$$S_i(t+1) = \text{sign}\left[\frac{1}{n}\sum_{j=1}^{n}\sum_{\mu=1}^{p}\xi_i^\mu\,\xi_j^\mu\,S_j(t)\right]$$

m_μ

n variables

$$\xi_i^\mu = \begin{cases} 1 & p = 1/2 \\ -1 & p = 1/2 \end{cases}$$

$$m_\mu(t) = \frac{1}{n}\sum_{j=1}^{n}\xi_j^\mu\,S_j(t)$$

guess: $S_j(t) = \xi_j^\mu$

$m_\mu = 1$

$m_{\nu \neq \mu} \sim \frac{1}{\sqrt{n}}$

$$m_\nu(t+1) = \frac{1}{n}\sum_i \xi_i^\nu\,\text{sign}\left[\sum_\mu \xi_i^\mu\,m_\mu\right]$$

p variables

$$= \frac{1}{n}\sum_i \xi_i^\nu\,\text{sign}\left[\xi_i^\nu m_\nu + \underbrace{\sum_{\mu\neq\nu}\xi_i^\mu m_\mu}_{\text{noise}}\right]$$

$z_i \sim N(0,1)$

σz_i

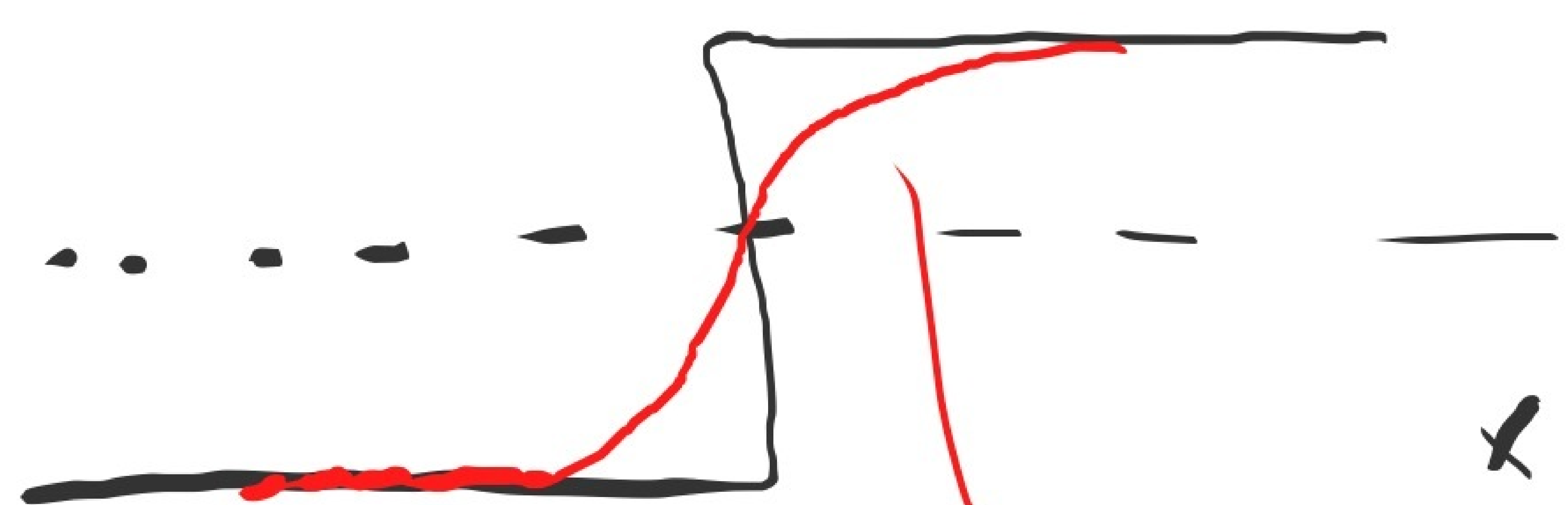
$$\to \frac{1}{n}\sum_i \xi_i^\nu\,\text{sign}\left[\xi_i^\nu m_\nu + \sigma z_i\right]$$

uncorrelated

$$m_\nu = \frac{1}{n}\sum_{i=1}^{n} \xi_i^\nu \,\mathrm{sign}\left[\xi_i^\nu m_\nu + z_i \sigma\right]$$

$$\to \frac{1}{n}\sum_{i=1}^{n} \xi_i^\nu \int dz \,\frac{e^{-z^2/2}}{\sqrt{2\pi}} \,\mathrm{sign}\left[\xi_i^\nu m_\nu + z\sigma\right]$$

Sign(x)



larger $\sigma \Rightarrow$ more smoothing

$$m_\nu = \frac{1}{n}\sum_{i=1}^{n} \xi_i^\nu \,\tilde{\Phi}\left(\xi_i^\nu m_\nu ; \sigma\right)$$

