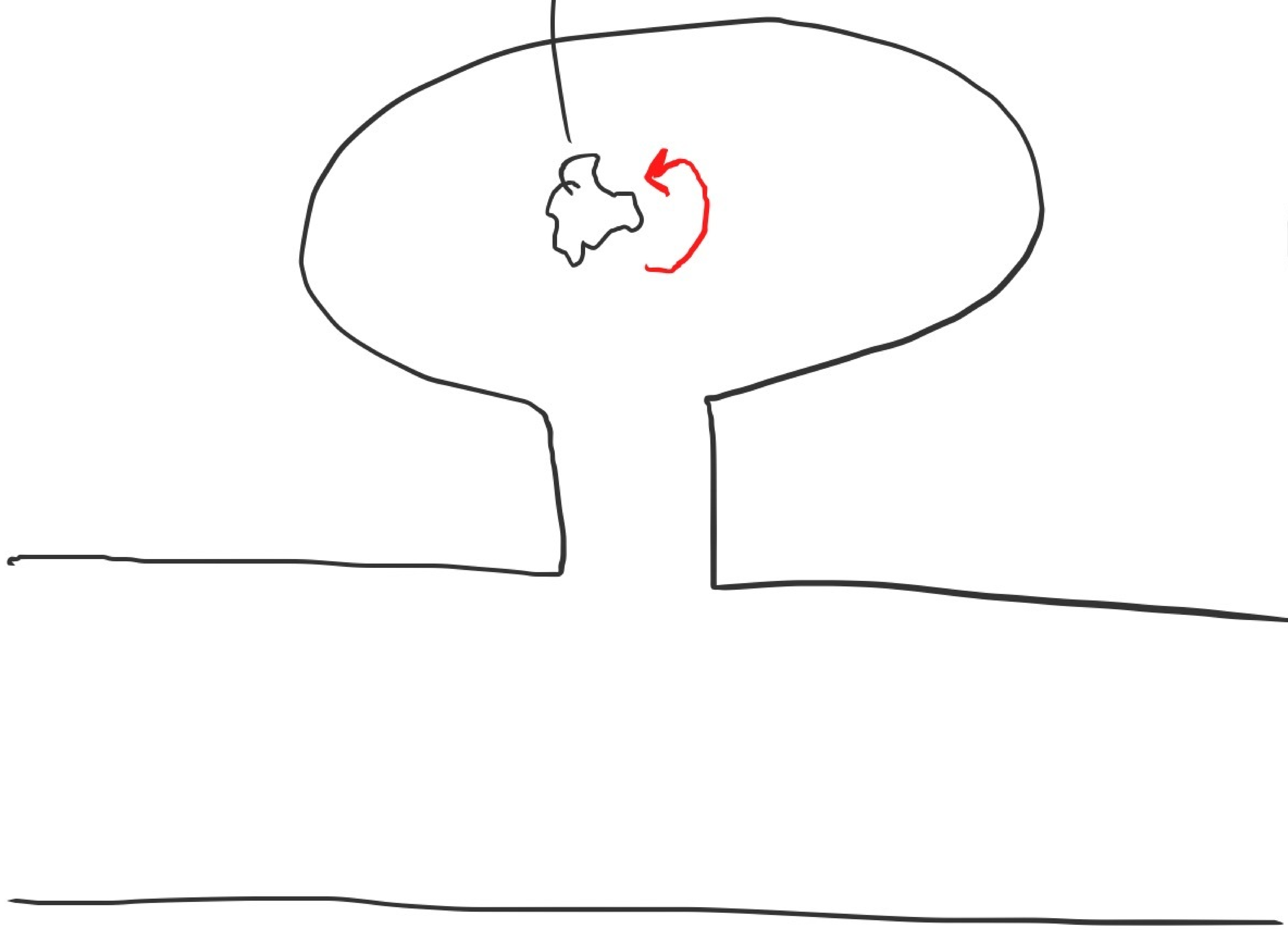
1. pre + post gave stereotyped Ca^{++} influx
2. CaMKII
3. plast. in 2-state system

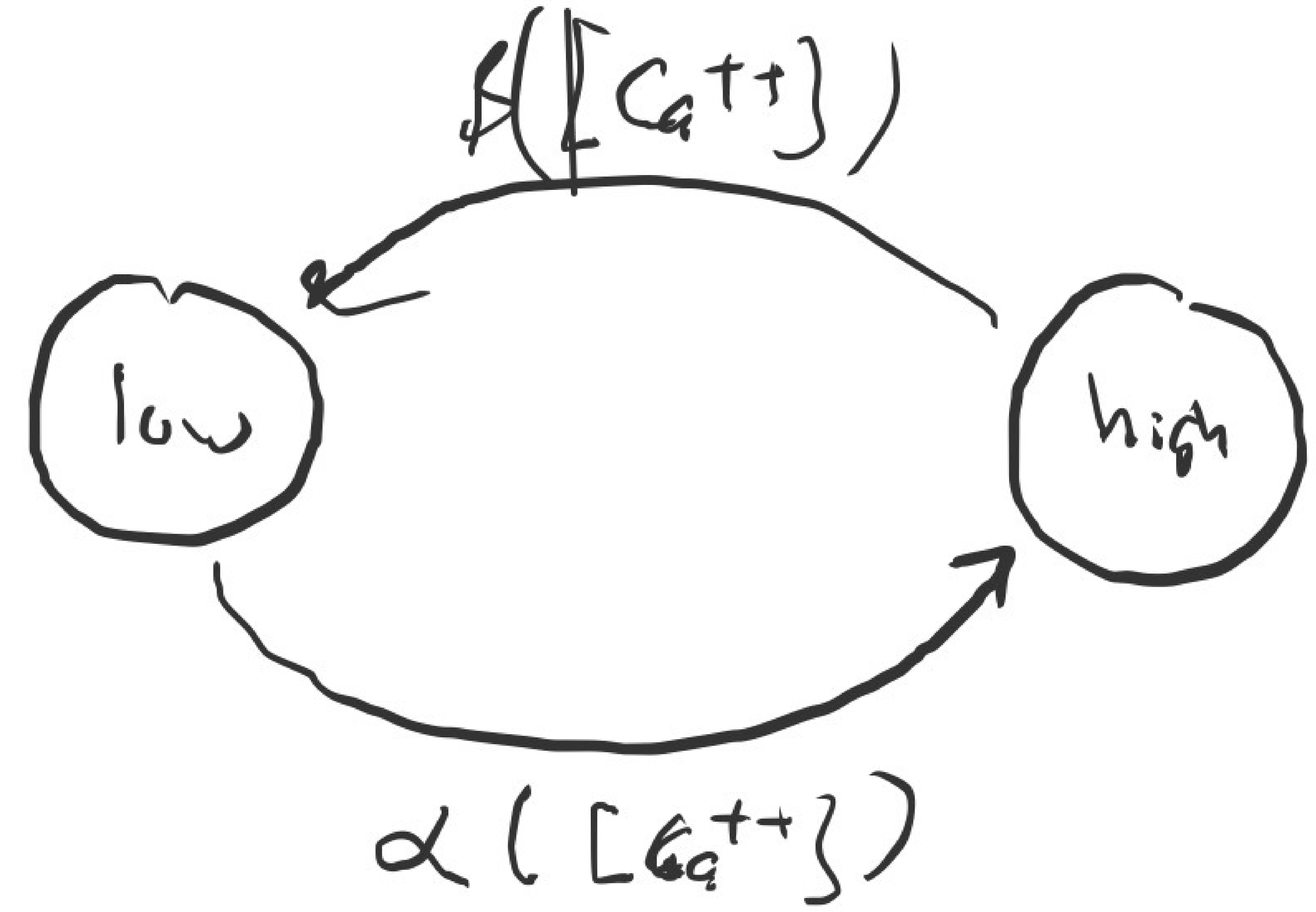
prob (low → high) $\propto$ time above yellow threshold

