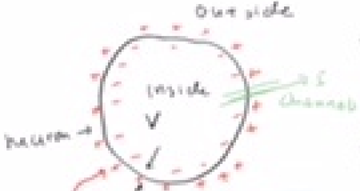
4 lectures: biophysics

Scientists: → explain behavior in terms of neural activity

1. figure out the equations
2. solve them
   ↳ an approximate

biophysics is about the equations



V: membrane potential on inside relative to outside

goal: write down differential equation for V

$$\frac{dV}{dt} = \ldots$$

V = voltage drop across membrane

dendrite

electric field → voltage difference

$Q = CV$ $V = \frac{Q}{C}$ physics

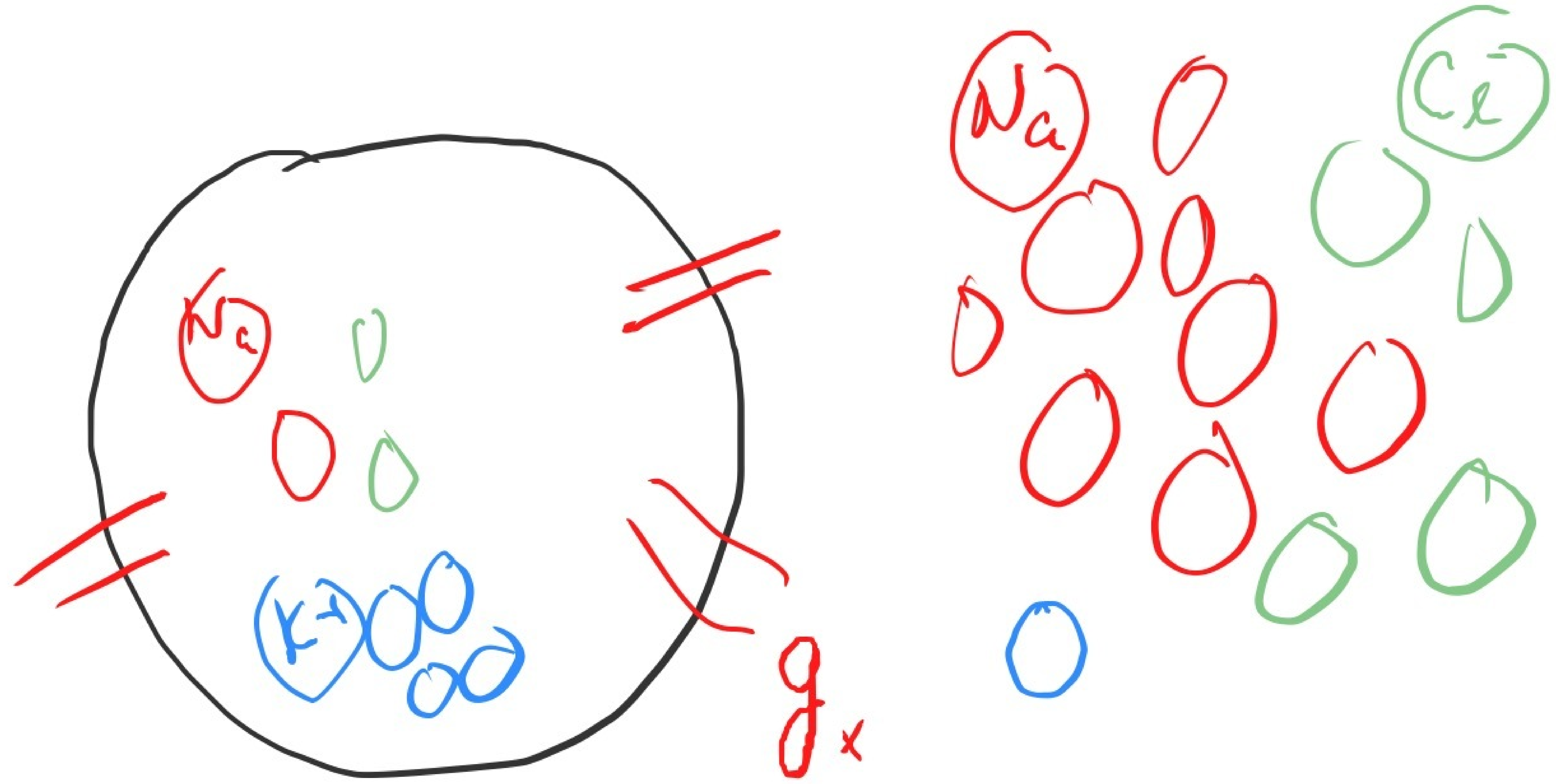
$(V - E) = IR$ $I = \frac{V - E}{R} = g(V - E) = I = \frac{dQ}{dt}$

↑ reversal potential

conductance

I ext

$$\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{I}{C}$$



ion specific

outside: lots of Na^+ + Cl^-

inside: lots of K^+

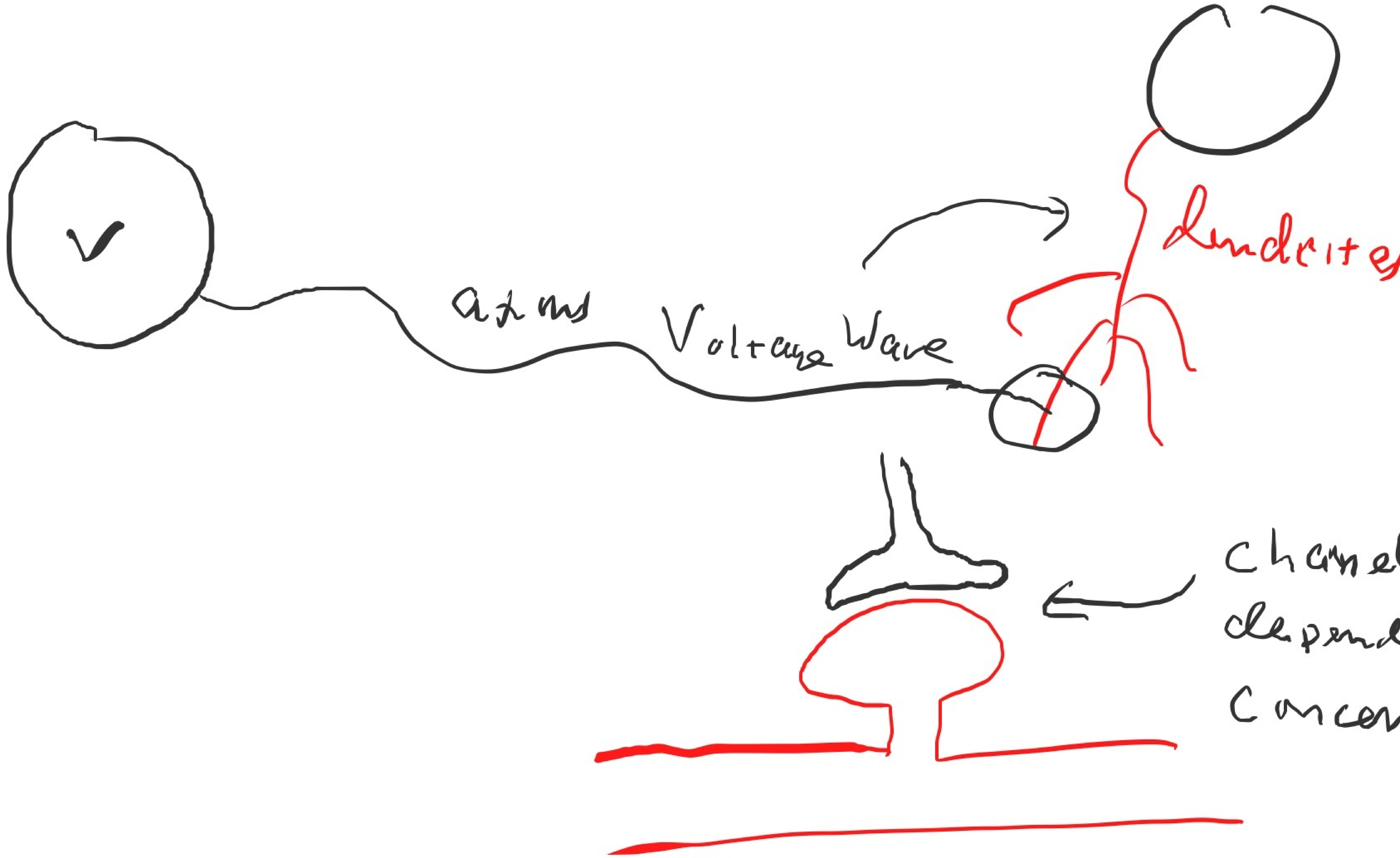
$$C\frac{dV}{dt} = -\sum_{\ell} g_{\ell}(V - E_{\ell})$$

- ion specific and fixed
- depend on V

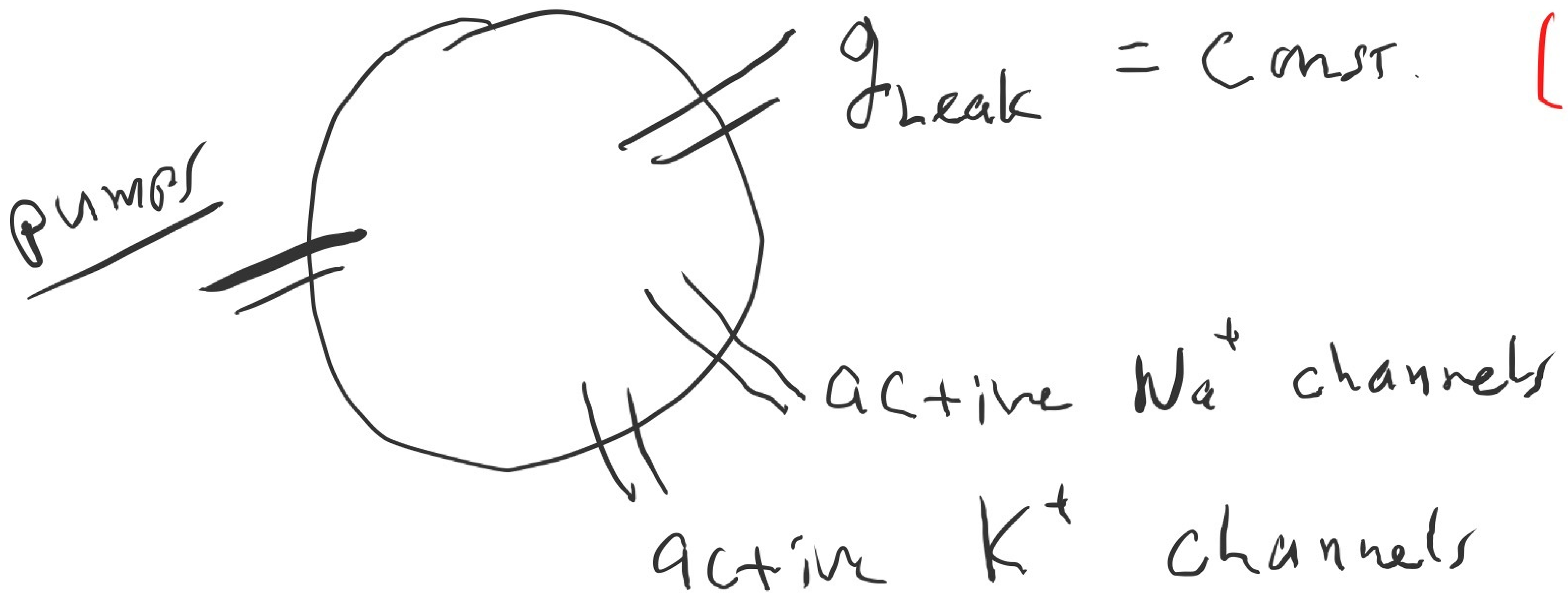
Hodgkin-Huxley equations, dendrites axons