$(\pm 1)\,\tilde{\Phi}(x)$
$= \pm\,\tilde{\Phi}(x)$

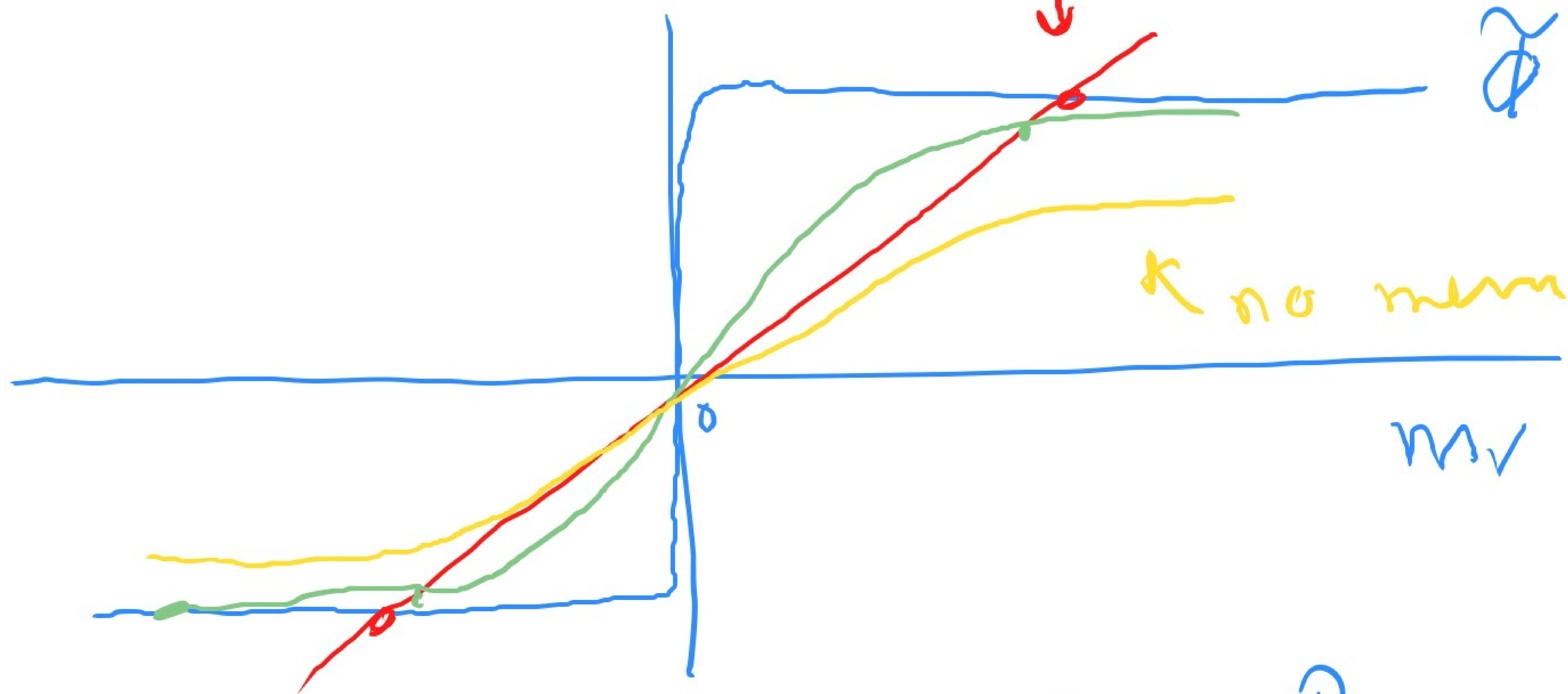
$$m_\nu = \tilde{\Phi}(m_\nu ; \sigma) \Rightarrow \left\langle \xi \,\tilde{\Phi}(\xi ; \sigma) \right\rangle$$

$\xi = \begin{cases} 1 & \text{prob } 1/2 \\ -1 & \text{prob } 1/2 \end{cases}$

$$m_\nu = \tilde{\phi}(m_\nu ; \sigma)$$

memories : $m_\nu \cong 1$

$\tilde{\phi}(m_\nu, \sigma \text{ small})$



no memories

$\frac{p}{n} \cong 0.138$

$$\sigma^2 \propto \frac{p}{n}$$

$$\sigma^2 = \mathrm{Var}\left[\sum_{\mu \neq \nu}^{p} \xi_i^\mu m_\mu\right]$$

more realistic network

$$v_E = \frac{1}{n}\sum_i v_{Ei}$$

$$\tau_E \frac{dv_{Ei}}{dt} = \tilde{\Phi}_E\left(\sqrt{n}\left(w_{EE} v_E - w_{EI} v_I\right) + \frac{g}{n(n-1)} \sum_j \eta_i(\eta_j - 1) v_j\right) - v_{Ei}$$

$-\delta v_E$

$$\tau_I \frac{dv_I}{dt} = \tilde{\Phi}_I\left(\sqrt{n}\left(w_{IE} v_E - w_{II} v_I\right)\right) - v_I$$

$\epsilon = O\left(\frac{1}{\sqrt{n}}\right)$

$\tau_I \to 0 \Rightarrow v_I \approx \frac{w_{IE} v_E}{w_{II}} + O\left(\frac{1}{n}\right)$

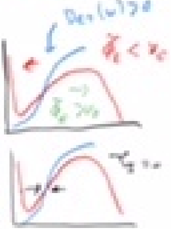
bad approx!!