prob (high → low) $\propto$ time above green threshold

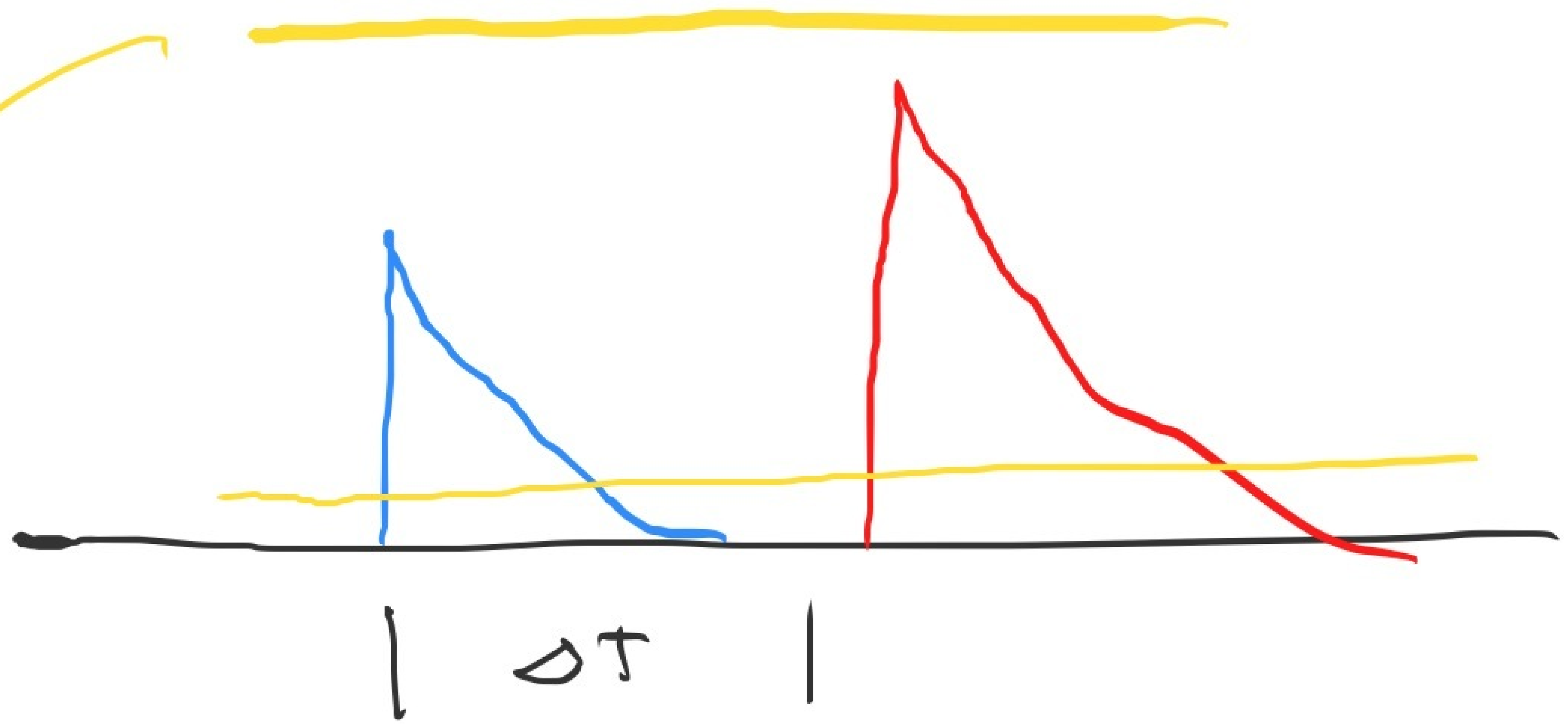
State of molecule depends on Ca^{++}

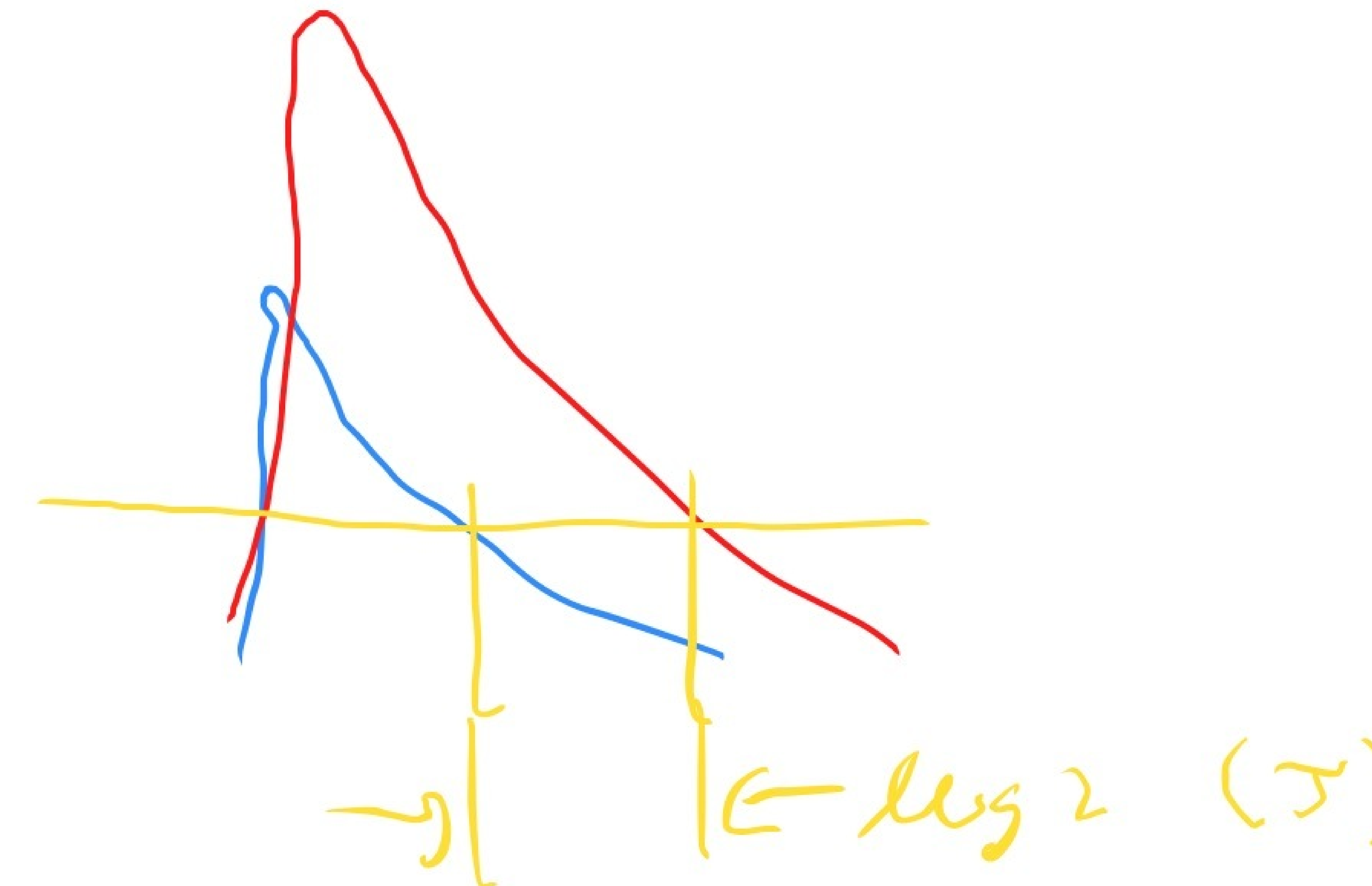
Ca^{++}/calmodulin-dependent kinase II CaMKII



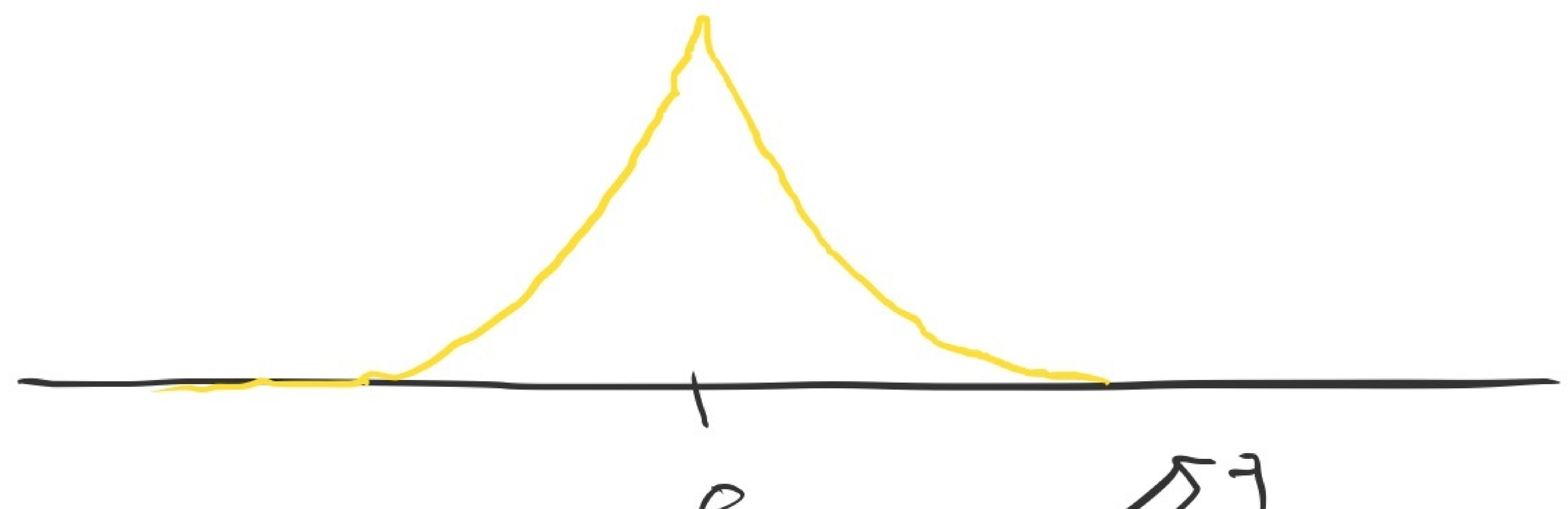


- 2-state synapse
  bistable
- very useful property

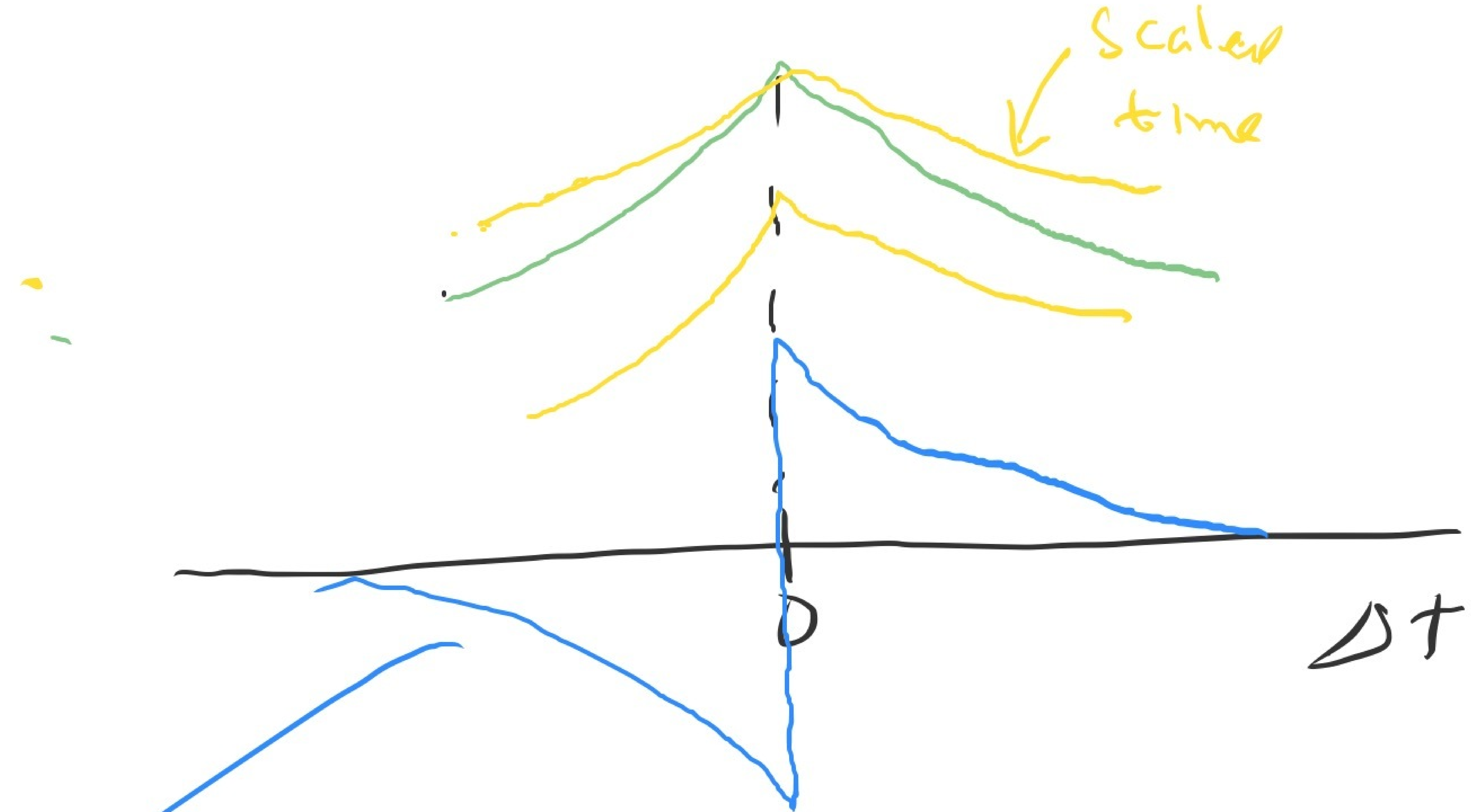




p(low→high)



p(low→high)



time spent above threshold for high→low

time above th. for low→high

$\Delta w \propto$ time above th.