- depend on concentration of neurotransmitters

synapses



chanels
depend on
concentrations



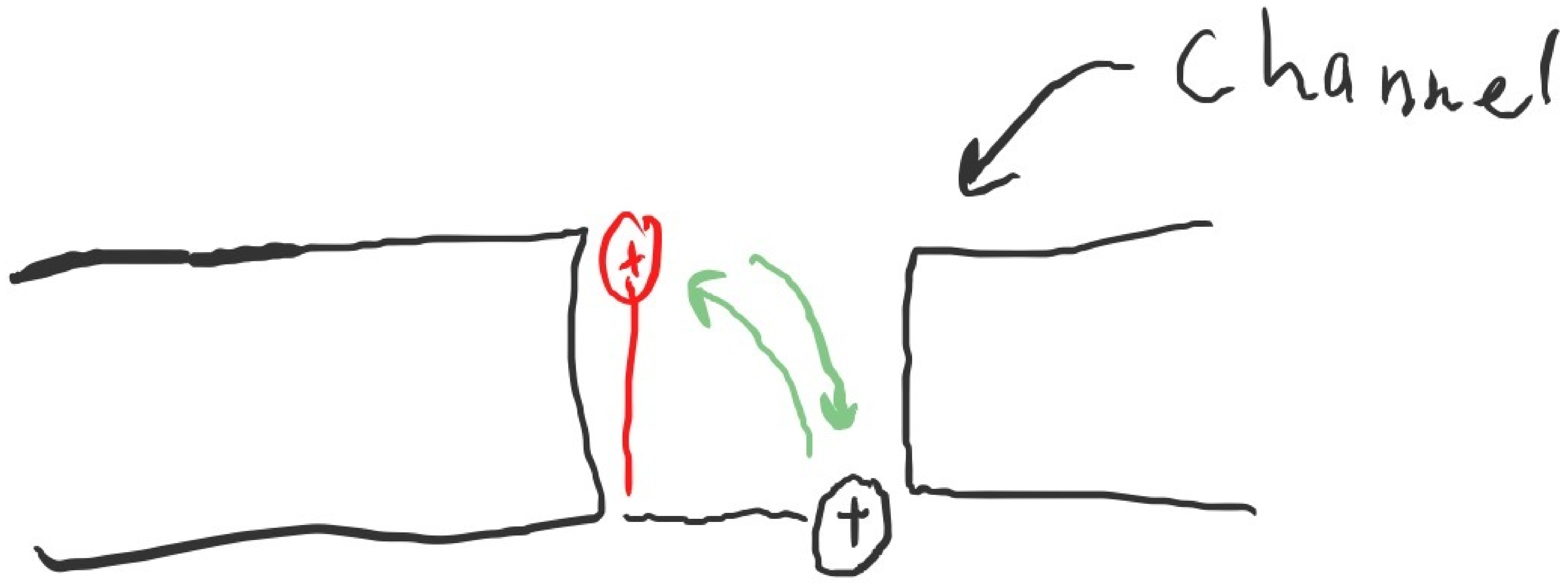
(Na^+, K^+, Cl^-)

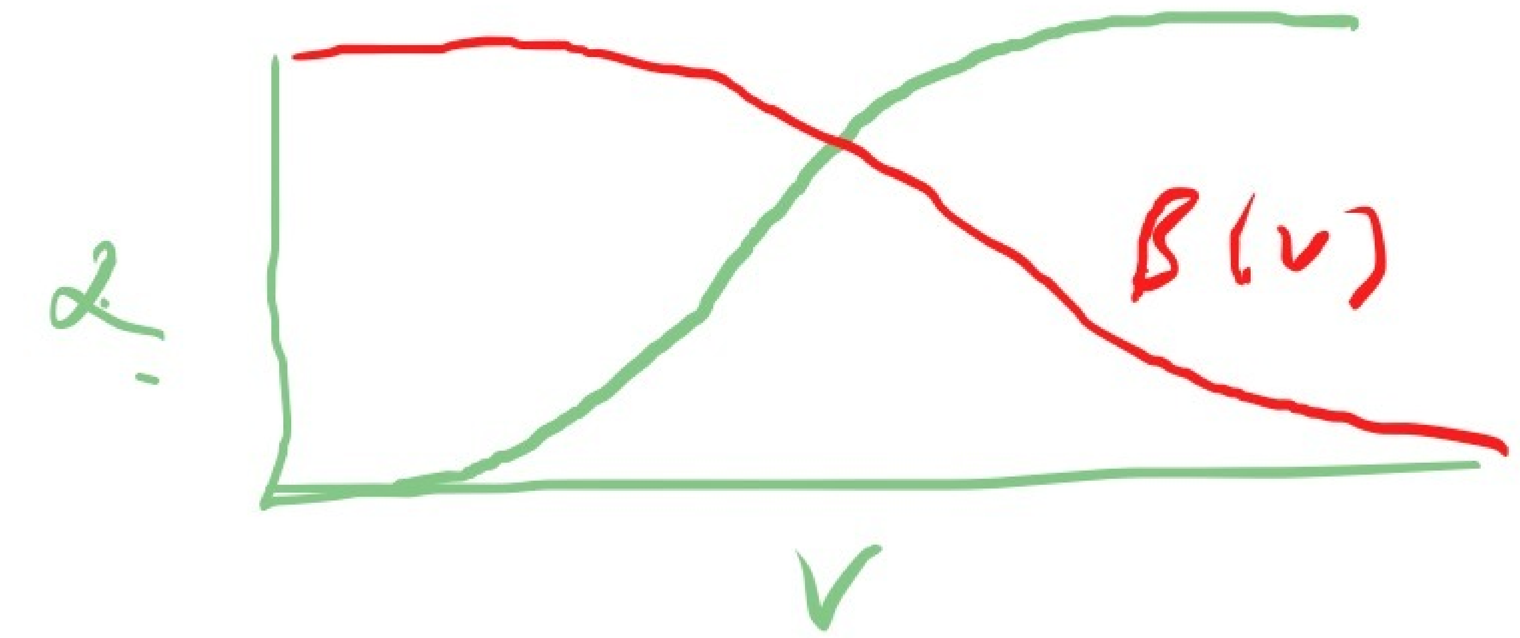
$$C\frac{dV}{dt} = -g_L(V-E_L) - g_{Na}(t)(V-E_{Na}) - g_K(t)(V-E_K)$$

E_L: -65 mV E_{Na}: $+20$ E_K: -80

$$\frac{C}{g_L}\frac{dV}{dt} = -(V-E_L) - \frac{g_{Na}(t)}{g_L}(V-E_{Na}) - \frac{g_K(t)}{g_K}(V-E_K)$$