$-\epsilon$

$$\sqrt{n}\left(w_{EE} - \frac{w_{EI} w_{IE}}{w_{II}}\right) v_E$$

$$\frac{1}{w_{II}}\left(w_{EE} w_{II} - w_{EI} w_{IE}\right)$$

$$= -\frac{\det(w)}{w_{II}}$$



$$\tau_E \frac{dV_{Ei}}{dt} = \tilde{\phi}_E \left(-\gamma V_E + \frac{\beta}{nf(1-f)} \sum_j \eta_i (\eta_j - f) V_{Ej} \right]$$

$$\eta_i = \begin{cases} 1 & \text{prob } f \\ 0 & \text{prob } 1-f \end{cases}$$

Overlap: $$m = \frac{1}{nf(1-f)} \sum_j (\eta_j - f) V_{Ej}$$

$$\tau_E \frac{dV_{Ei}}{dt} = \tilde{\phi}_E \left(-\gamma V_E + \eta_i \beta m \right) - V_{Ei}$$

$$\tau_E \frac{dm}{dt} = \frac{1}{nf(1-f)} \sum_i (\eta_i - f) \, \tilde{\phi}_E \left(-\gamma V_E + \eta_j \beta m \right)$$

$$\tau_E \frac{dV_E}{dt} = \frac{1}{n} \sum_i \tilde{\phi}_E \left(-\gamma V_E + \eta_i \beta m \right) - V_E$$

$$\tau_E \frac{d\nu_E}{dt} = \left\langle \tilde{\Phi}_E(-\gamma\nu_E + \eta\beta m) \right\rangle - \nu_E$$

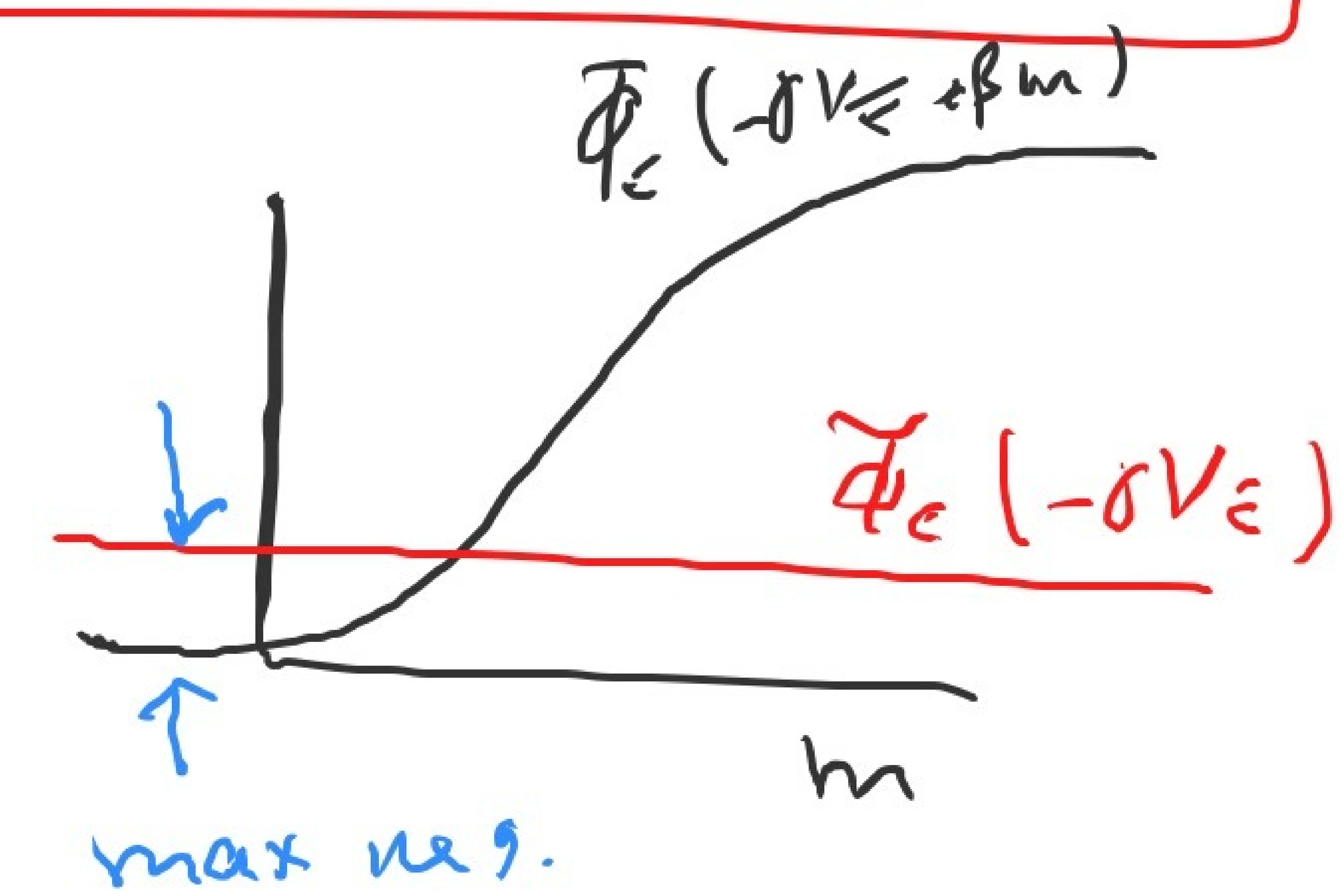
$$\tau_E \frac{dm}{dt} = \frac{1}{f(1-f)} \left\langle (\eta - f)\, \tilde{\Phi}_E(-\gamma\nu_E + \eta\beta m) \right\rangle - m$$

$$\tau_E \frac{d\nu_E}{dt} = \tilde{\Phi}_E(-\gamma\nu_E) + f\left[\tilde{\Phi}_E(-\gamma\nu_E + \beta m) - \tilde{\Phi}_E(-\gamma\nu_E)\right] - \nu_E$$

$$\tau \frac{dm}{dt} = \tilde{\Phi}_E(-\gamma\nu_E + \beta m) - \tilde{\Phi}_E(-\gamma\nu_E) - m$$