can find params that give you a classic STDP curve

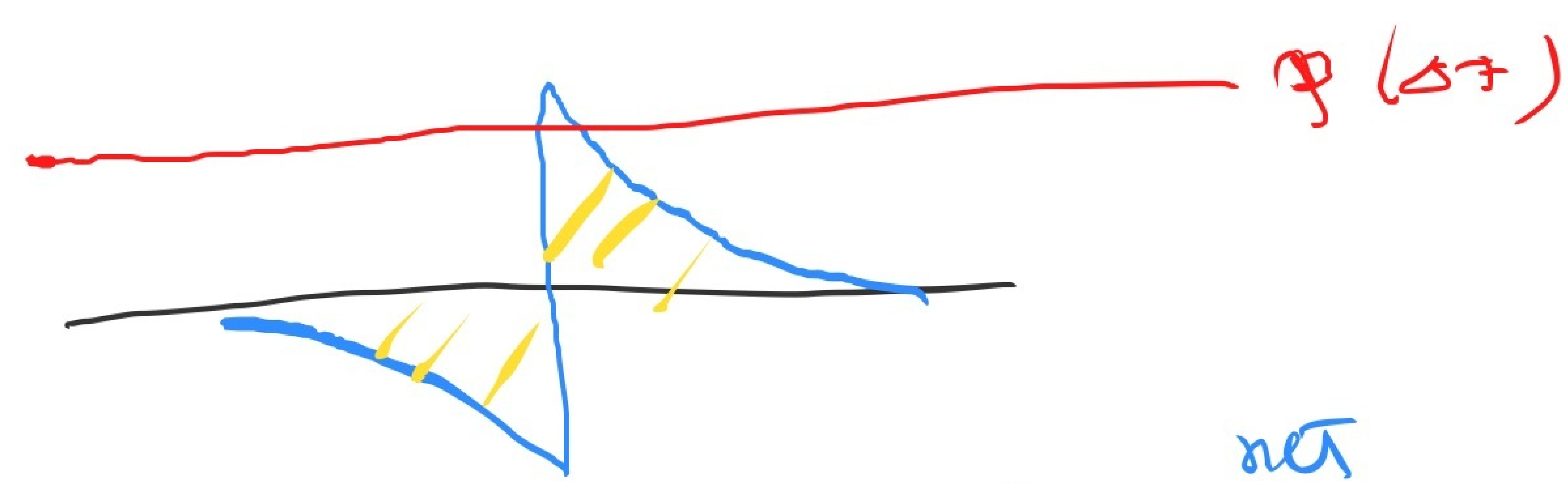
- plasticity
- unstable ✓ change learning rule
- prone to forgetting ✓ complex synapses
- learning something useful ← no error signal