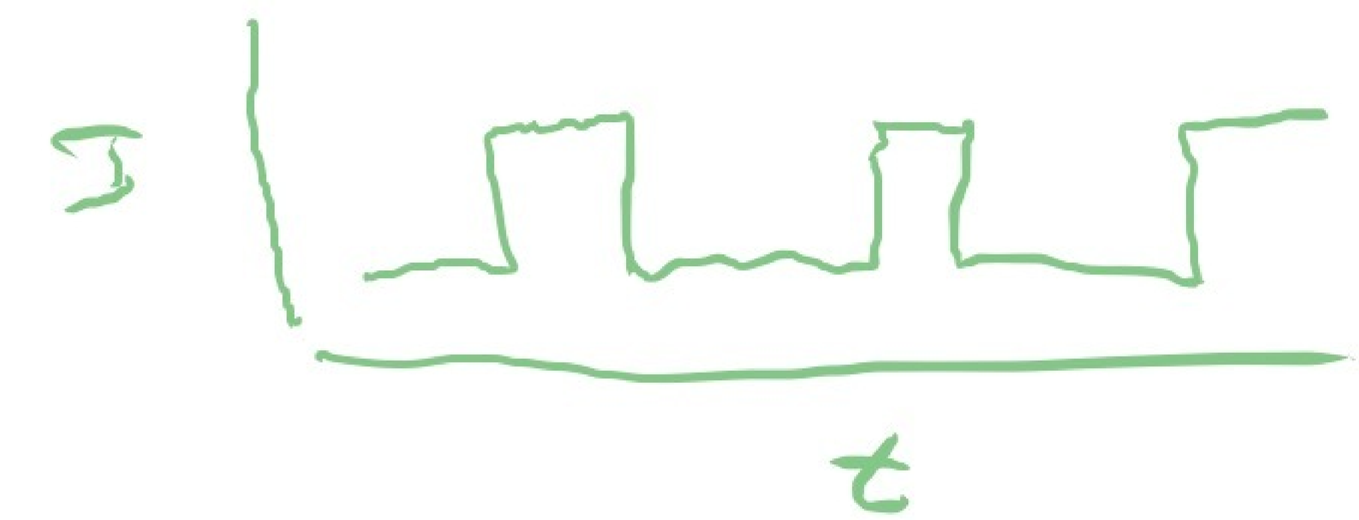
$\frac{C}{g_L}$: $\tau_m \approx 10$ ms

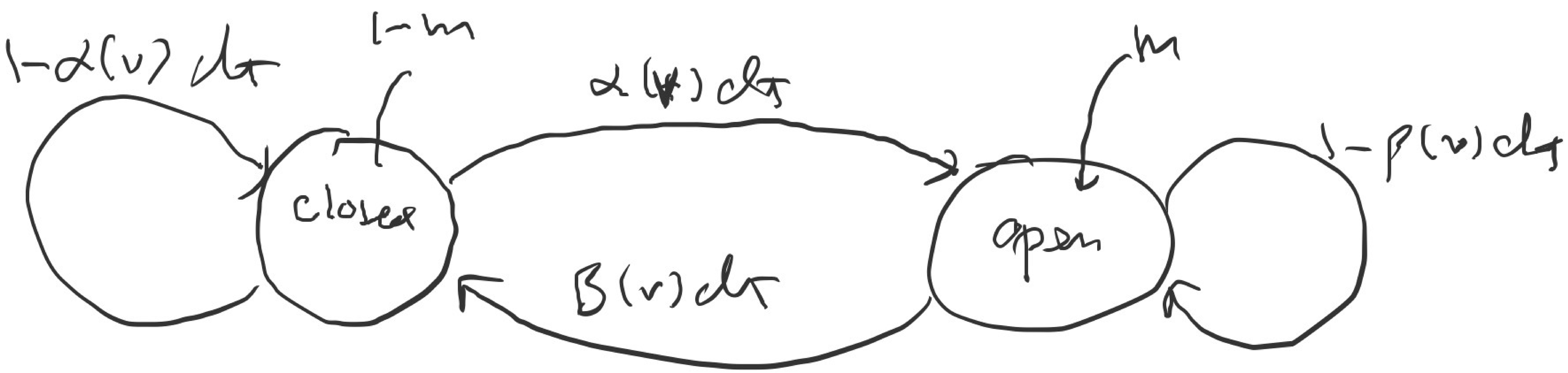


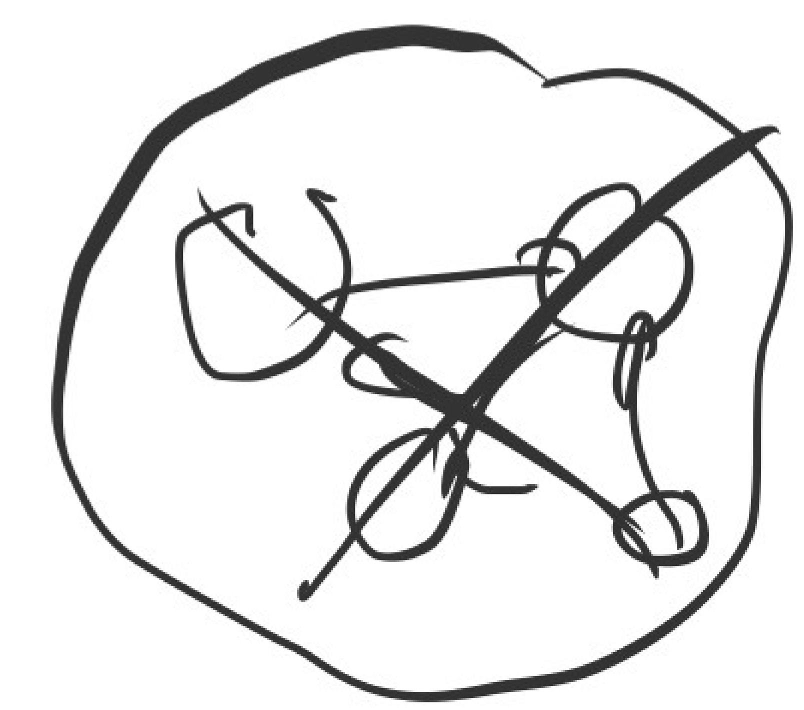


p(closed to open in time dt) $= \alpha(v)\,dt$

p(open to close in time dt) $= \beta(v)\,dt$







$m(t)$ = open prob. at time t

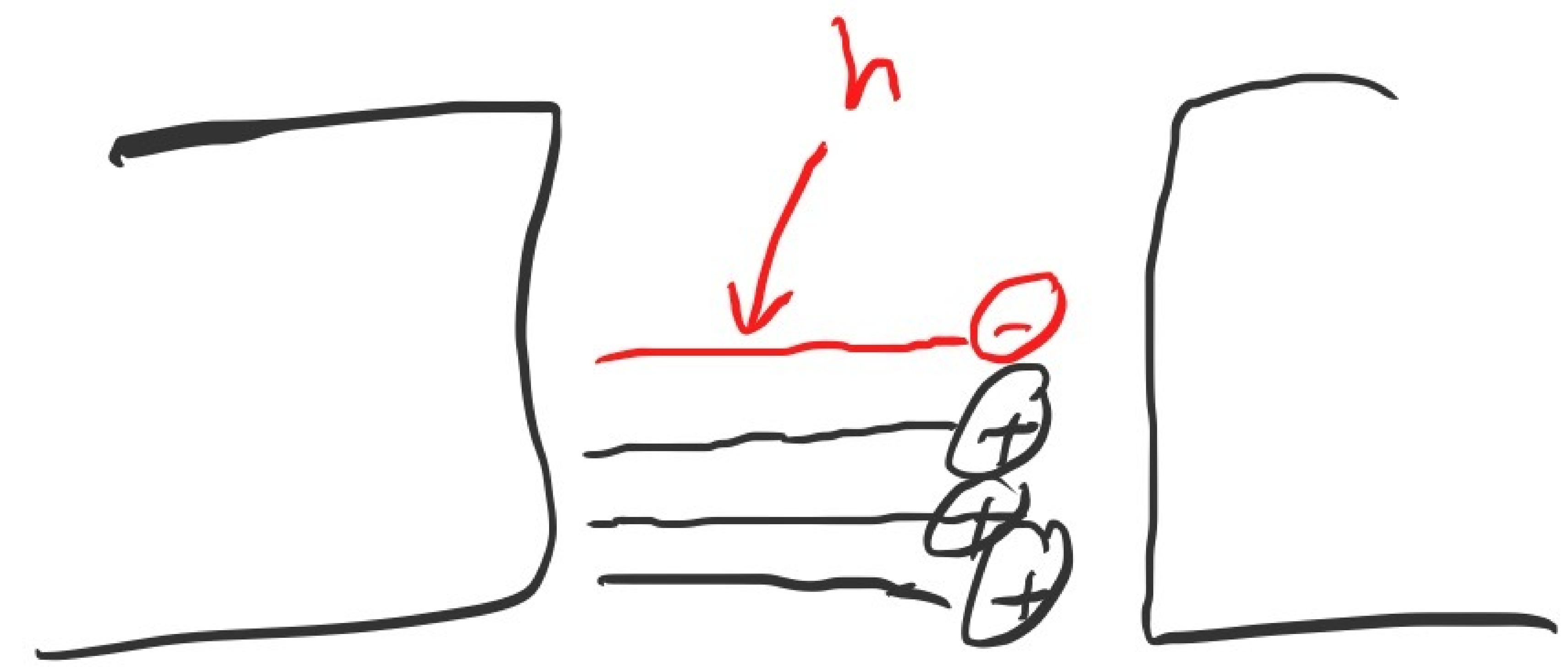
$$m(t+dt) = m(t)\left(1-\beta\,dt\right) + \left(1-m(t)\right)\alpha\,dt$$

$$= m(t) + \underbrace{dt\left(\alpha - (\alpha+\beta)m(t)\right)}_{\downarrow}$$

$$m(t+dt) - m(t) \equiv dt\,\frac{dm}{dt} =$$

$$\frac{dm}{dt} = \alpha - (\alpha + \beta) m$$

$$\underbrace{\frac{1}{\alpha(v)+\beta(v)}}_{\tau_m(v)} \frac{dm}{dt} = \underbrace{\frac{\alpha(v)}{\alpha(v)+\beta(v)}}_{m_\infty(v)} - m$$



$$g = g_0 m$$

$$\tau_m(v) \frac{dm}{dt} = m_\infty(v) - v$$

$$P_{open} = m^3 h$$

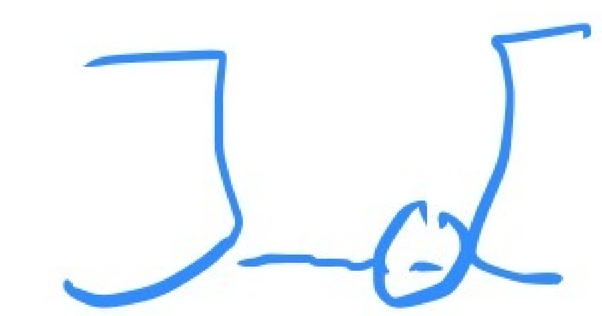
Na: $g_{Na}(t) = \bar{g}_{Na} m^3 h \longrightarrow \frac{\bar{g}_{Na}}{g_L} \equiv \rho_{Na} \approx 400$

K: $g_K = \bar{g}_K n^4 \longrightarrow \frac{\bar{g}_K}{g_L} \equiv \rho_K \approx$ big

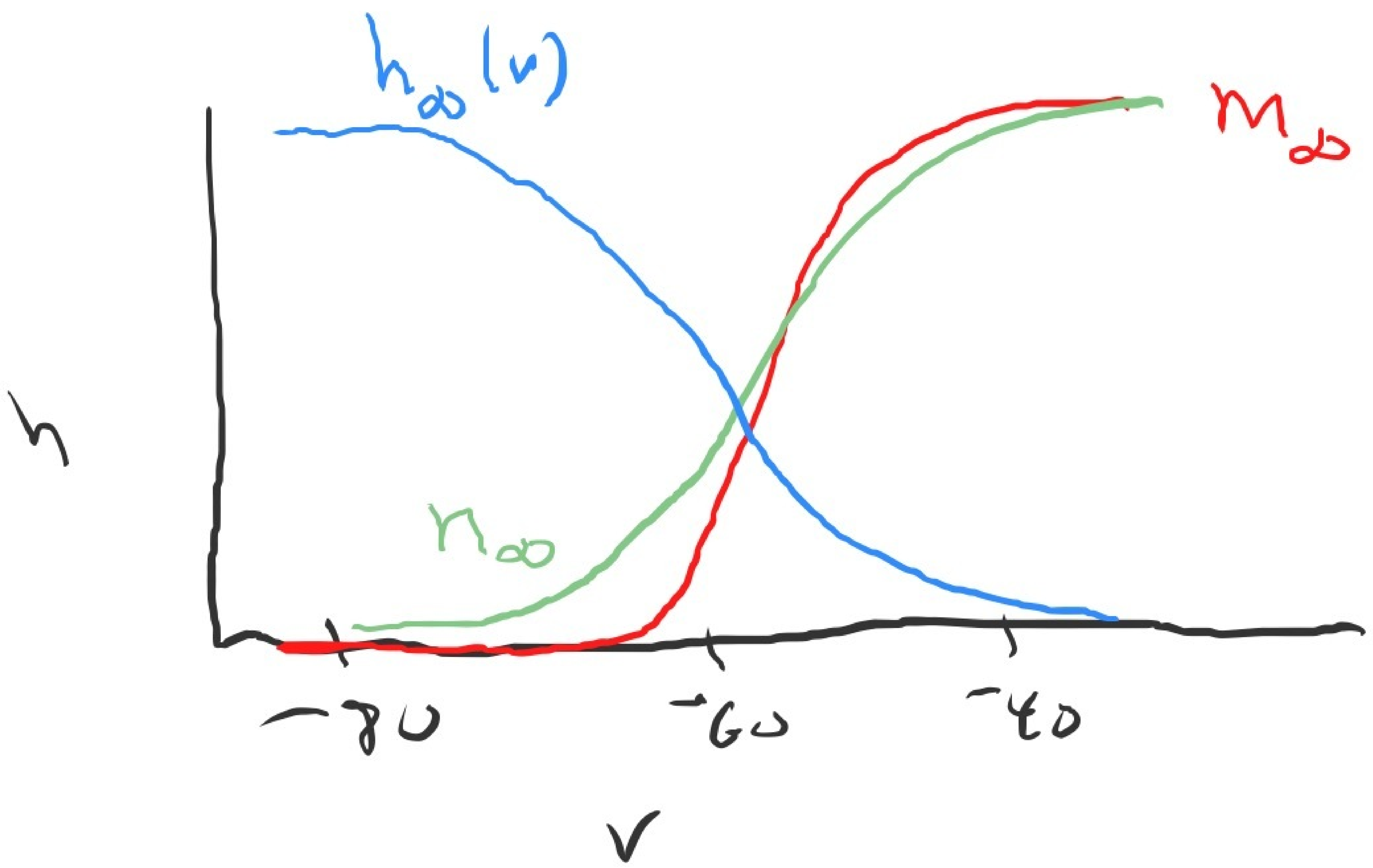
HH:

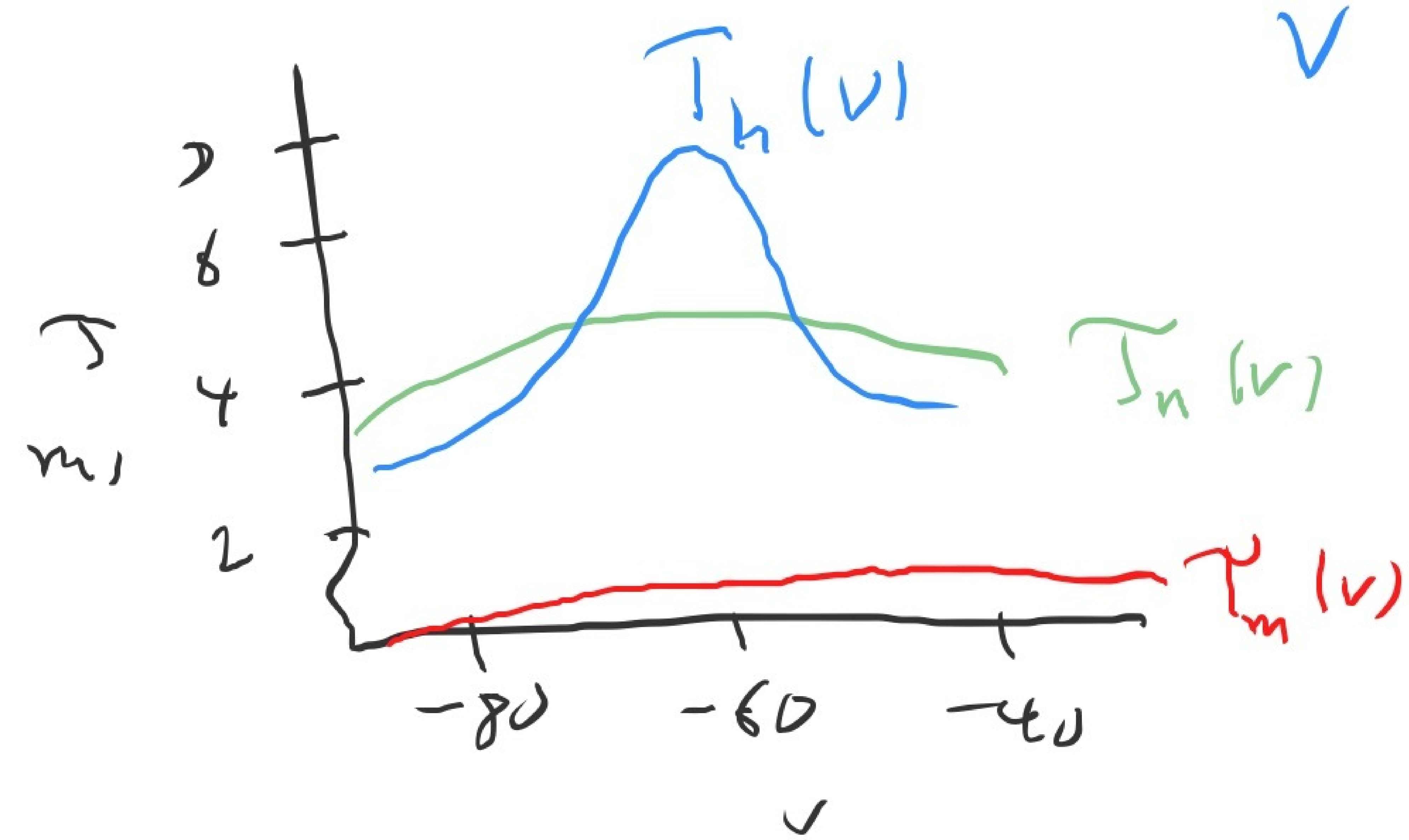
$$\tau \frac{dv}{dt} = -(v - \overset{-65}{E_L}) - \overset{400}{\rho_{Na}}\, m^3 h (v - \overset{+20}{E_{Na}}) - \rho_K n^4 (v - \overset{-80}{E_K}) + \underline{I_{ext}}$$