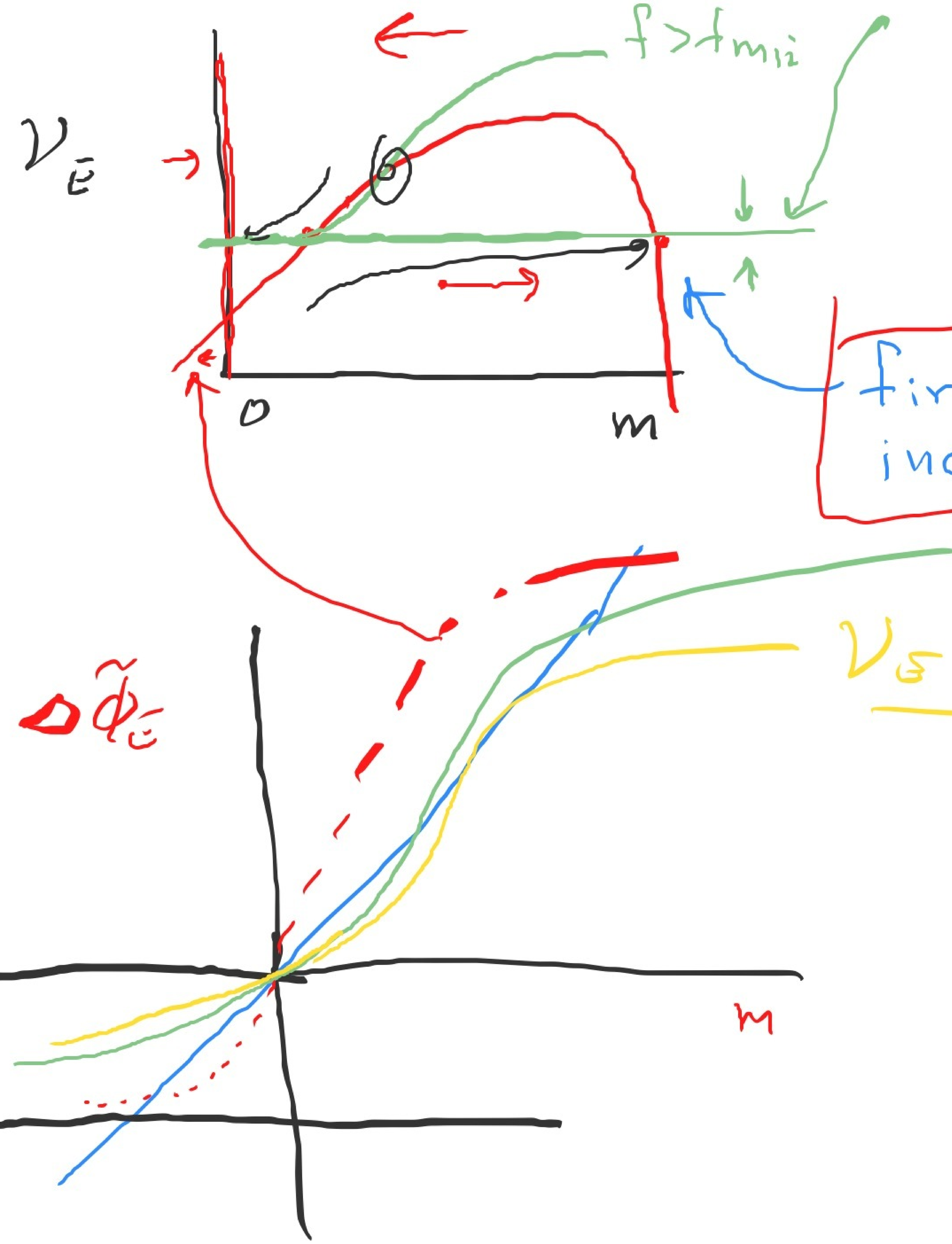
$$\Delta\tilde{\Phi}_E(\nu_E, \beta m)$$

$$\Delta\tilde{\Phi}_E(\nu_E, 0) = 0$$



m-nullcline: $m = \Delta \tilde{\Phi}_E(\nu_E, m)$

ν_E-nullcline: $\nu_E = \tilde{\Phi}_E(-\gamma \nu_E) + f \Delta \tilde{\Phi}_E(-\gamma \nu_E, \beta m)$



try: $f \to 0$ ← ruled out by exps.

firing rate is near ν_{max} ~100 Hz
inconsistent with experiments

~5-10 Hz

very smooth $\tilde{\Phi}_E$

lots of memories
~p

problem w/ classic Hopfield networks:

$p \propto K$ ← # connections/neuron

fix: $p \propto \frac{K}{f}$ ← # neurons involved in memory

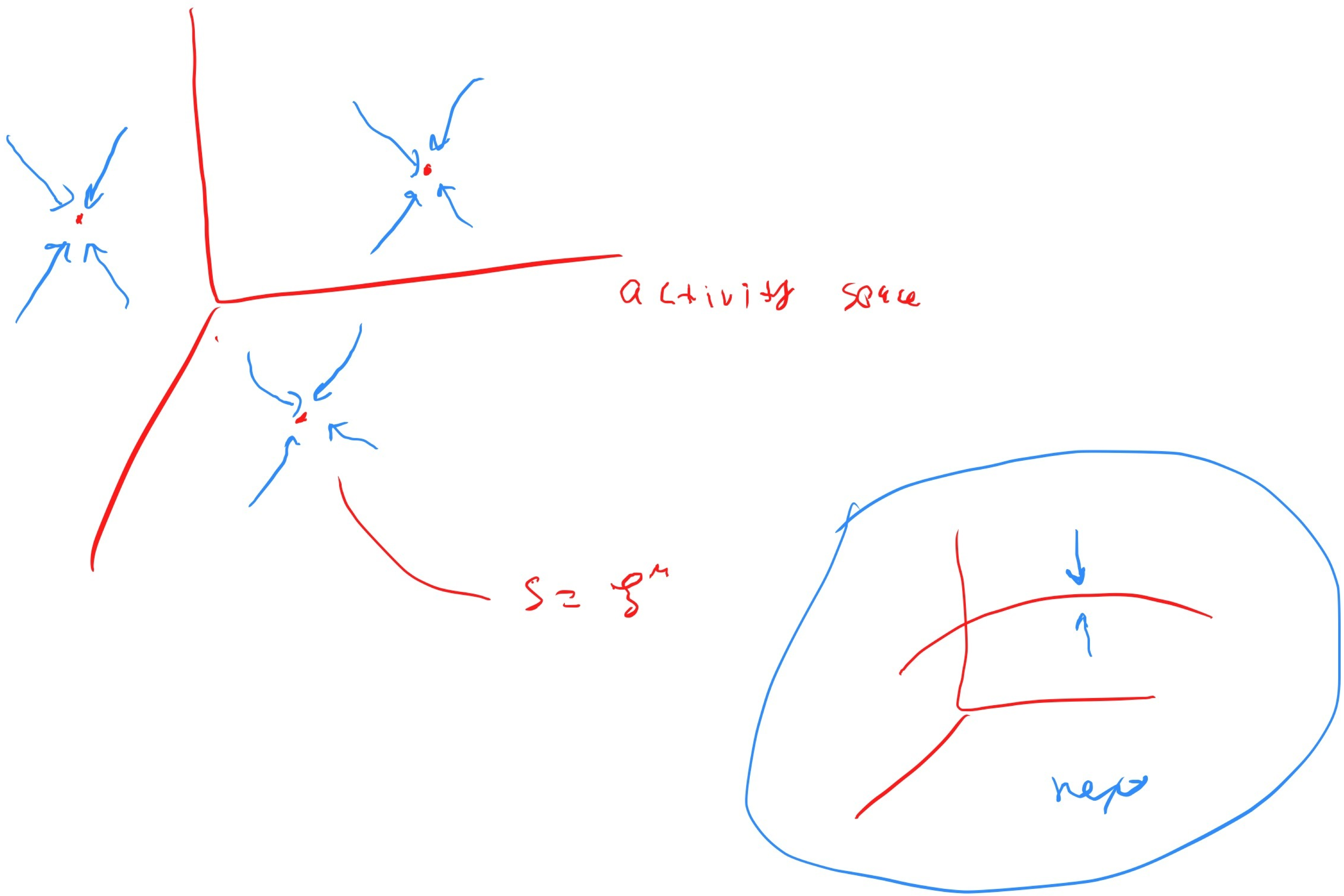
choose $f \sim K/n$

recover $p \propto n$

fix is dead for realistic memories

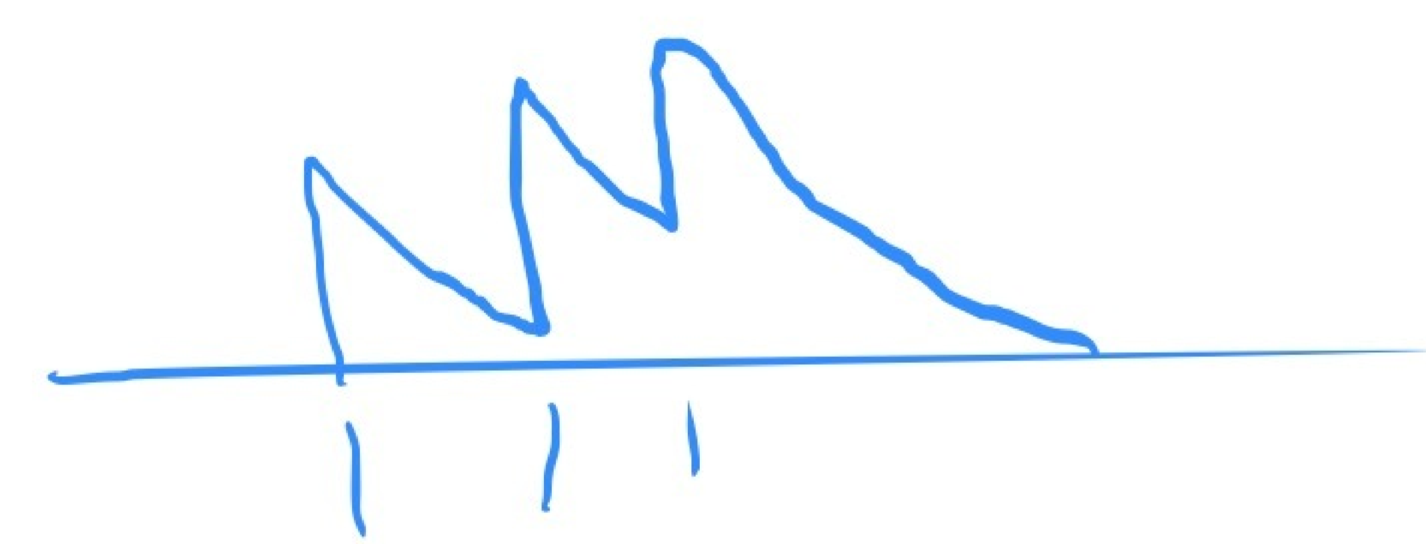
conclusion:

we have absolutely no idea how associative memory works in the brain


activity space

networks of neurons:

$$\tau \frac{dV_i}{dt} = (\text{single neuron model}) - \sum_j \overbrace{A_{ij}(V_i - \mathcal{E}_j)}^{\tilde{W}_{ij}} g_j(t)$$



time of spikes on neuron j

→ techiques for dealing with $\sum_j \tilde{W}_{ij}\, g_j(t)$

→ main techique:

a. large sums are turned into Gaussian random variables

b. replace sums by Gaussian integrals

→ gain funcsions are smoothed

→ added structure; still replace sums with integrals, but not Gaussian → $\frac{1}{n}\sum_i \xi_i f(\ldots \xi_i)$

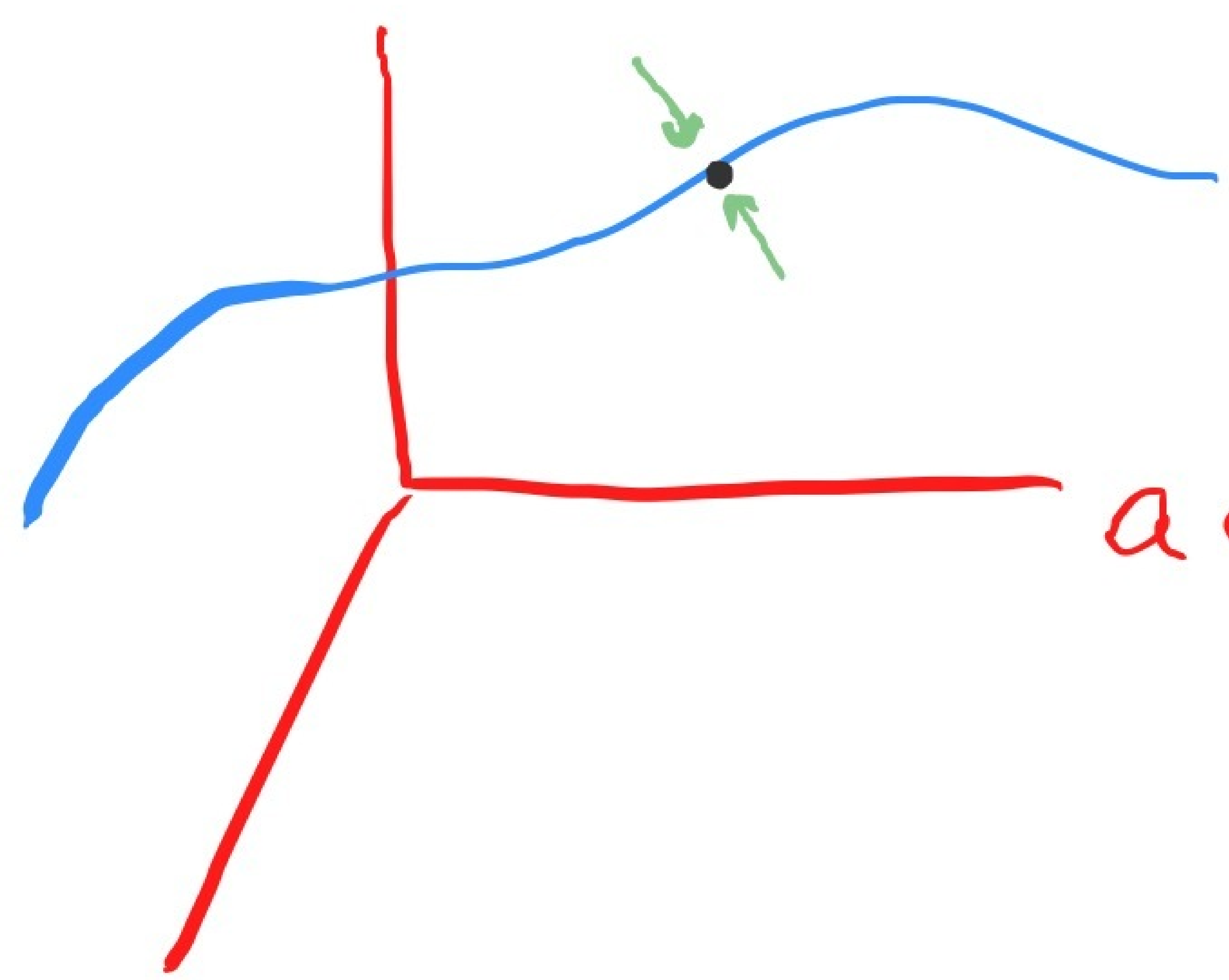
$\to \langle \xi f(\ldots \xi) \rangle$

ξ discrete

- line attractor networks

ignore { - point attractors are useful for storing discrete items.
- not good for continuous items.