$$\Delta w_{ij} = \sum_{ij} K\left(t_i^{post} - t_j^{pre}\right)$$

spike times

1. assume pre & post spike times are ~~not~~ independent

$K(\Delta t)$

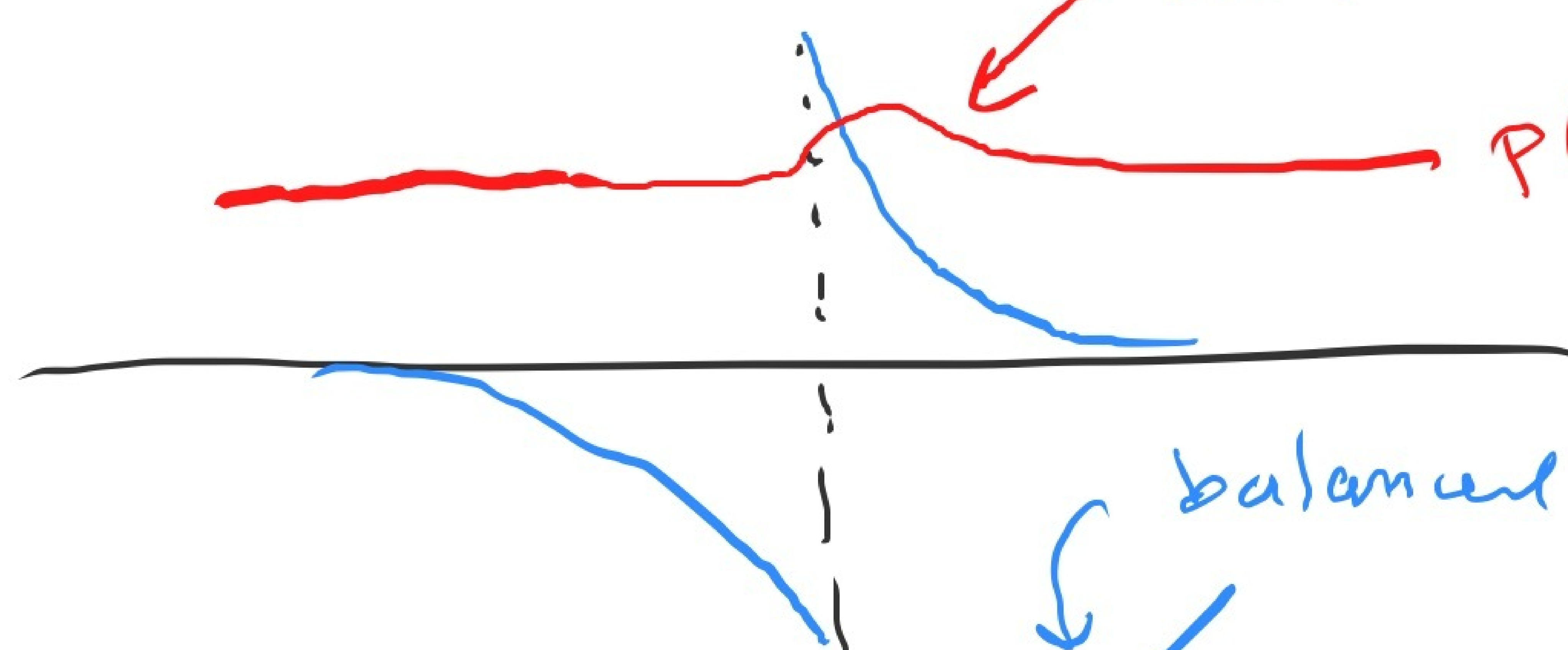


net rate of change is determined by net area under STDP curve

$$\Delta w = \sum_{ij} K(t_i^{post} - t_j^{pre})$$

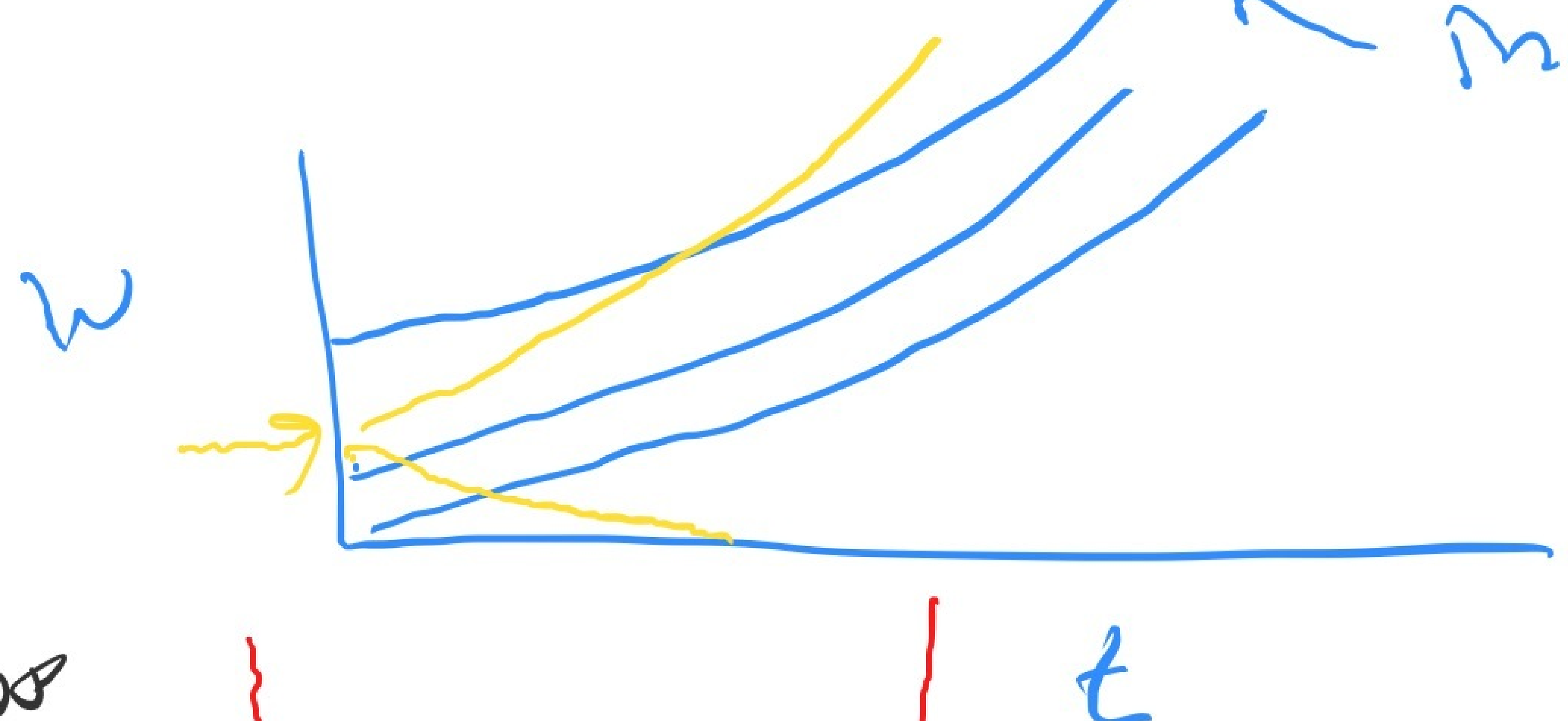
because post causes pre

height depends on weight



balanced $\Longleftrightarrow$ $\int d\Delta t \, K(\Delta t) = 0$

increase exponentially



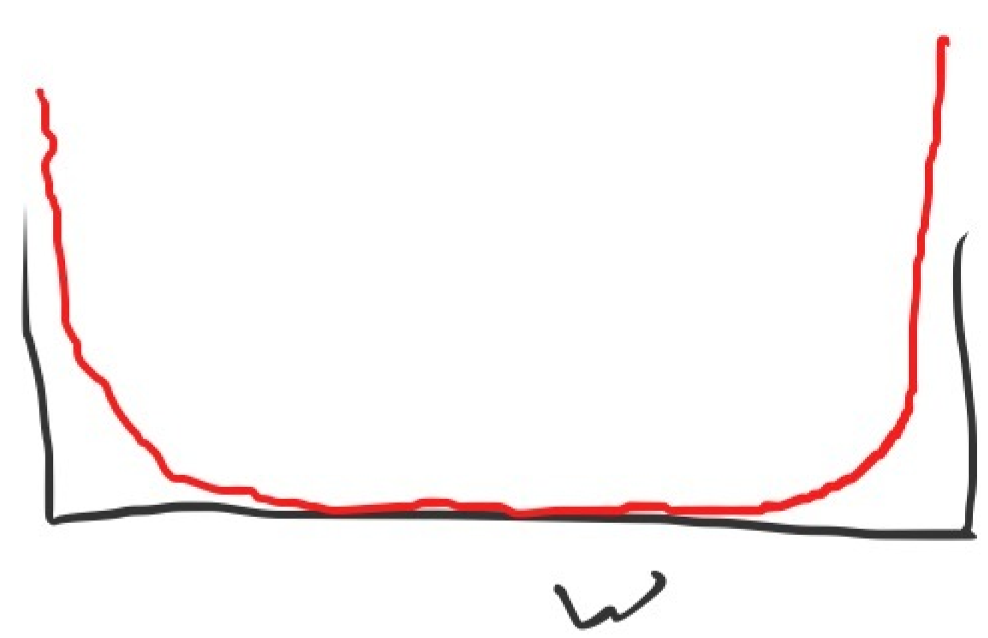
Unbalanced

$\int d\Delta t \, K(\Delta t) < 0$

$\int d\Delta t \, K(\Delta t) \, p(\Delta t | w)$

> 0 when w large

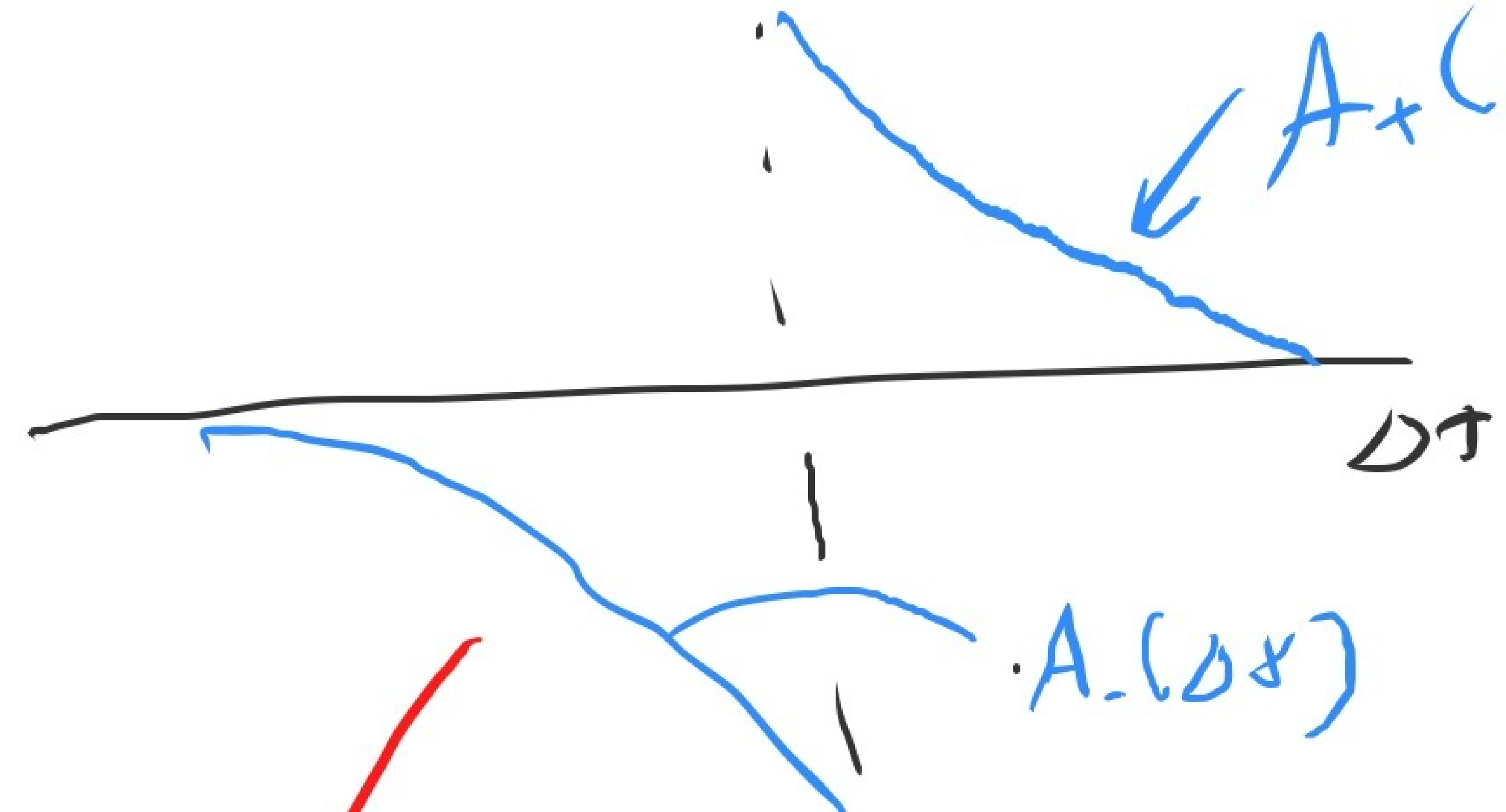
Vanilla STDP



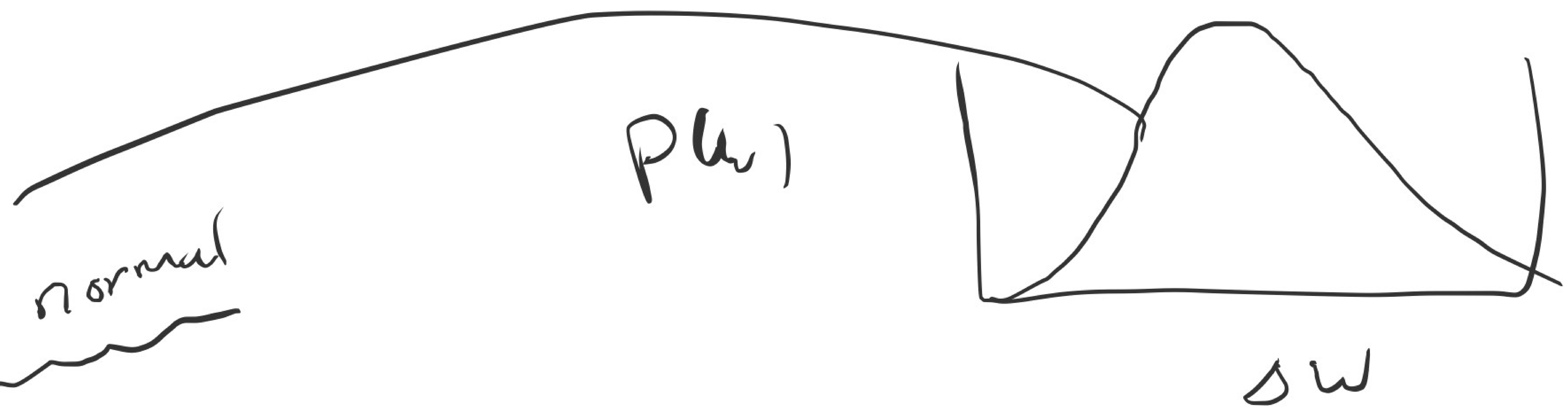
A fix (Mark van Rossum)

potentiation is const

depression is multiplicative



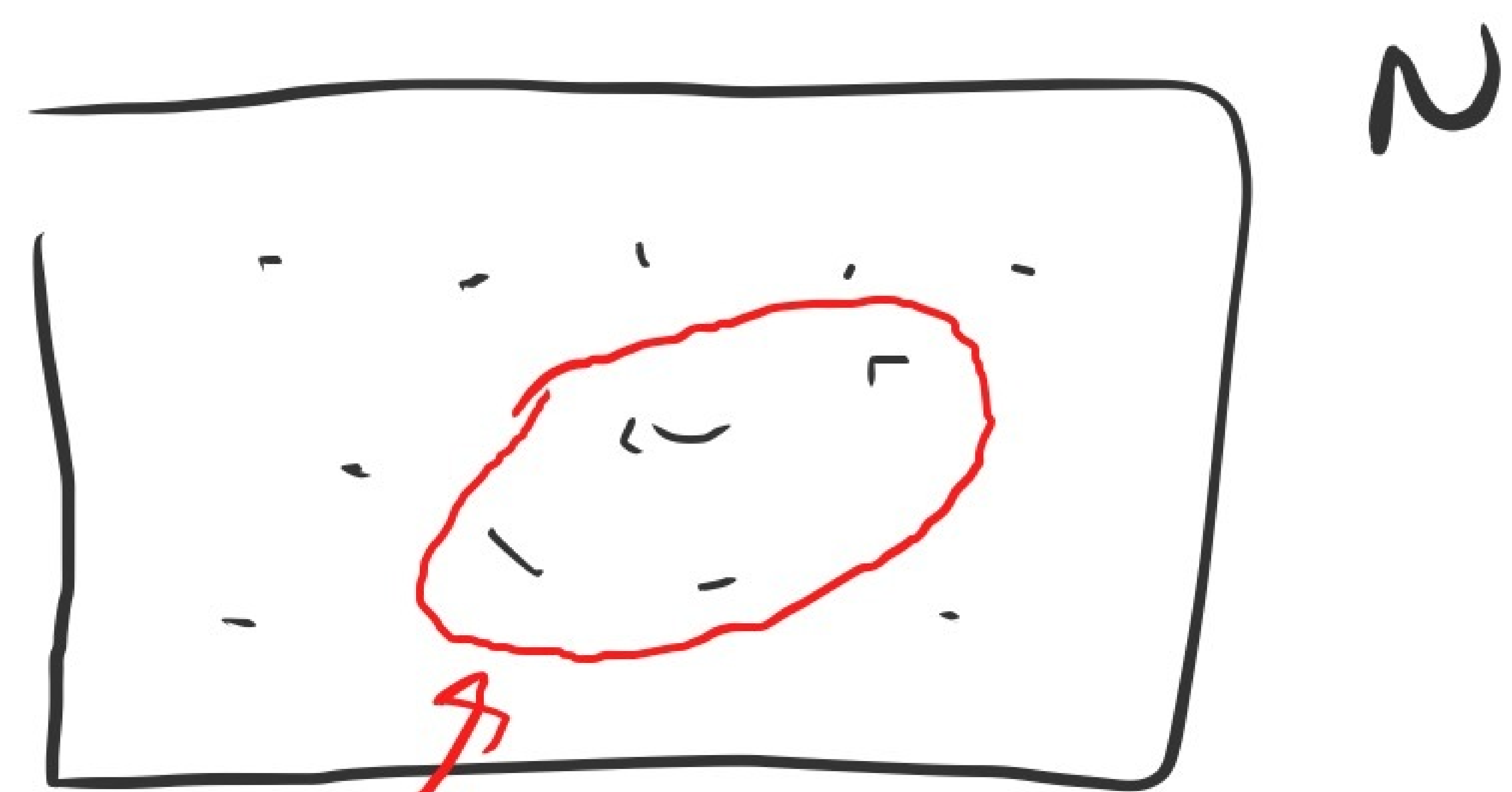
$$\Delta W = \underline{A_-(\Delta t)\cdot W} + \underline{A_+(\Delta t)}$$