m, h, n $$\underline{\tau_x(v)} \frac{dx}{dt} = \underline{x_\infty(v)} - x$$



V high

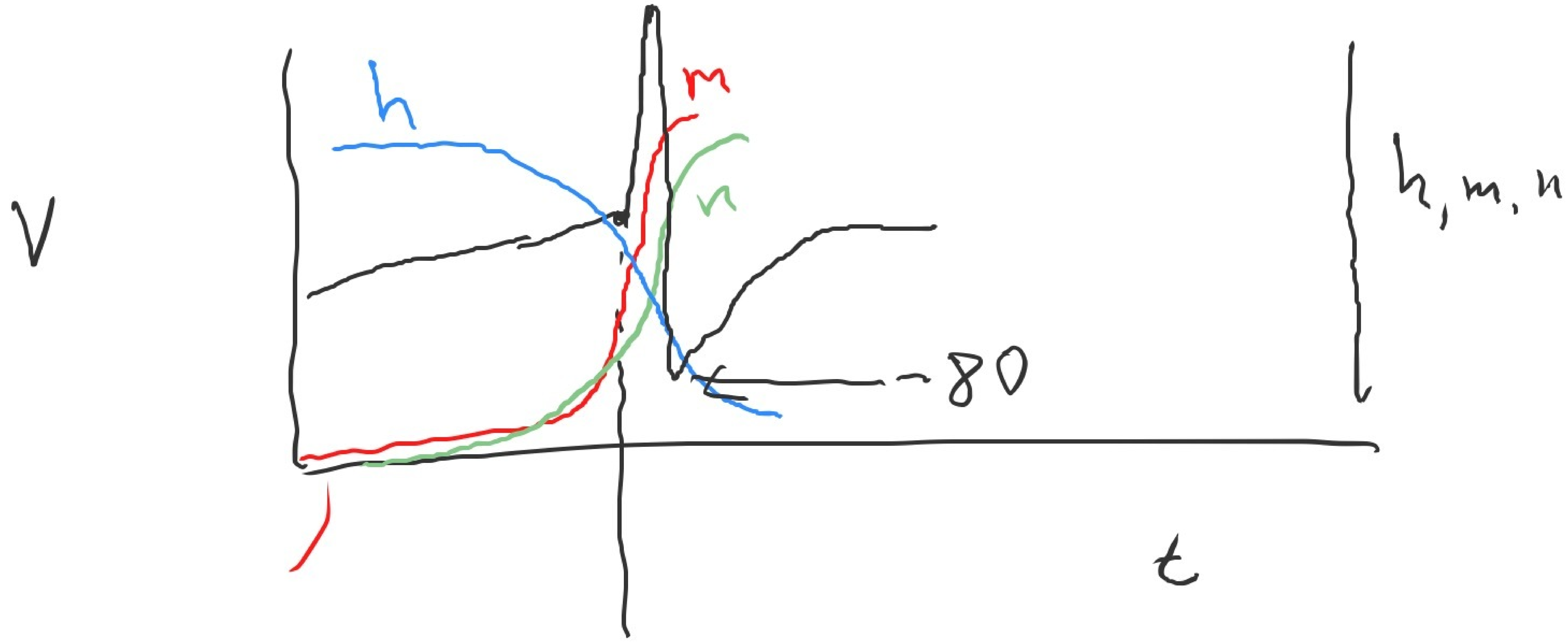




HH: 4-dimensional equation

$$\frac{dx_i}{dt} = f_i(x_1 \dots x_n)$$

n-dim ODE



fast

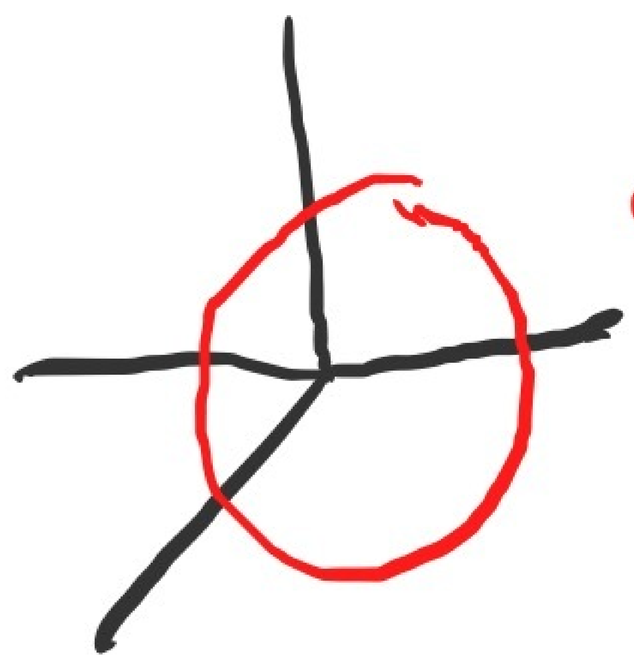
Na^+ channels open

$h \downarrow \Rightarrow Na^+$ close $\Rightarrow V \downarrow$

$n \uparrow \Rightarrow K^+$ open $\Rightarrow V \downarrow$

repetitive spiking

4-D space

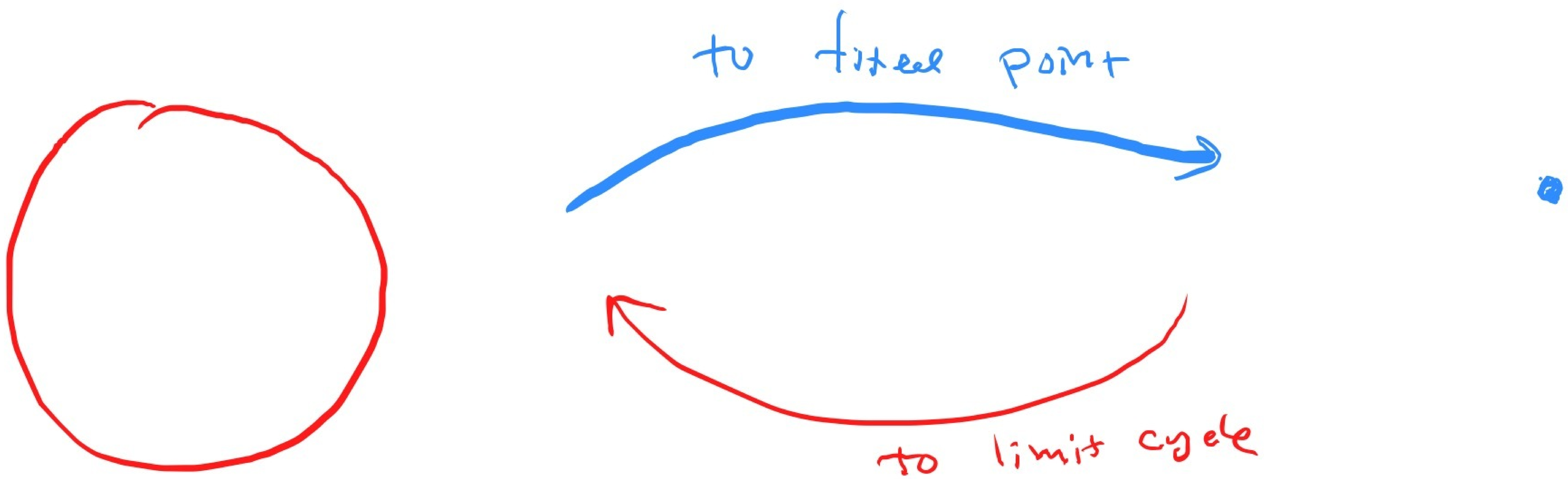


← limit cycle ⟸ regular repetitive spiking

Q: how do we go from firing to silence?

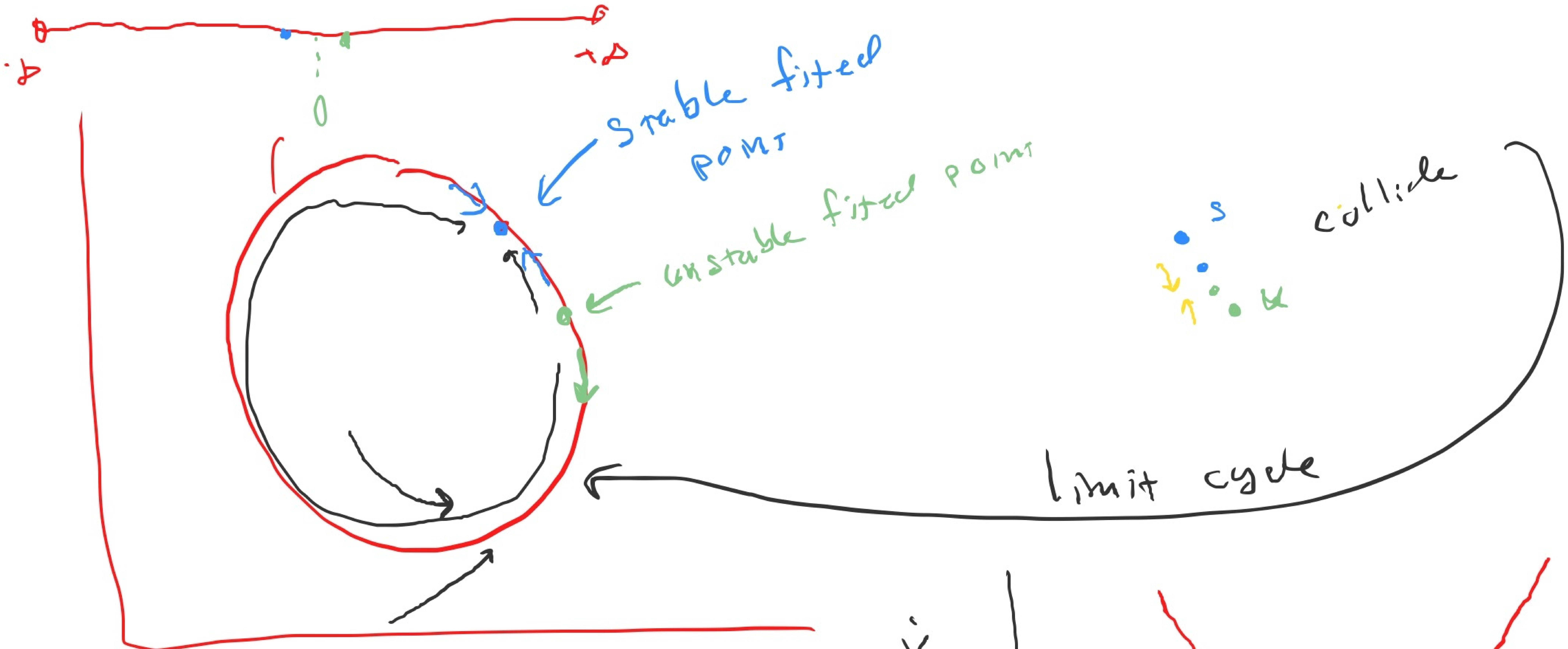
Why do we care?