- how do you remember an analog variable for short times.



activity space

$$\dot{r}_i = \phi\left(\sum_j W_{ij} r_j\right) - r_i$$

$$\boxed{f_i(\theta) = \phi\left(\sum_j W_{ij} f_j(\theta)\right)}$$

for a range of θ

- line attractor

Shift if

$W_{ij} = W_{i-j}$

$$f_i(\theta) = \phi\left(\sum_j W_{i-j}\, f_j(\theta)\right)$$

one sol: $f_i(\theta)$

guess another: $f_i(\theta) \to f_{i+k}(\theta)$

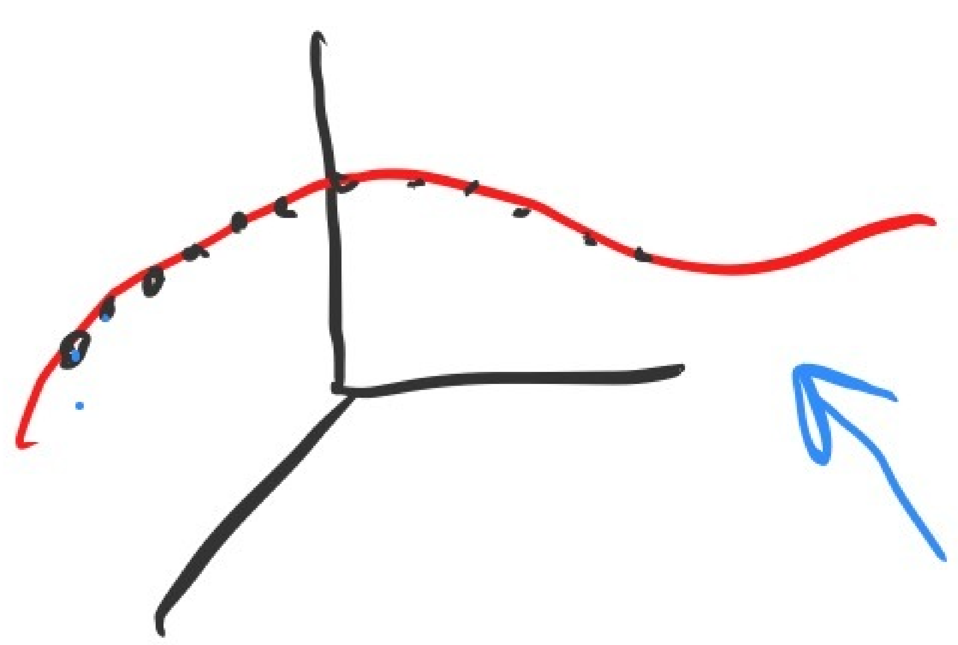
$$f_{i+k}(\theta) = \phi\left(\sum_j W_{i-j}\, f_{j+k}(\theta)\right)$$

$i = i' - k$

$j = j' - k$

$$f_{i'}(\theta) = \phi\left(\sum_{j'} W_{i'-k-(j'-k)}\, f_{j'}\right)$$

$$= \phi\left(\sum_{j'} W_{i'-j'}\, f_{j'}(\theta)\right)$$



large n limit, collapses onto line

1. Stability

1a. can we control drift along line attractor

- Circuitry that tells you where to point your eyes.
- you need to be able to provide a signal to move your eye.

2. can we embed multiple line attractors?

we'll see that this is hard

$$\dot{r}_i = \phi\Big(W_{ij} r_j + \sum_j V_{ij} r_j + h_i\Big) - r_i$$

external signal

purturbation because weights cant be set perfectly

V_{ij} small
h_i small
$\Big\}$ compared to W_{ij}

firing rate

$$r_i = f_i(\theta_0) + \delta r_i$$

fixed

$$\delta\dot{r}_i = \phi\Big(\sum_j W_{ij} f_j(\theta_0) + \sum_j V_{ij} f_j(\theta_0) + h_i + \sum_j W_{ij}\,\delta r_j + \cancel{\sum_j V_{ij}\,\delta r_j}\Big) - f_i(\theta_0) - \delta r_i$$

small

$$\simeq \cancel{\phi\Big(\sum_j W_{ij} f_j(\theta_0)\Big) - f_i(\theta_0)} + \phi'_i \Big[\sum_j W_{ij}\,\delta r_j + \sum_j V_{ij} f_j(\theta_0) + h_i\Big] - \delta r_i$$

$\phi'\Big(\sum_j W_{ij} f_j(\theta_0)\Big)$

$$\delta \dot{r}_i = \phi_i' \sum_j W_{ij}\, \delta r_j - \delta r_i + \phi_i' \left(\sum_j V_{ij} f_j(\theta_0) + h_i \right)$$