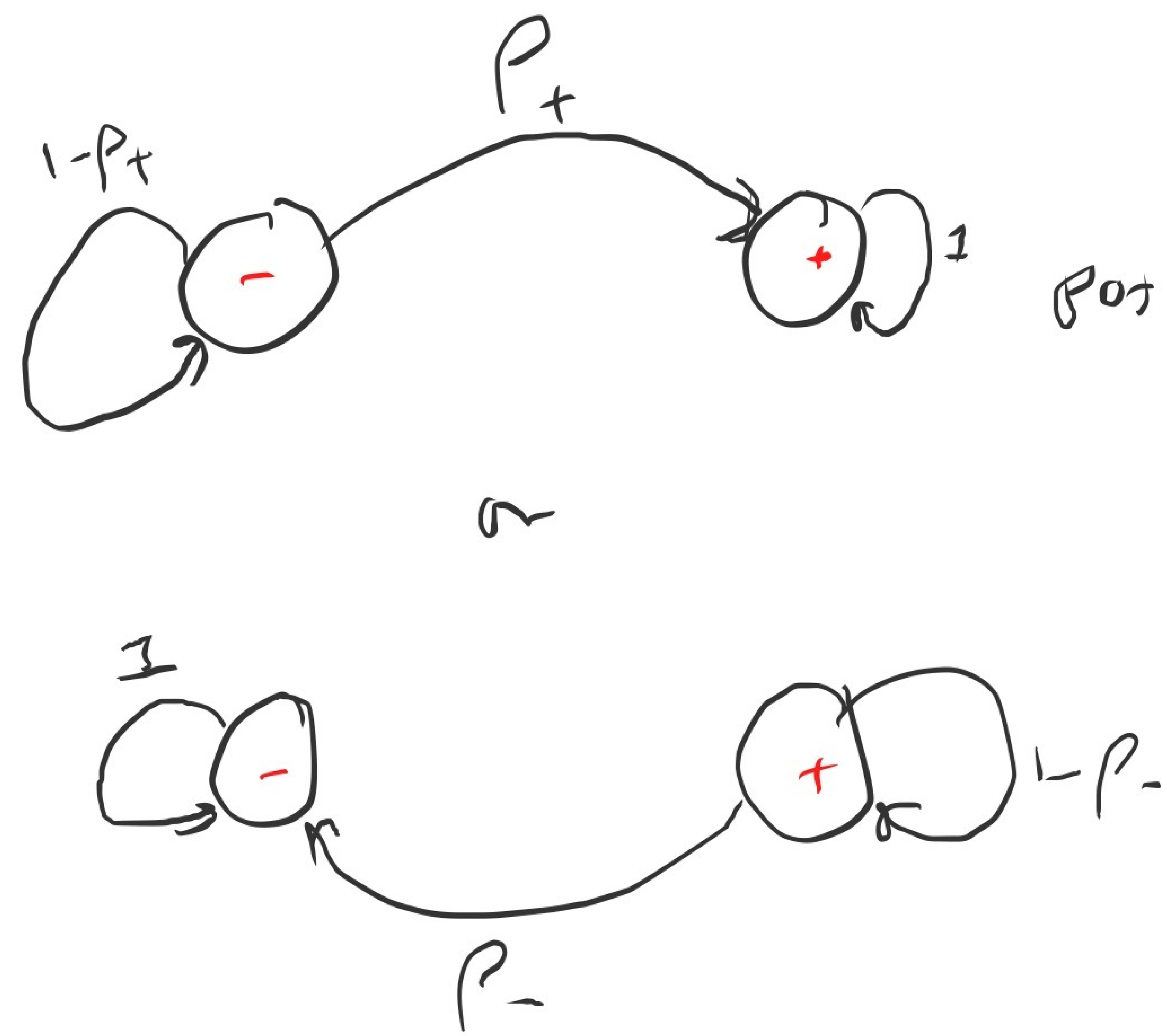


log normal

- there's an over-writing problem
- Larry Abbott + Stephano Fusi
- bag of {binary synapses} model

fast: $p_+ = p_- = 1$





- potentiation event on n synapses
- subsequent events are random
- how long before we forget about our potentiating event?