- eventually we're going to make reduced models of neurons.



1. Saddle-node bifurcation

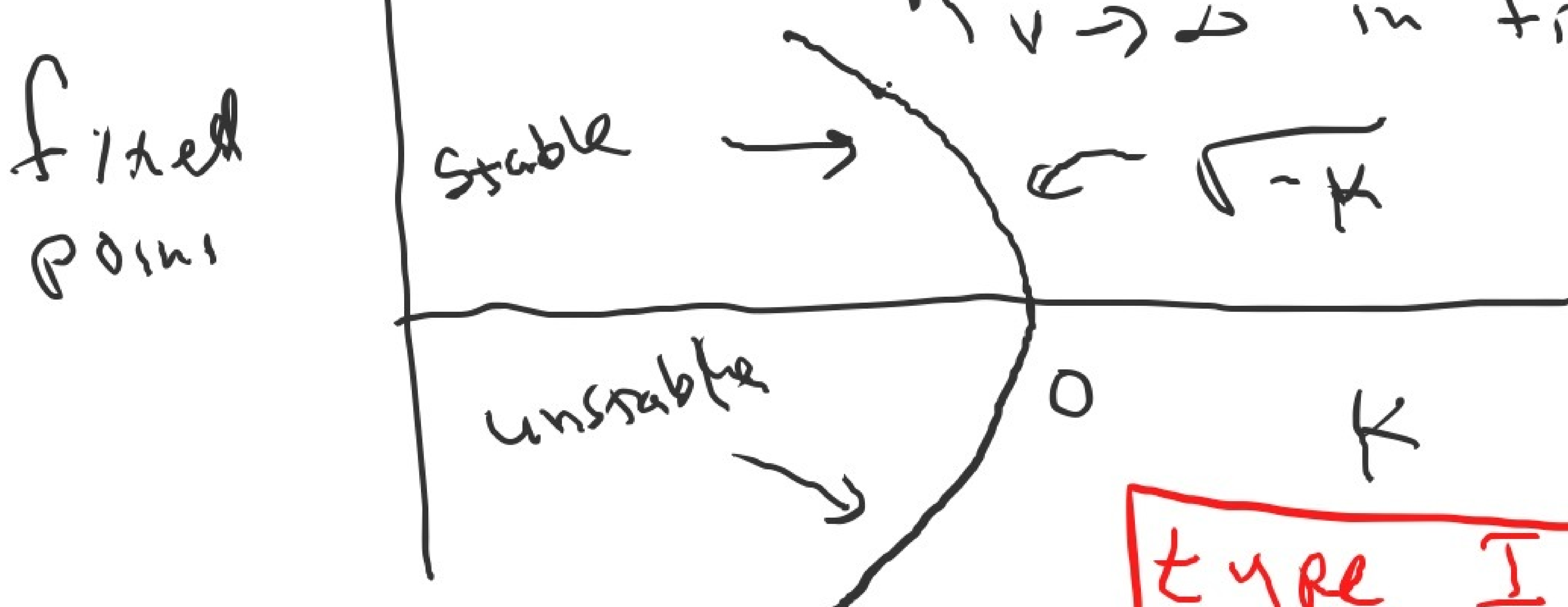
2. Hopf bifurcation



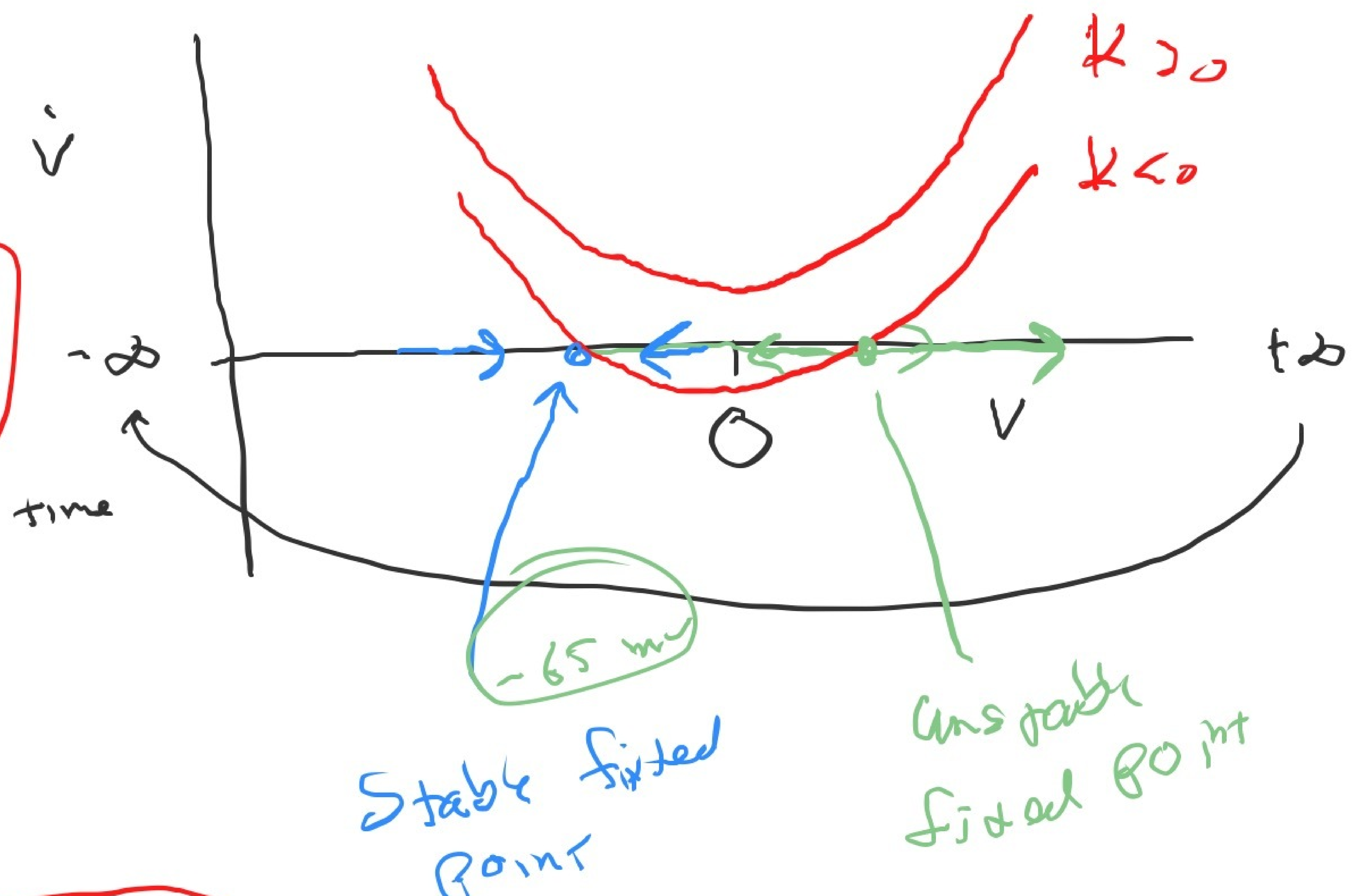
QIF = quadratic integrate and fire

$$\boxed{\dot{V} = k + V^2}$$

↖ $V \to \infty$ in finite time



$\boxed{\text{type I neurons}}$



$$\dot{V} = V^2 + K \qquad K > 0$$

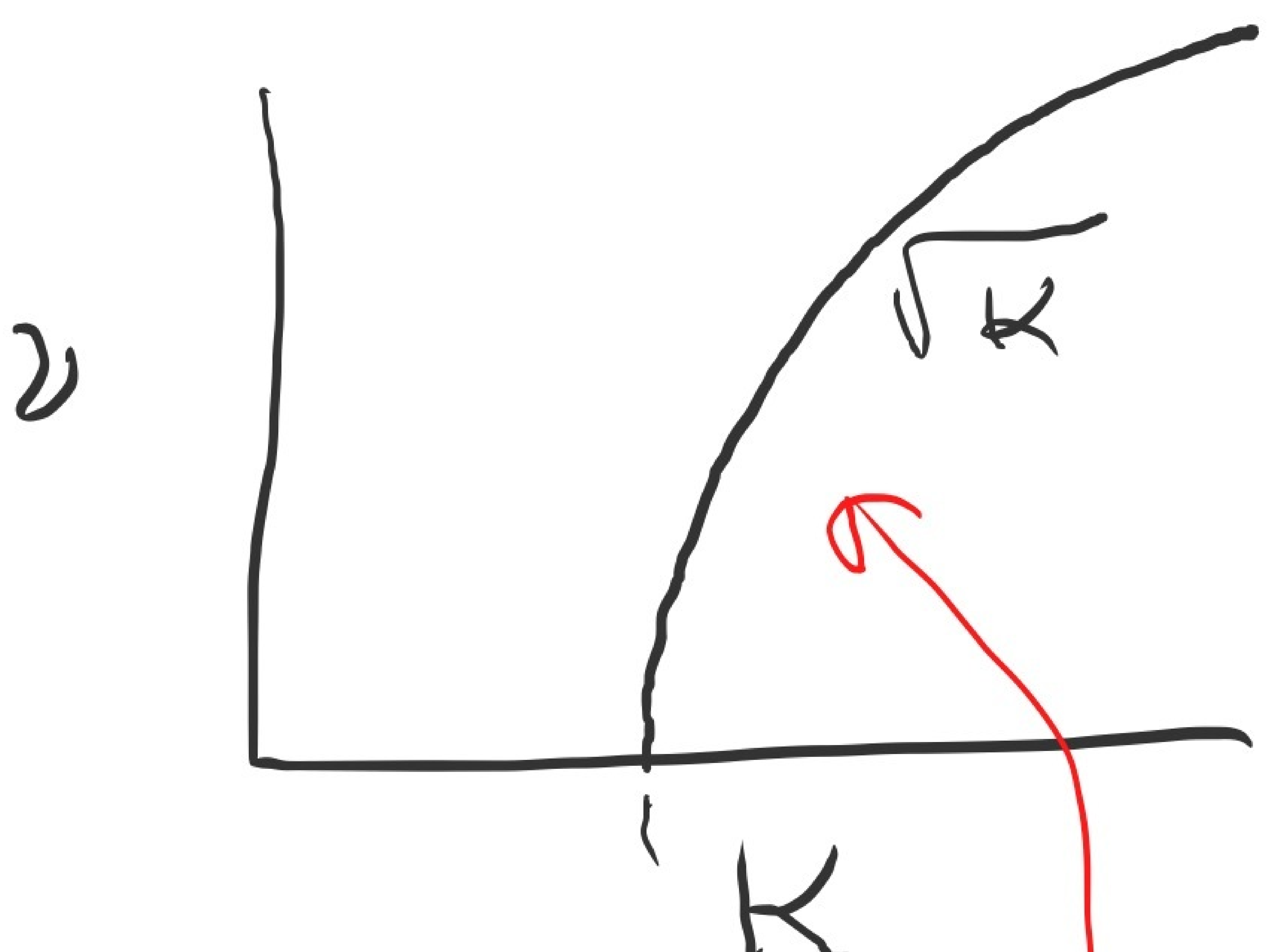
$$V = +\infty \longrightarrow -\infty$$

$$\tan\frac{\theta}{2} = v$$

$$\dot{\theta}$$

θ-neuron

$$\frac{dv}{v^2+K} = dt \qquad T = \int_{-\infty}^{\infty} \frac{dv}{v^2+K}$$



$$v = \sqrt{K}\, u$$

$$dv = \sqrt{K}\, du$$

π?