$$= \sum_j J_{ij}(\theta_0)\, \delta r_j + S_i(\theta_0)$$

J_{ij} = Jacobian

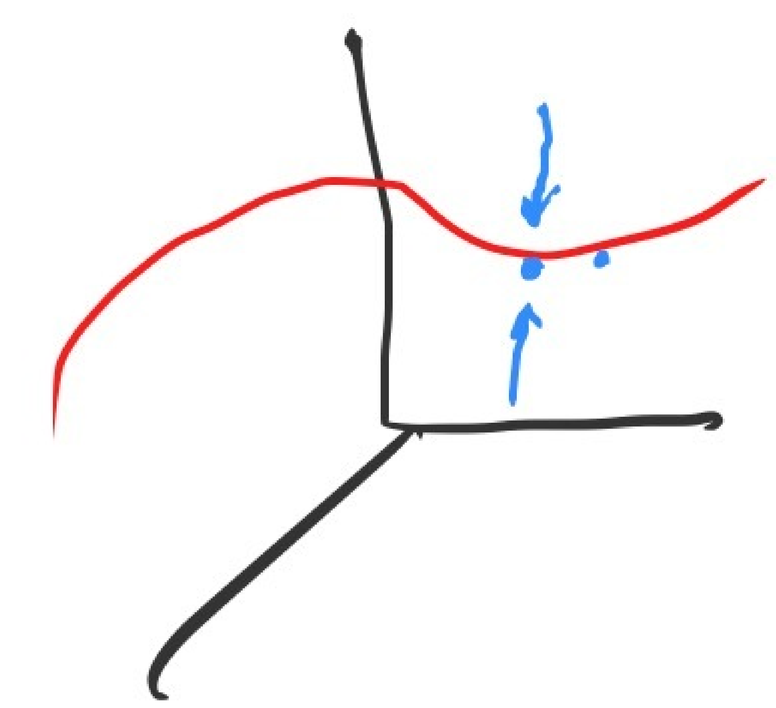
Kronecker delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$= \phi_i' W_{ij} - \delta_{ij}$$

$$S_i = \phi_i' \left[\sum_j V_{ij} f_j(\theta_0) + h_i \right]$$

$$\boxed{\delta \underline{r} = \underline{\underline{J}} \cdot \delta \underline{r} + \underline{S}}$$



$$f_i(\theta) = \phi_i\left(\sum_j W_{ij} f_j(\theta)\right) \qquad \forall \theta$$

$$\frac{d f_i(\theta)}{d\theta} = \phi_i' \sum_j W_{ij} \frac{d f_j(\theta)}{d\theta}$$

$$0 = \underbrace{\left[\phi_i' \sum_j W_{ij} - \delta_{ij}\right]}_{J_{ij}} \frac{d f_j(\theta)}{d\theta}$$

$$= \sum_j J_{ij} f_j'(\theta) = 0$$

eigenvector with eigenvalue $= 0$

$\underline{\underline{J}} \cdot \underline{f}' = 0$

$\underline{\underline{J}} \cdot \underline{v}_k = \lambda_k \underline{v}_k$

$\underline{v}_k = \underline{f}' \Rightarrow \lambda_k = 0$

$$\delta \dot{\underset{\sim}{r}} = \underline{\underline{J}} \cdot \delta \underline{r} + \underline{S}$$

$$\underline{\underline{J}} \cdot \underline{V}_k = \lambda_k \underline{V}_k$$

$$\underline{V}_k^{\dagger} \cdot \underline{\underline{J}} = \lambda_k \underline{V}_k^{\dagger}$$

$$\delta \underline{r}(t) = \sum_k a_k^{(t)} \underline{V}_k$$

$$\underline{V}_k^{\dagger} \cdot \underline{V}_e = \delta_{ke}$$

$$\sum_k \dot{a}_k \underline{V}_k = \underline{\underline{J}} \cdot \sum_k a_k \underline{V}_k + \underline{S}$$

$$\underline{V}_e^{\dagger} \cdot \sum_k \dot{a}_k \underline{V}_k = \underline{V}_e^{\dagger} \cdot \sum_k a_k \lambda_k \underline{V}_k + \underline{V}_e^{\dagger} \cdot \underline{S}$$

$$\boxed{\dot{a}_e = \lambda_e a_e + \underline{V}_e^{\dagger} \cdot \underline{S}}$$

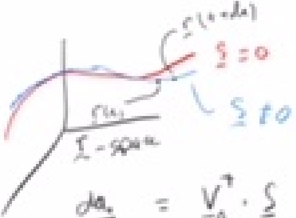
$\lambda_0 = 0$

$\lambda_{e>0} < 0$

asymptotically: $\lambda_{e>0} < 0$

$$\boxed{a_e = \frac{\underline{V}_e^{\dagger} \cdot \underline{S}}{-\lambda_e}}$$

$\lambda_0 = 0$



$$\frac{da_0}{dt} = \underline{V}_0^T \cdot \underline{\xi}$$

$$a_0(t) = a_0(0) + t\, V_0^T \cdot \underline{\xi}$$

$$a_0(t+dt) = a_0(t) + dt\, V_0^T \cdot \xi$$

$a_0(t) = 0$

$$r_i = f_i(\underline{a}) + \delta r_i$$

$t = dt \quad a_0(t+dt) = dt\, \underline{V}_0^T \cdot \xi$

$$\underline{r}(t+dt) = \sum^{r(a)} a_i(t+dt)\, \underline{V}_a = \sum_{a \neq 0} a_i(t+dt)\, V_a$$

$\xi(t+dt)$ + perturbations of ξ + the line section

$r(\cancel{t}) + dt\, \underline{V}_0^T \cdot \underline{\xi} \quad V_0 \cdot V_0$

$\approx \xi(t) + \frac{d\xi}{dt}\, dt \quad V_0$

$$\Rightarrow \frac{dt}{da_0} = \frac{V_0^T \cdot \xi}{V_0^T \cdot d\xi/dt}$$

$$\frac{d\theta}{dt} = \frac{\underline{v}_0^T \cdot \underline{s}}{\underline{v}_0^T \cdot d\underline{f}/d\theta}$$