] faster learning
=> faster forgetting

want: large ρ for storing a memory
small ρ to avoid forgetting

$\frac{S}{N}$ balanced case: $\frac{n}{2}$ $+$

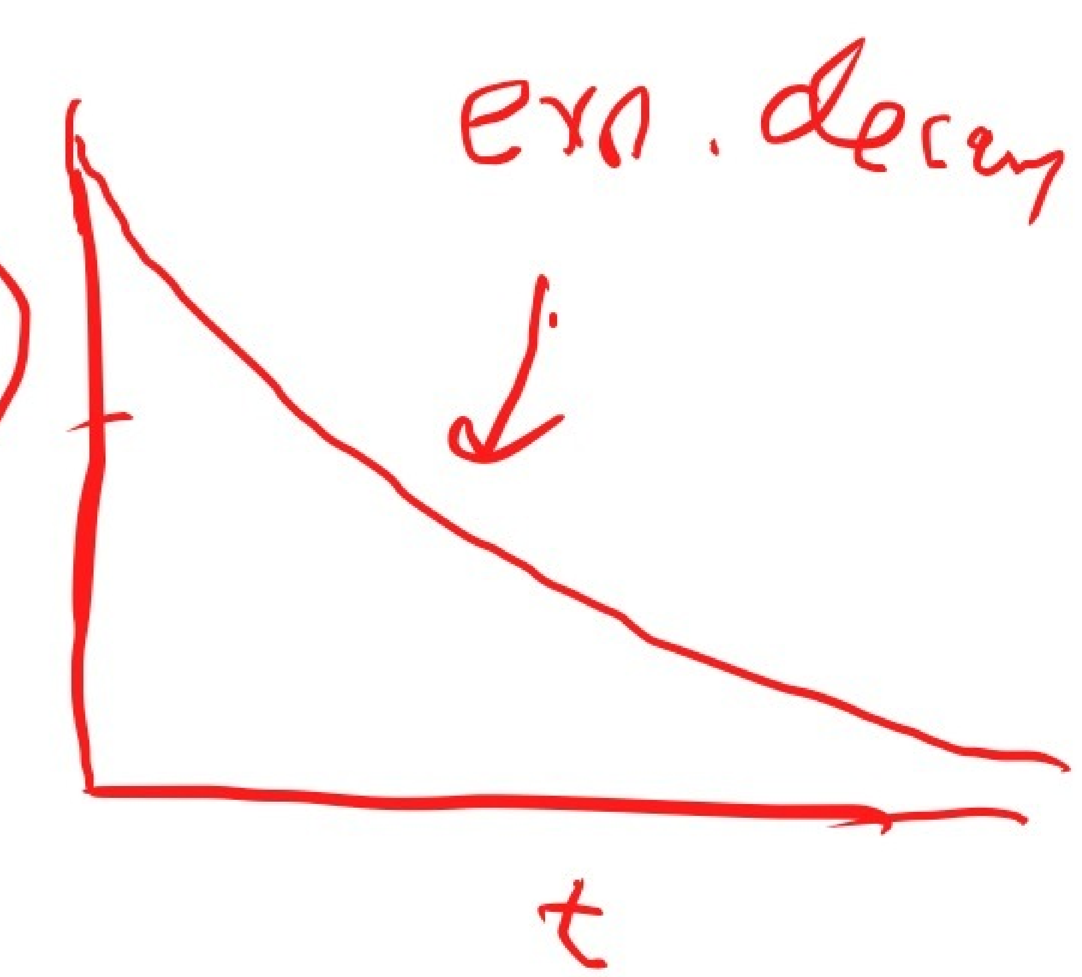
$\frac{n}{2}$ $-$

after potentiation $n\left(\frac{1+\rho}{2}\right)$ $+$

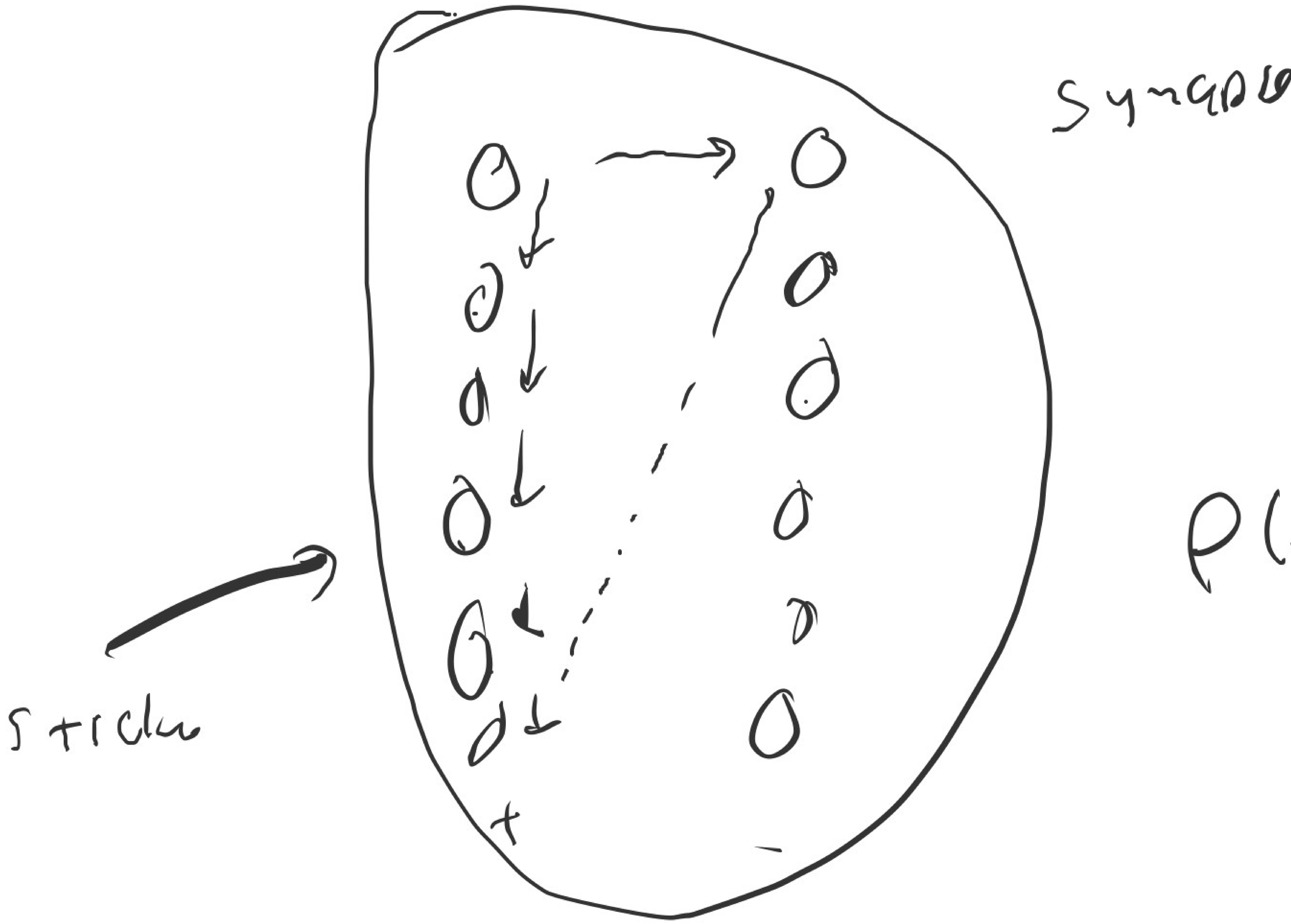
$n\left(\frac{1-\rho}{2}\right)$ $-$

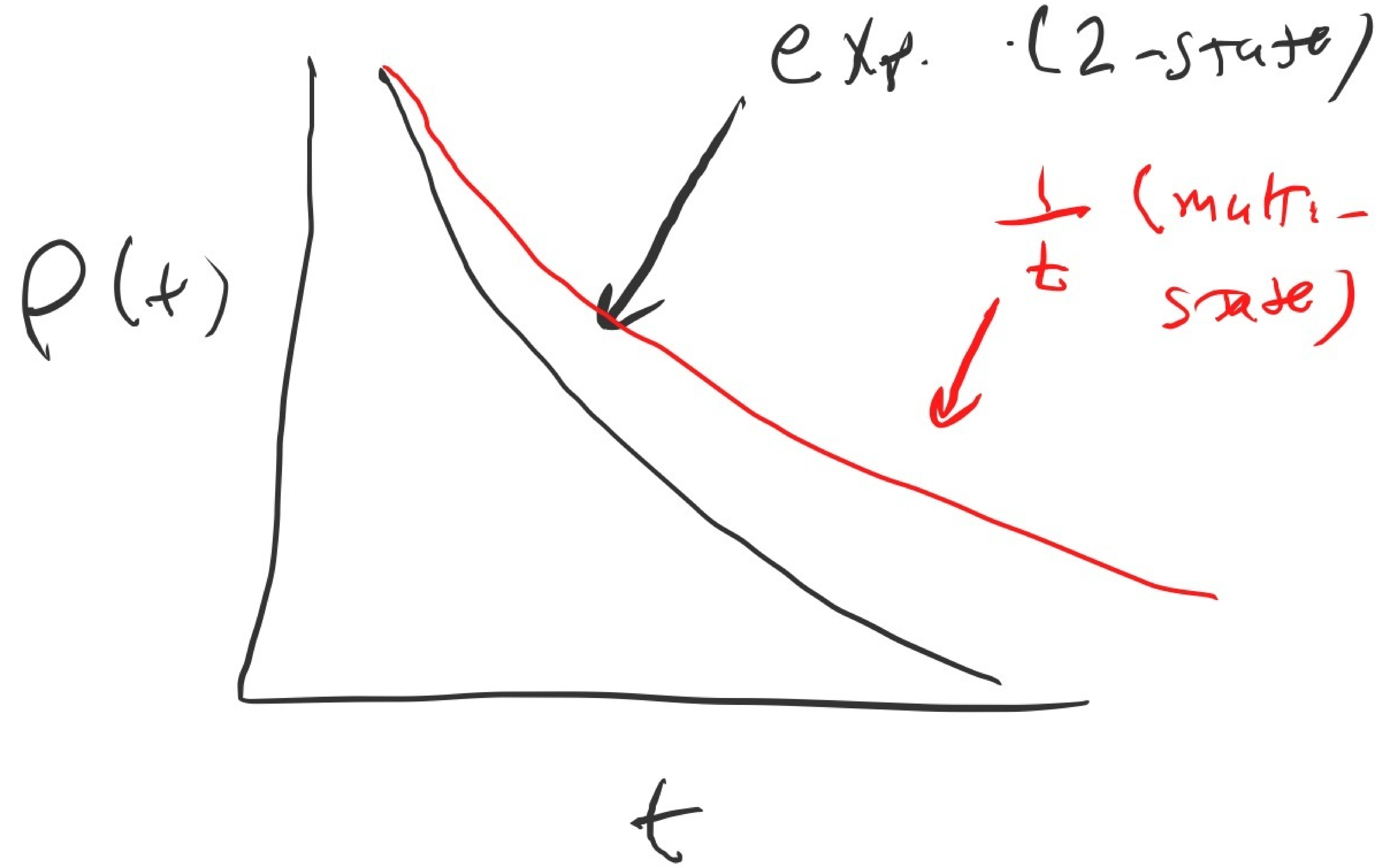
Signal $= S = \frac{n}{2}(1+\rho^{(t)}) - \frac{n}{2} = \frac{n\rho^{(t)}}{2}$ $\left(p(t) - \frac{1}{2}\right)$

Noise $= \sqrt{\frac{n}{4}}$ $\qquad \frac{S}{N} = \frac{\rho(t)}{\sqrt{n}}$



$\left(p(t)^{(t)} - \frac{1}{2}\right) =_{t=0} \underline{(1-2\rho)^t}$ $\quad \rho \sim \frac{1}{\sqrt{n}}$


synapse


exp. (2-state)
1/t (multi-state)
ρ(t)
t

$$\nu_i = f\left(\sum_j w_{ij} \nu_j\right) \qquad h_i \equiv \sum_j w_{ij} \nu_j$$

minimize loss

$$\frac{\partial \mathcal{L}}{\partial w_{ij}} = \frac{\partial \mathcal{L}}{\partial h_i} \nu_j \qquad \Delta w_{ij} = -\eta \frac{\partial \mathcal{L}}{\partial w_{ij}}$$

$$\frac{\partial \mathcal{L}}{\partial h_i} \frac{\partial h_i}{\partial w_{ij}}$$

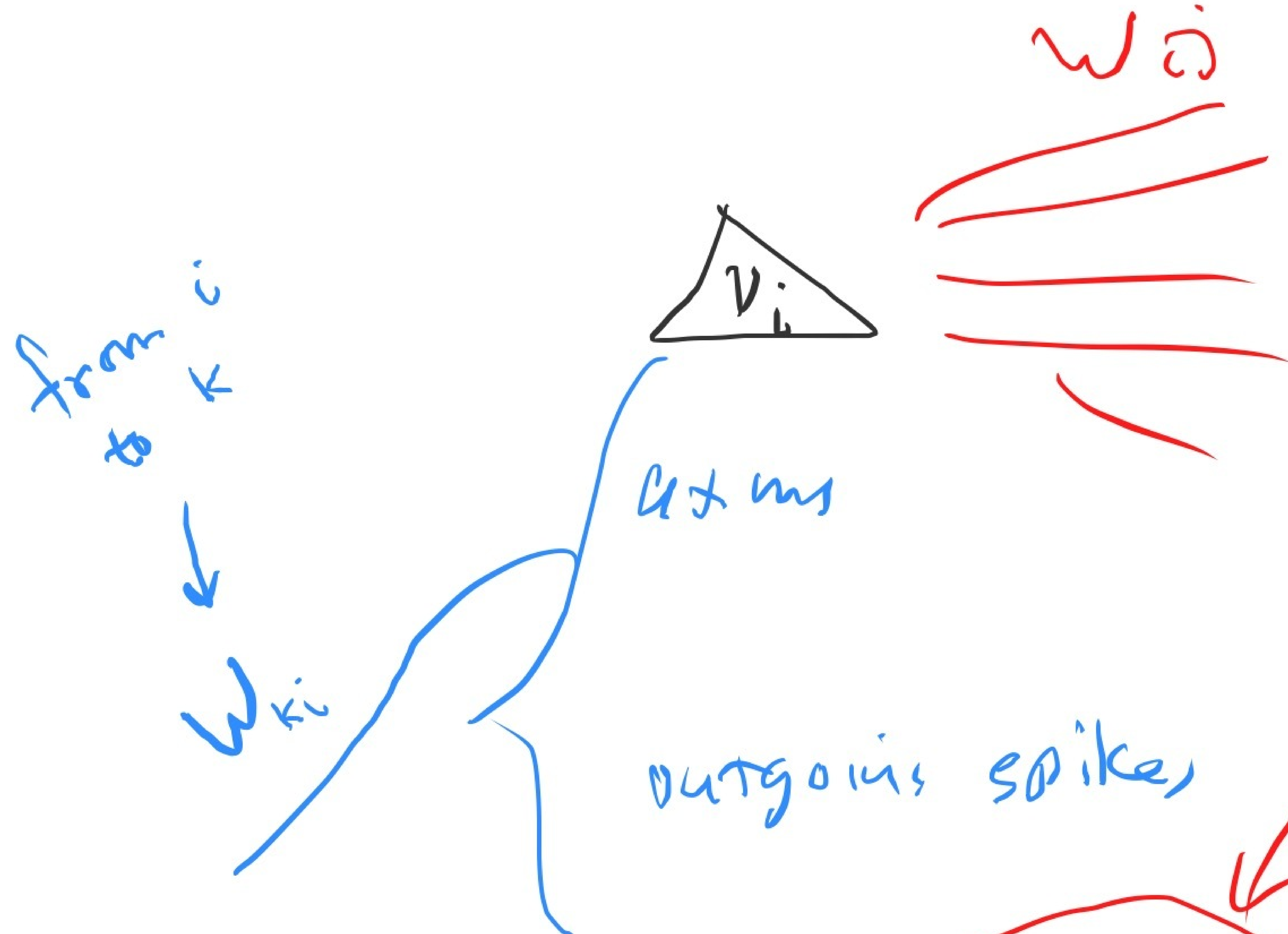
post-synaptic term

pre-synaptic – backprop

$$\frac{\partial \mathcal{L}}{\partial h_i} = \underline{\underline{\frac{\partial \mathcal{L}}{\partial \nu_i}}} \; \underbrace{f'(h_i)}_{\text{pre}}$$