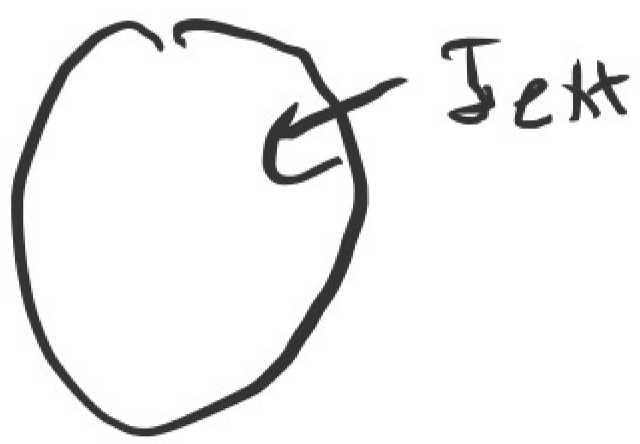
$$T = \frac{\sqrt{K}}{K} \int_{-\infty}^{\infty} \frac{du}{u^2+1} = \frac{1}{\sqrt{K}} \int_{-\infty}^{\infty} \frac{du}{u^2+1}$$

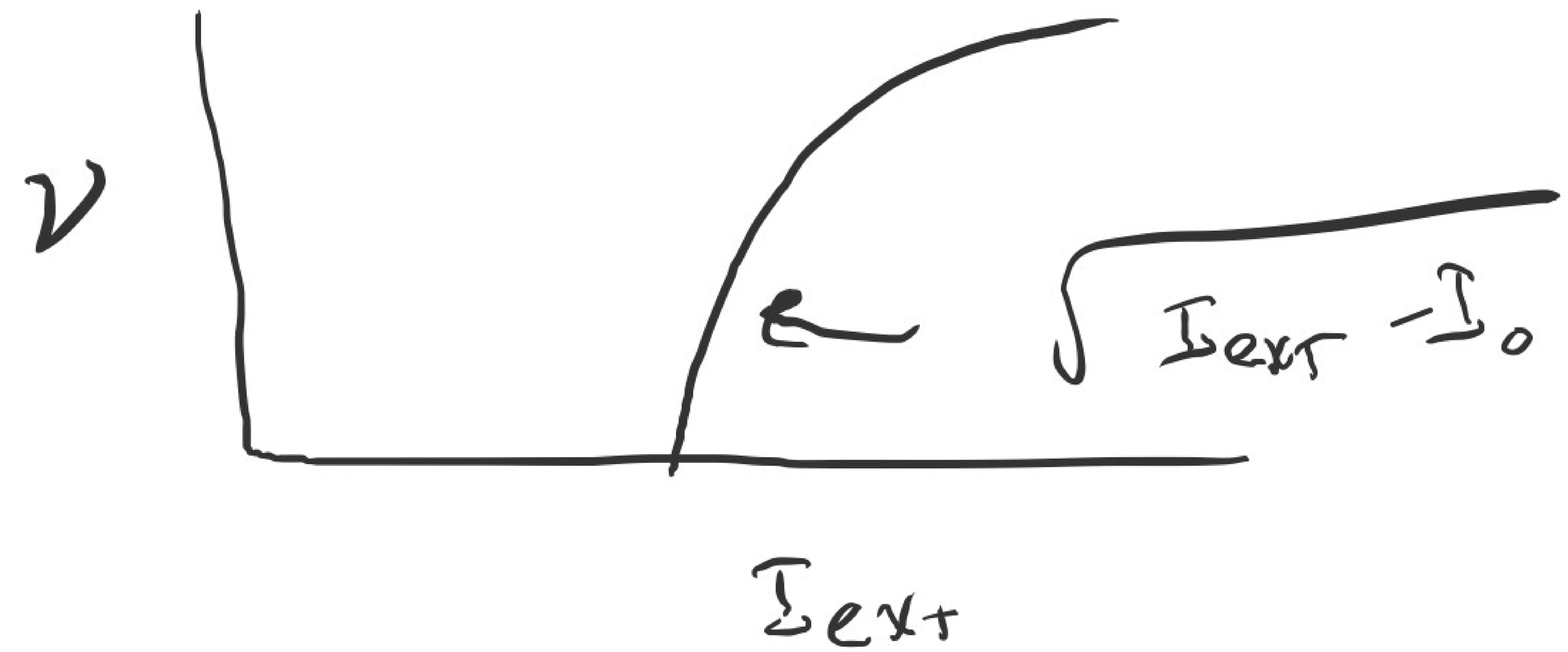
$$\nu = \frac{1}{T} \propto \sqrt{K}$$

$$\tau \frac{dv}{dt} = -(v - E_L) - \rho_{Na}\, m^3 h\, (v - E_{Na}) - \rho_K\, n^4 (v - E_K) + I_{ext}$$