FF : update rule is backprop

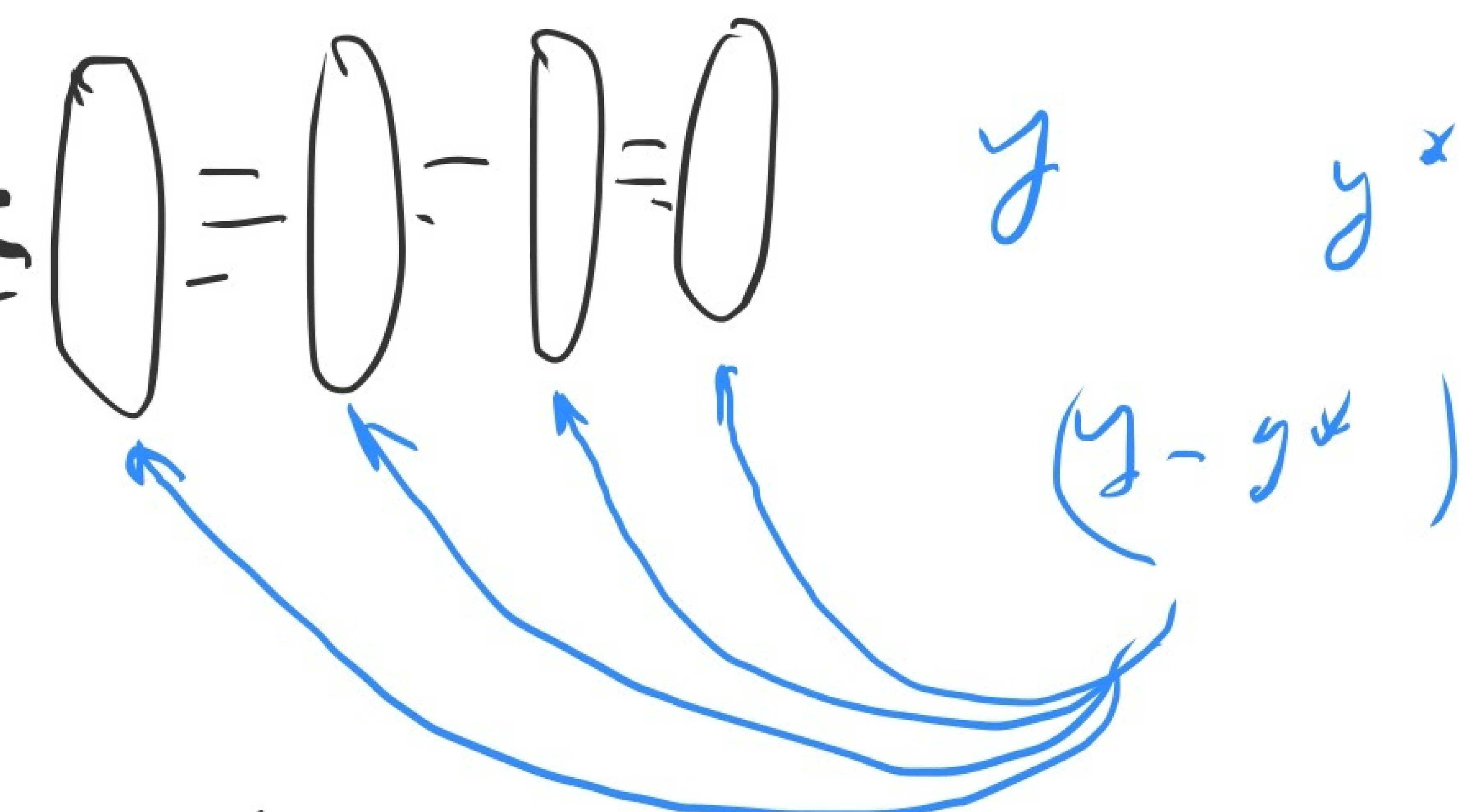
RNN : " " " " through time



$$\frac{\partial \mathcal{L}}{\partial \nu_i}$$

no easy way for the brain to compute this

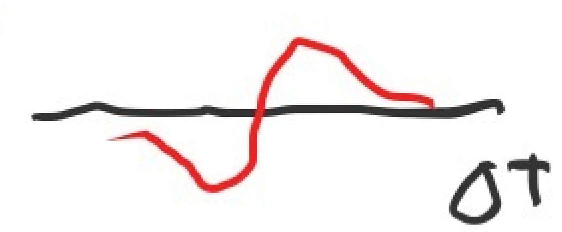
best characterized experimentally



pre

post

$$\Delta W_{ij} = f_3(y-y^*)\, g_i\, \nu_j$$

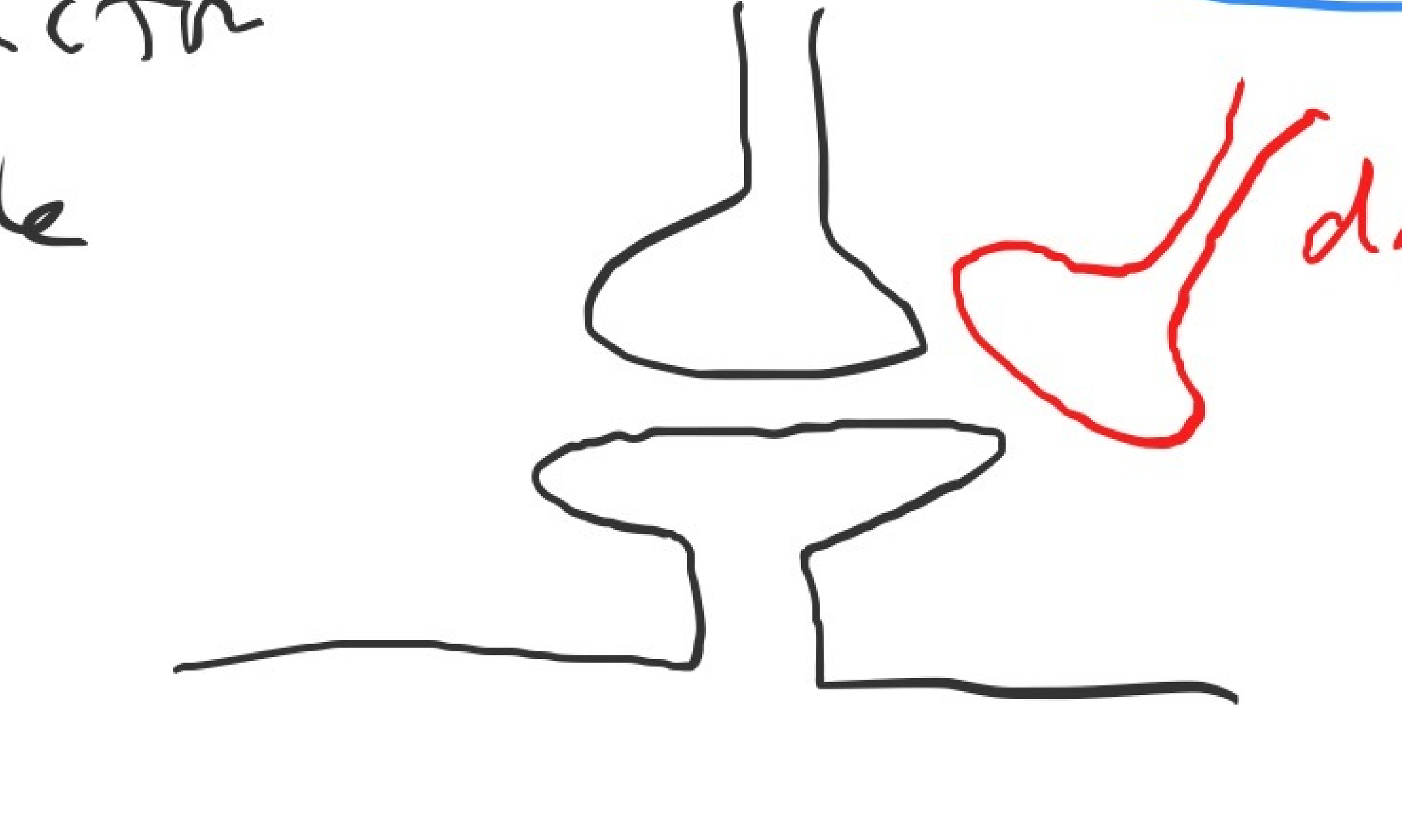


third - factor

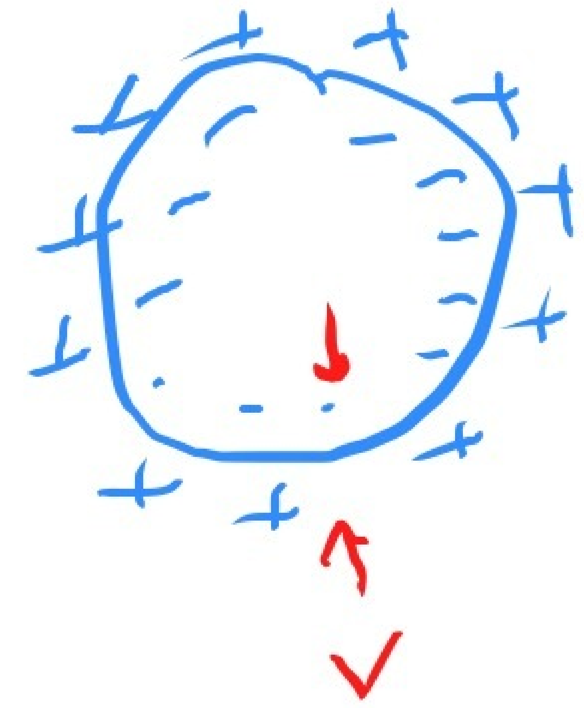
speculative

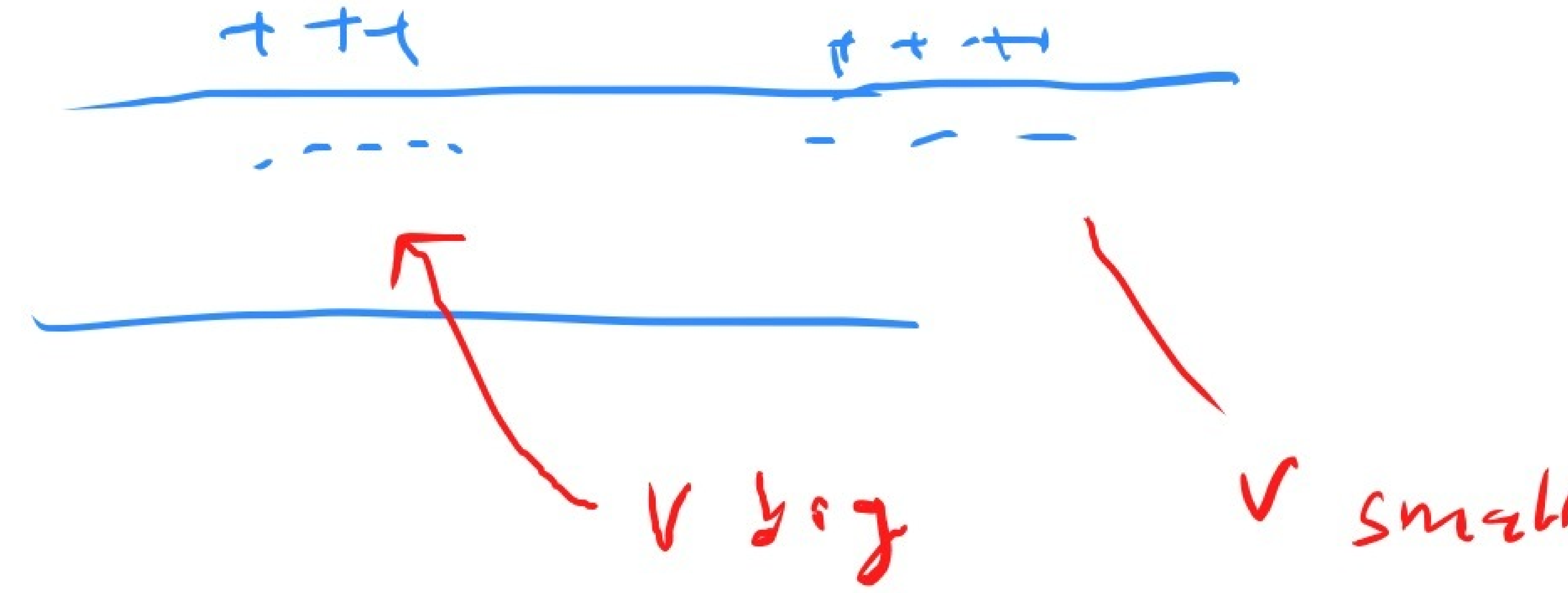
3 - factor Hebb rule

exp. evidence



$Q = CV$