m is fast

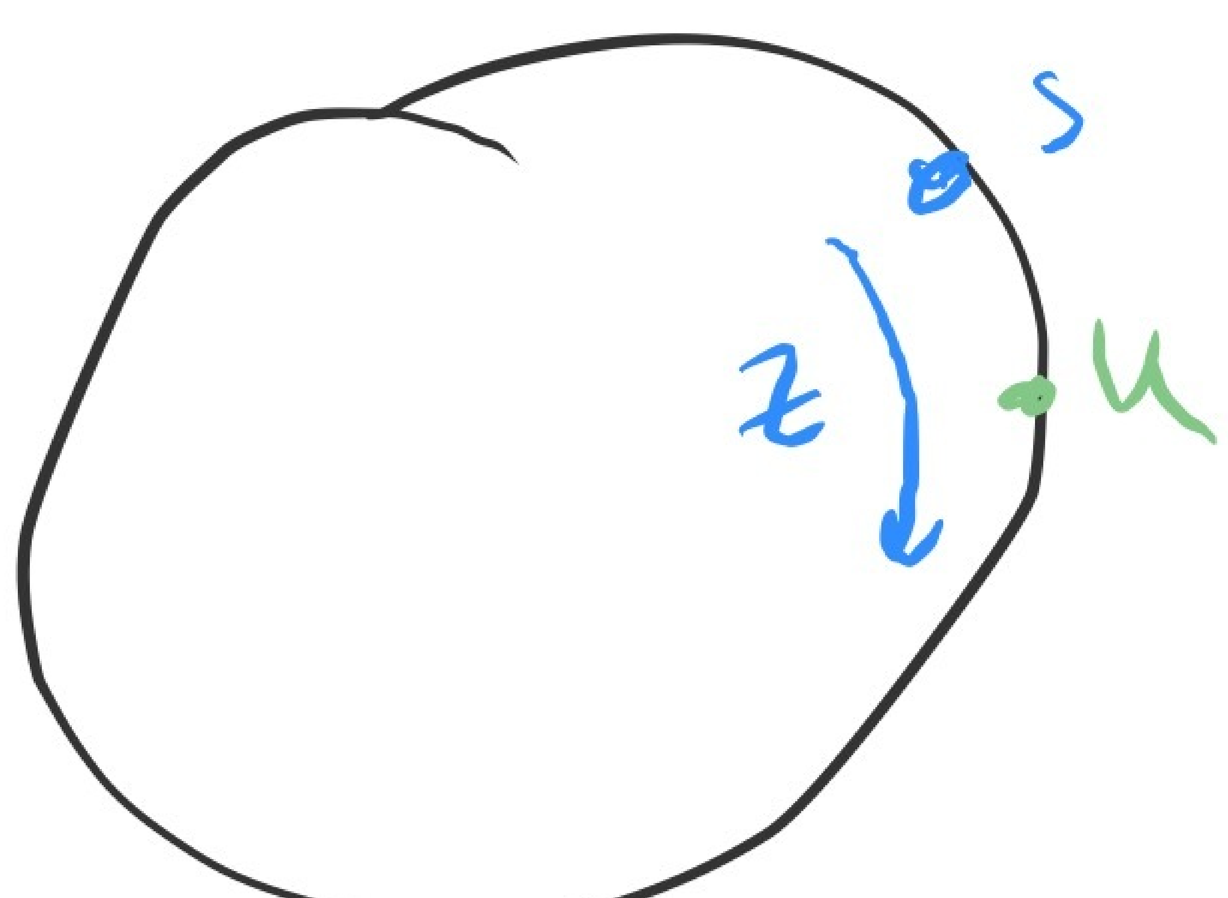
K

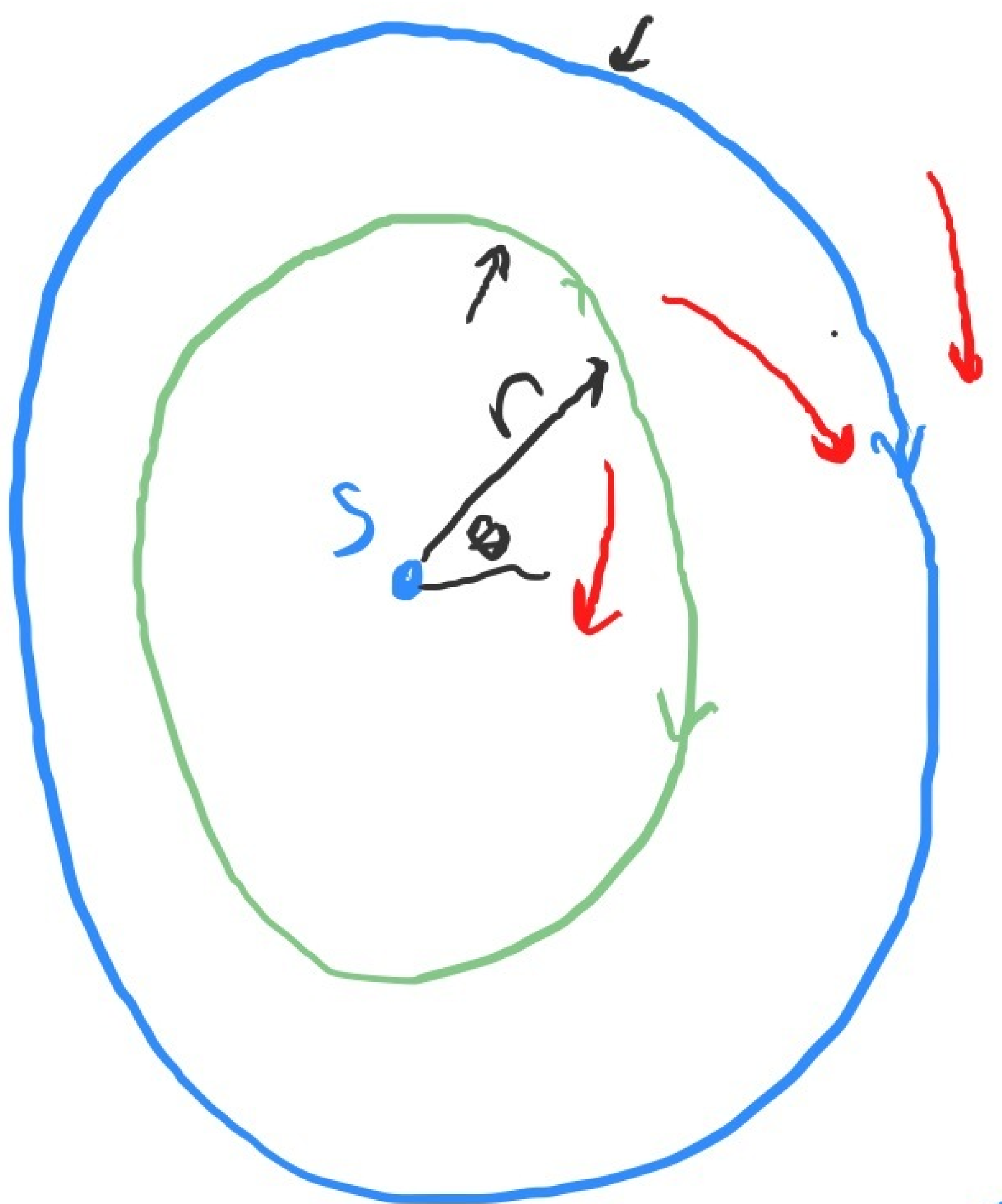




Bard Ermentrout : proved behavior at low freq. governed by

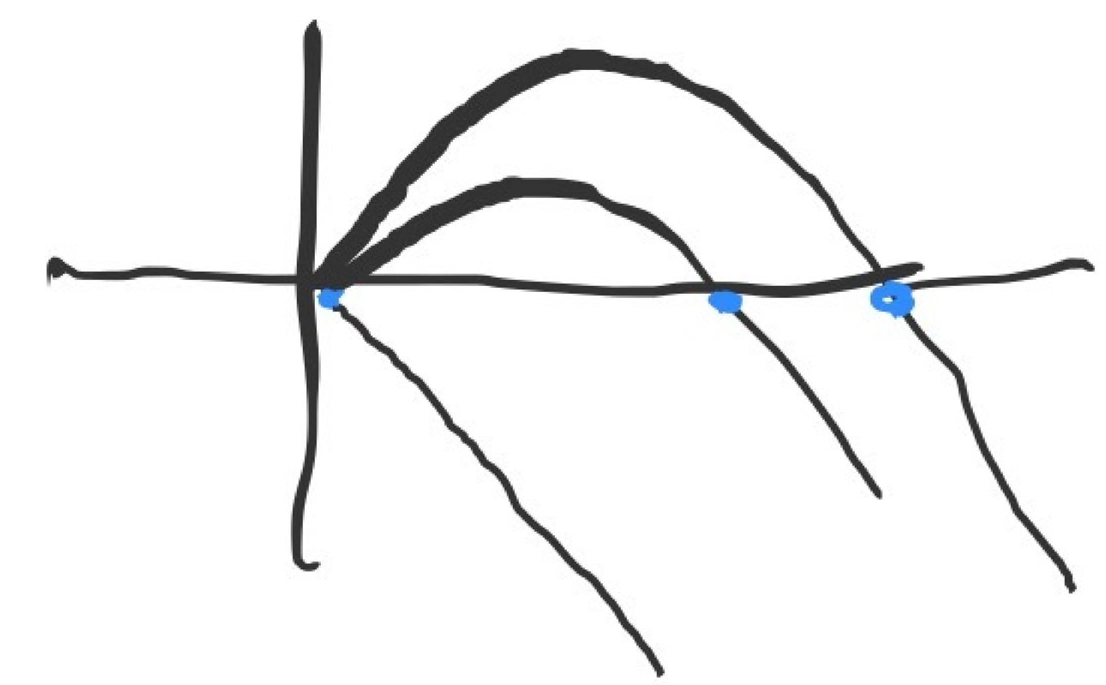
$$\dot{z} = K + z^2$$

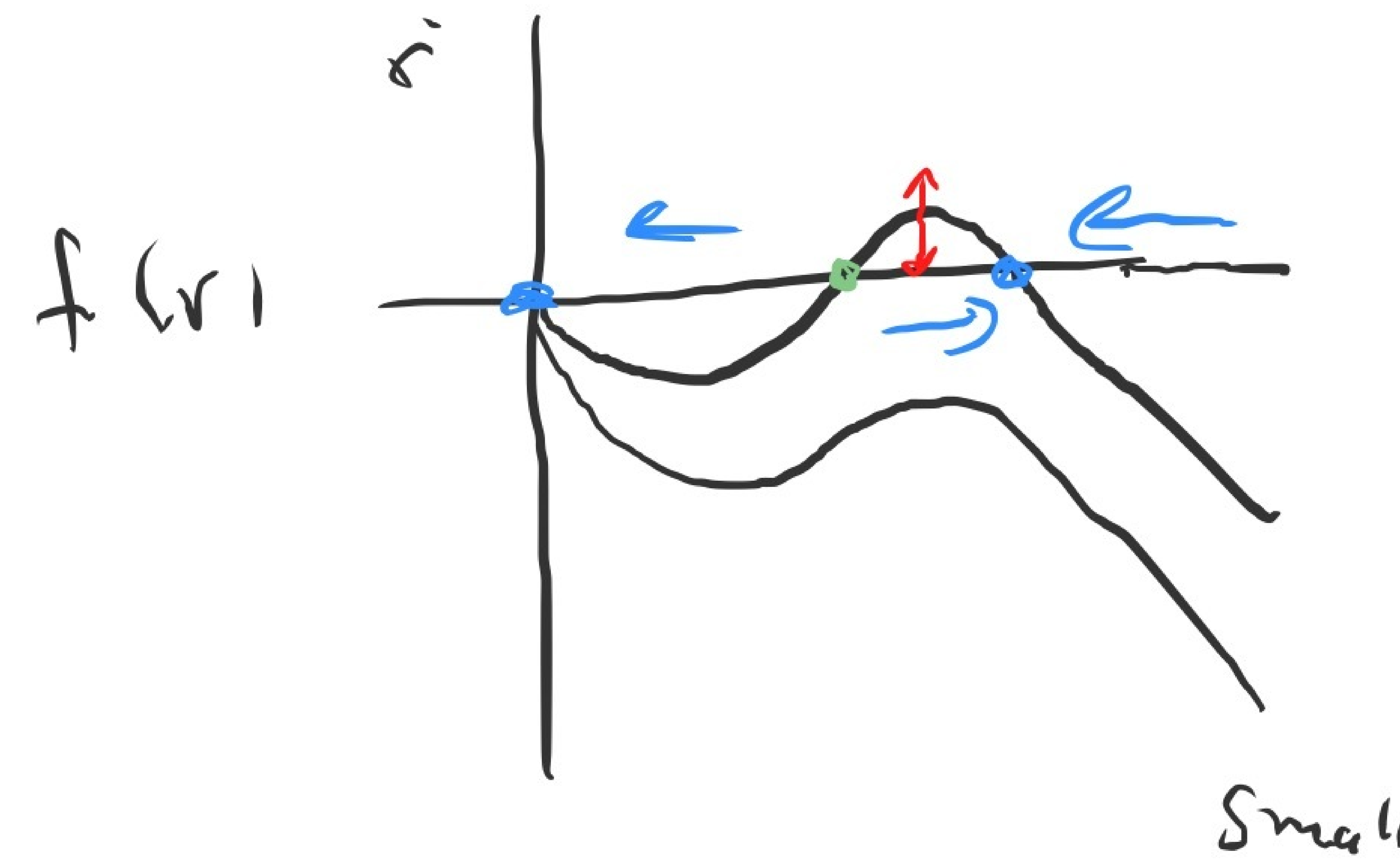


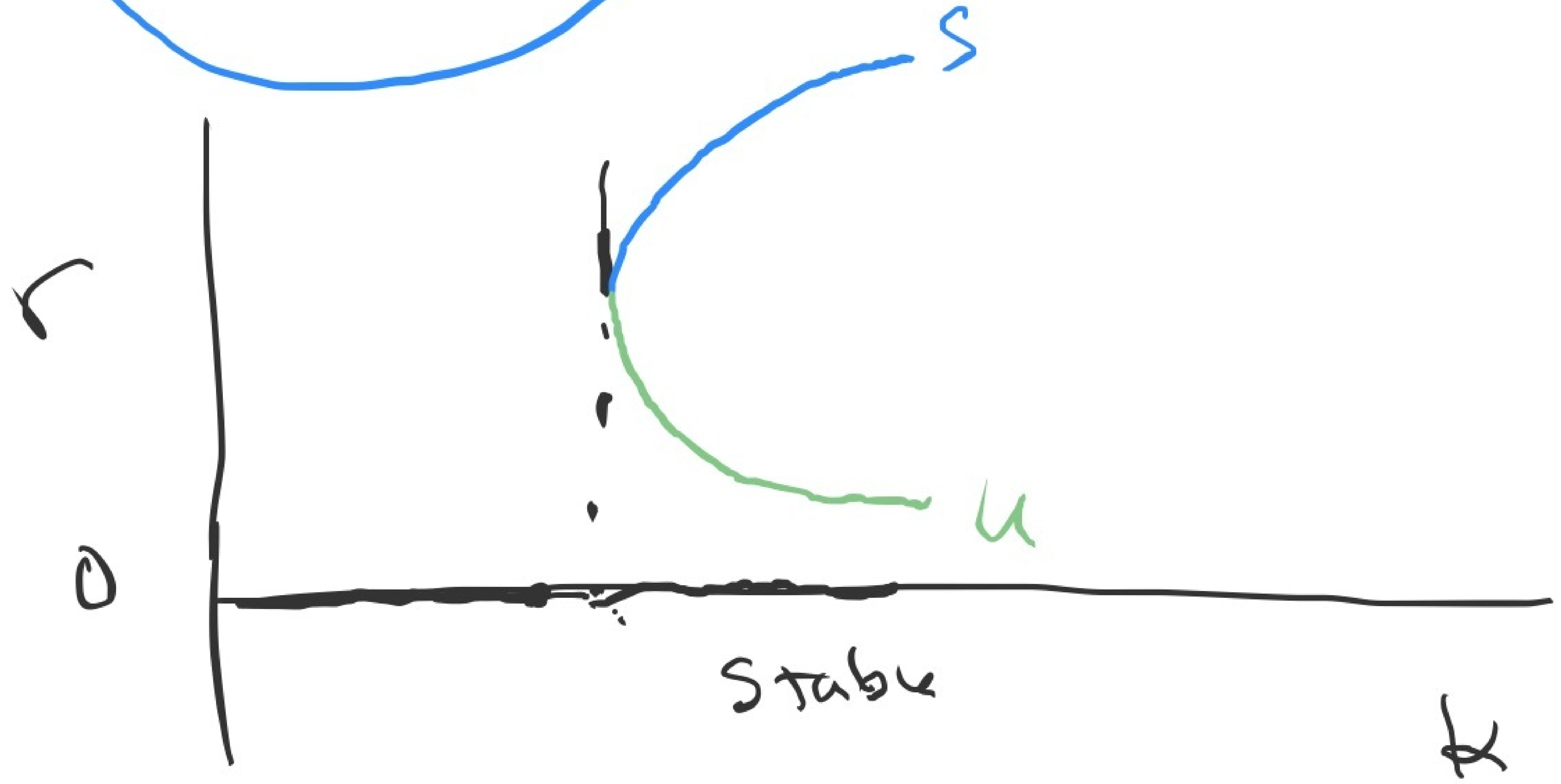


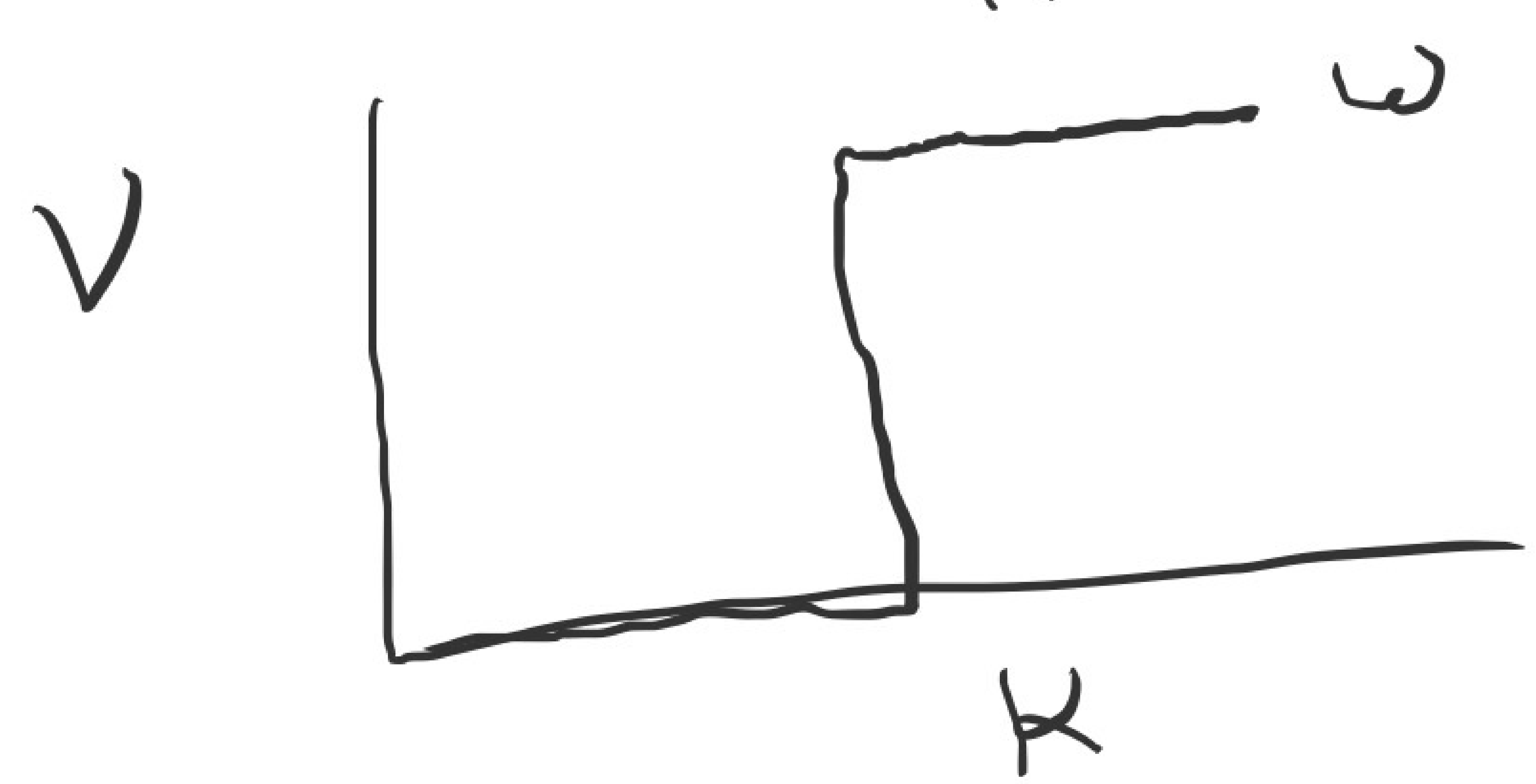
$$\dot{r} = f(r, \kappa)$$

$$\dot{\theta} = \omega$$









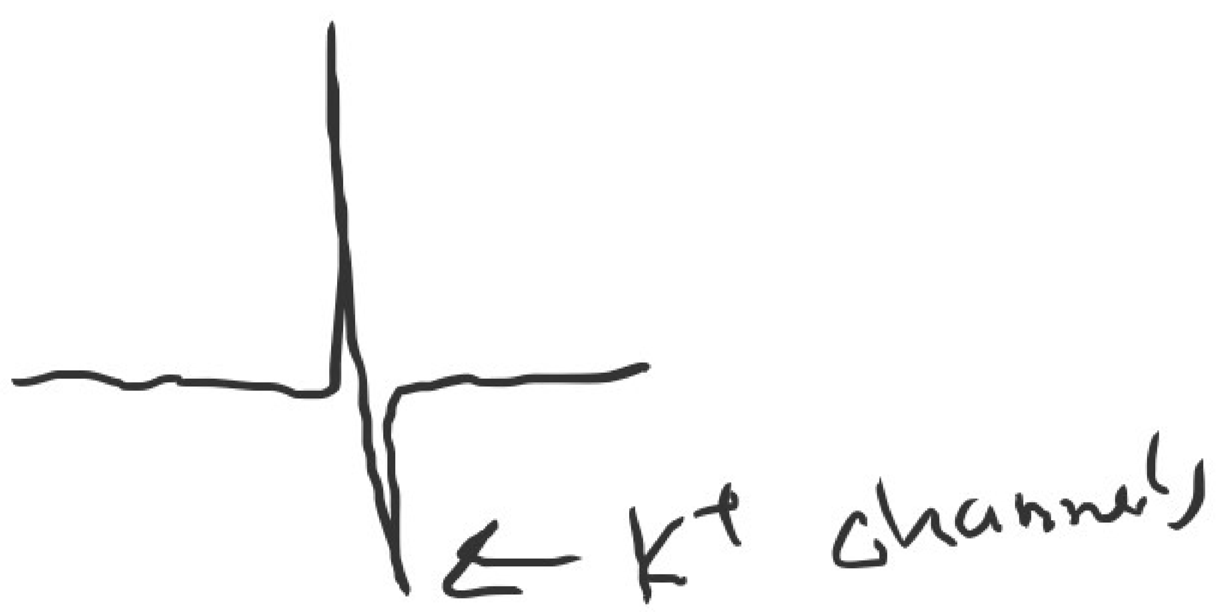
type II

Hopf

$$\tau \frac{dv}{dt} = -(v - E_L) - \rho_{Na} m^3 h (v - E_{Na}) - \rho_K n^4 (v - E_K) + I_{ext}$$

$\tau(v) \frac{dm}{dt} = m_\infty - m$ $m \approx m_\infty(v)$ appr #1 good