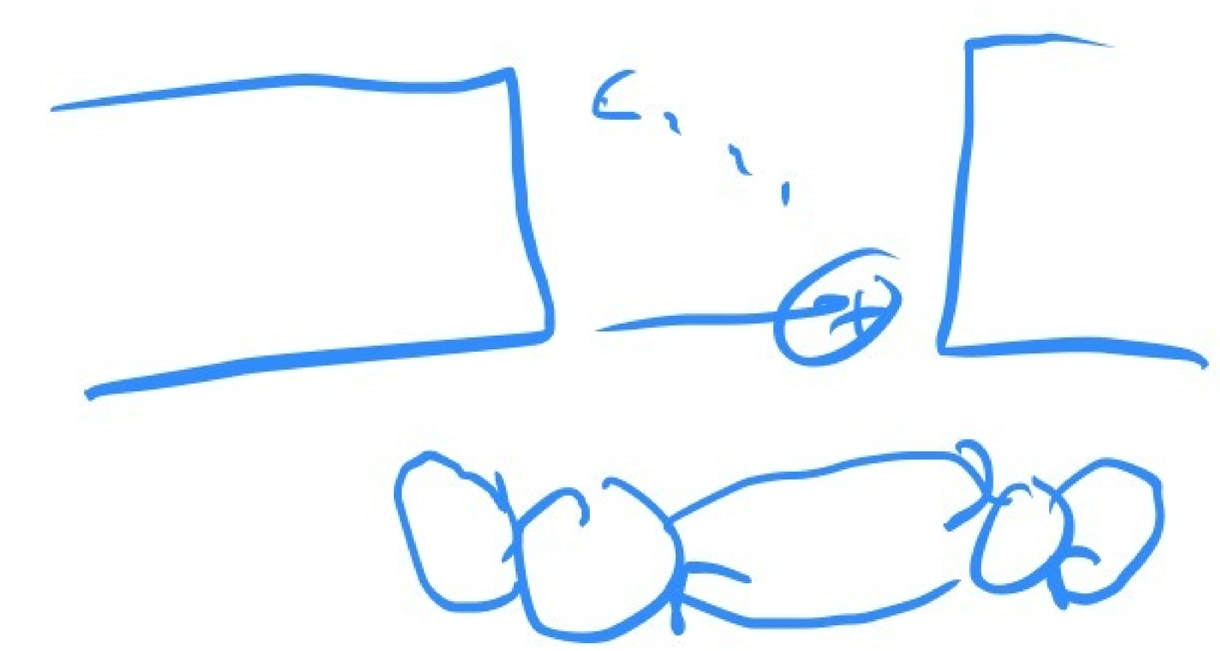
$$\frac{dQ}{dt} = I = C\frac{dV}{dt}$$

$$I = -\sum_x g_x (V - E_x)$$

reversal potential from ion pumps

conductance is ion specific

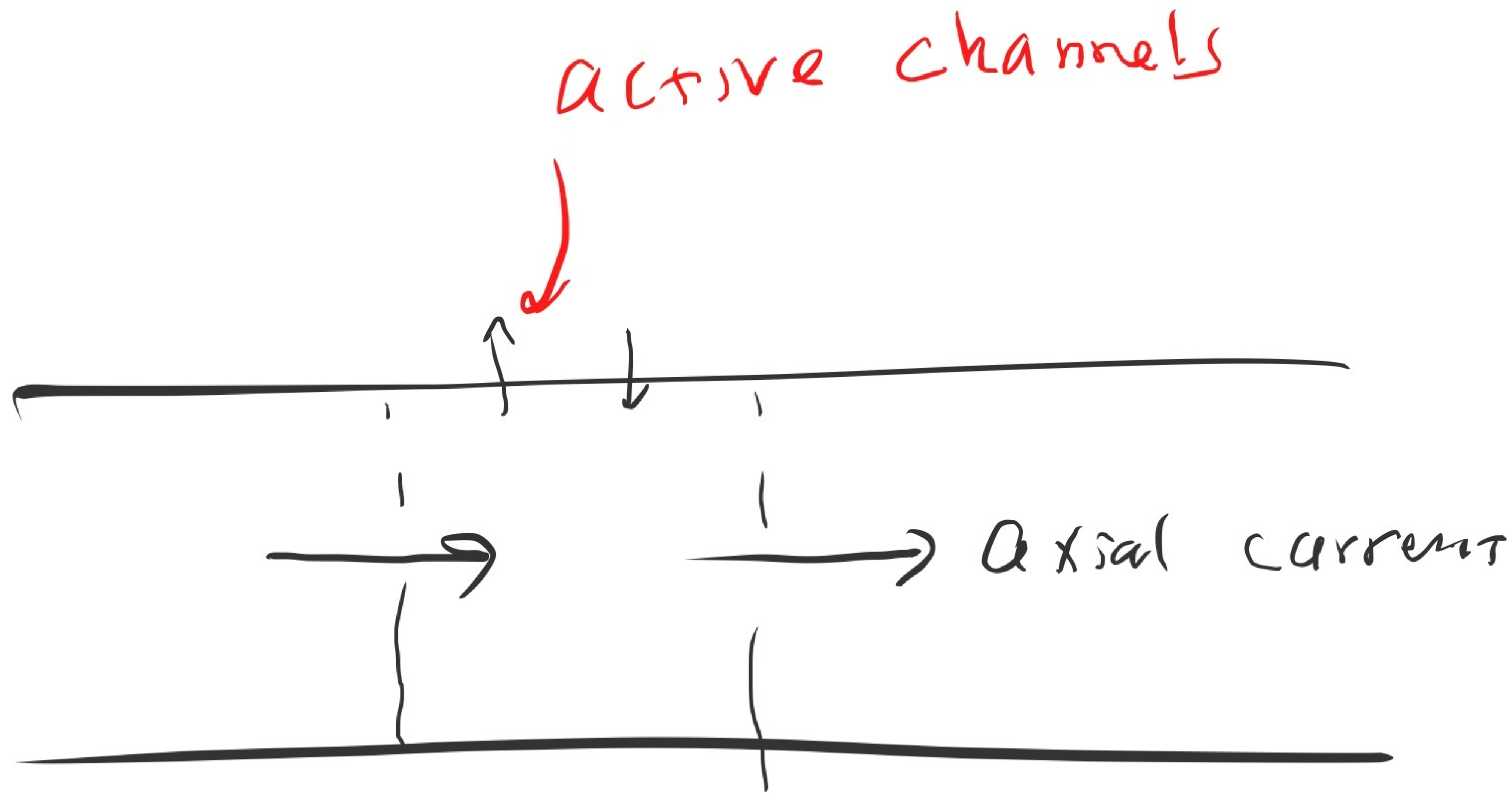
g_x : constant (leak)
voltage-dependent (HH)
concentration-dependent (synapses)



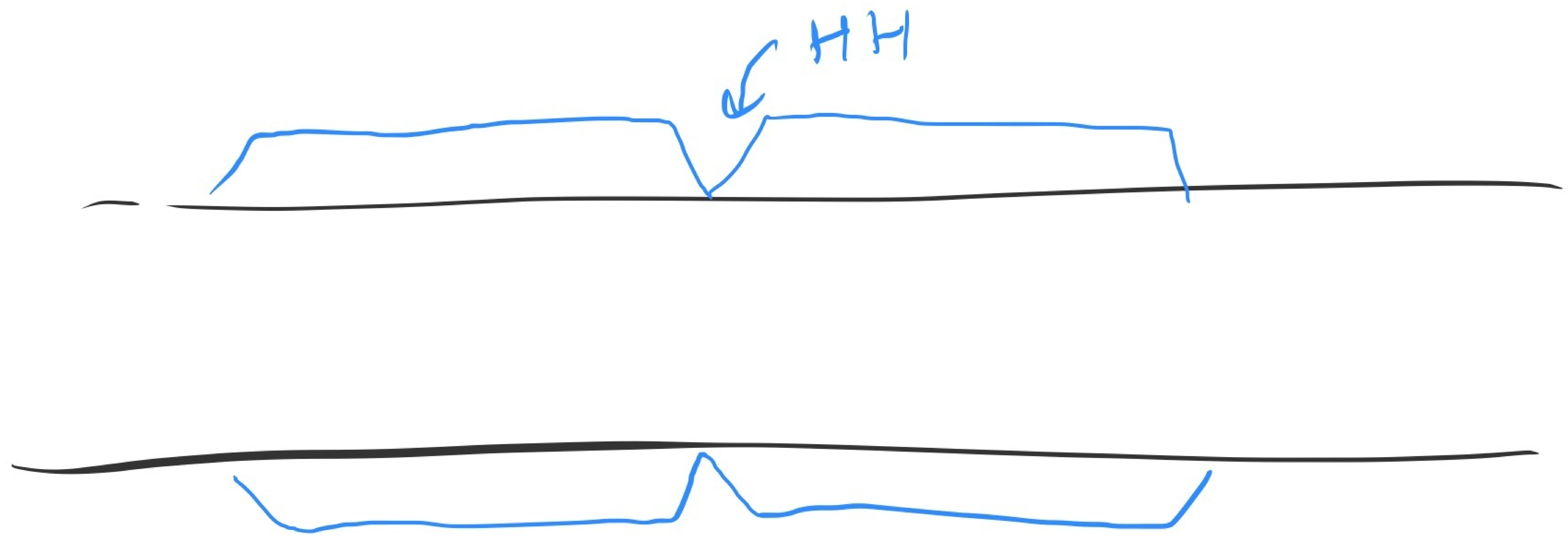
current flow causes biochemical changes

$[Ca^{++}]$


Ca++
AMPA
NMDA
CaMKII
Ca++
long term changes in weight
different models can give just about anything
ΔT



dendrite



axon
(active cable)