τ ↳ sub-ms $\rho_K = 0$ appr #2 less good



$$0 = \tau \frac{dv}{dt} = -(v - E_L) - \rho_{Na} m_\infty^3(v) h (v - E_{Na}) + \overbrace{R I_{ext}}^{\text{units of voltage}}$$

$$0 = \tau_h \frac{dh}{dt} = h_\infty(v) - h$$

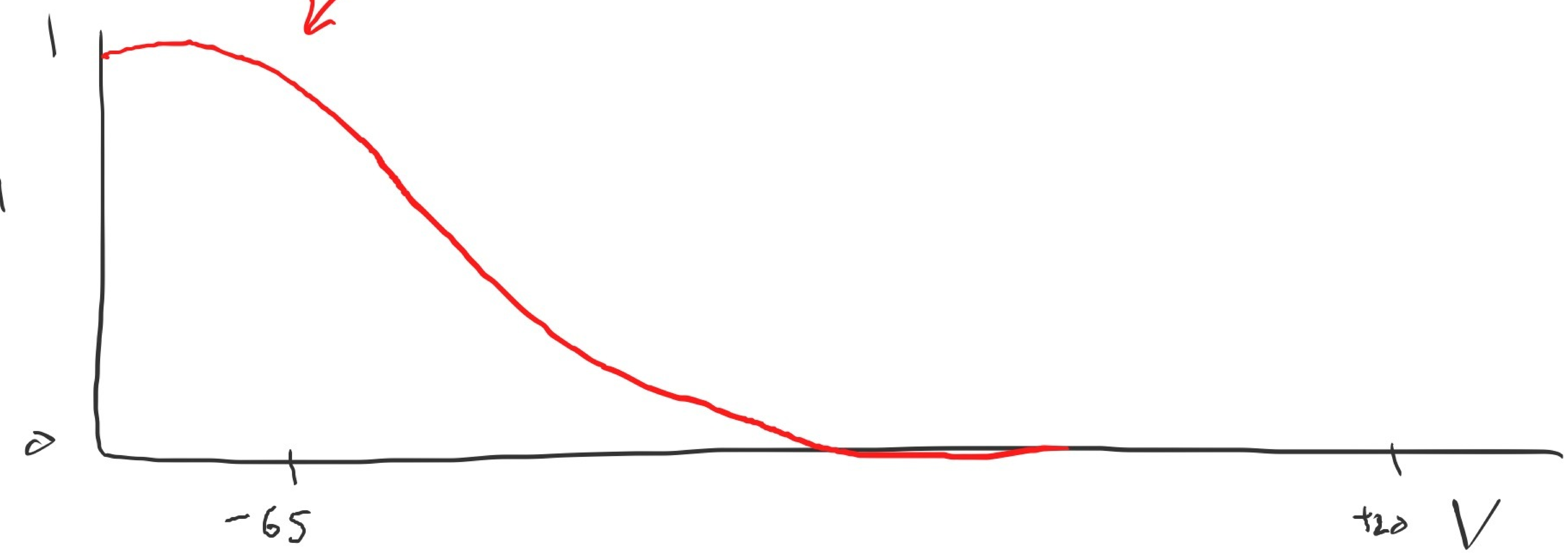
solve

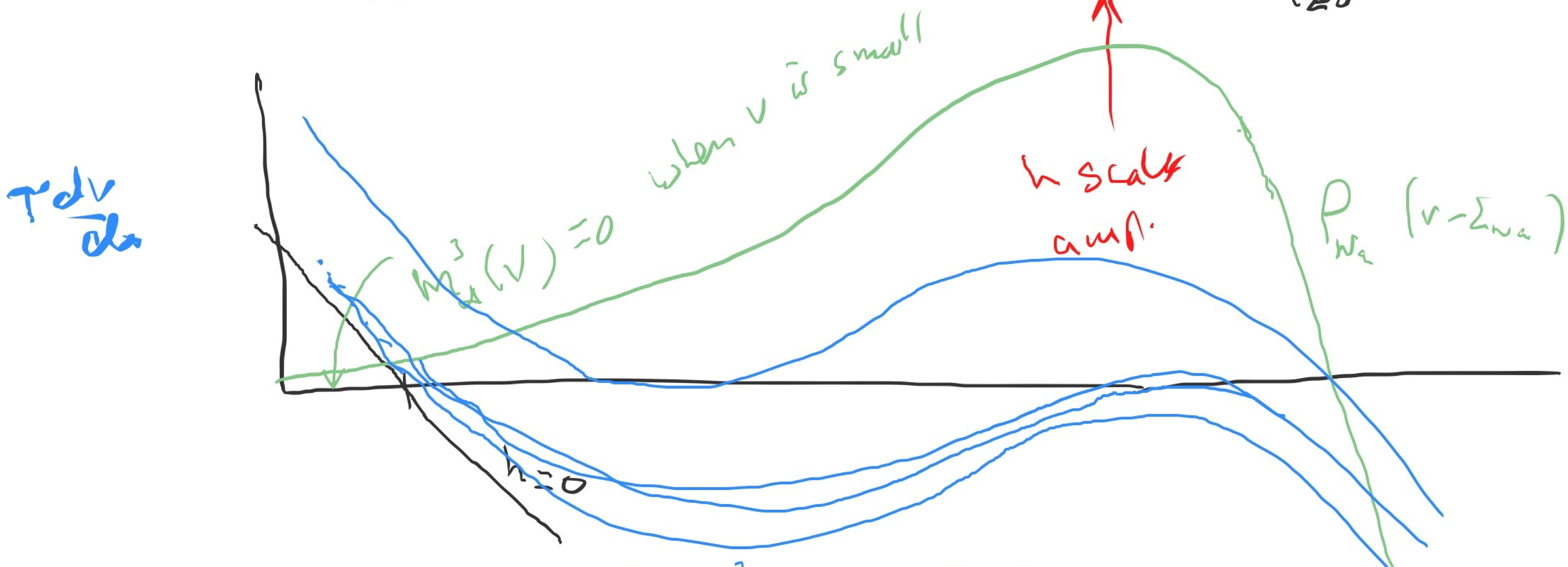
nullcline analysis (only works in 2-D)

$$0 = \tau \frac{dv}{dt} = -(v - E_L) - g_{Na} m_\infty^3 h (v - E_{Na}) = 0$$

$$0 = \tau_h \frac{dh}{dt} = h_\infty(v) - h$$

h-nullcline $h = h_\infty(v)$





$$0 = -(v-\Sigma_{L}) - \rho_{w_a} M_a^3 h (v - \Sigma_{w_a})$$

−65   +20