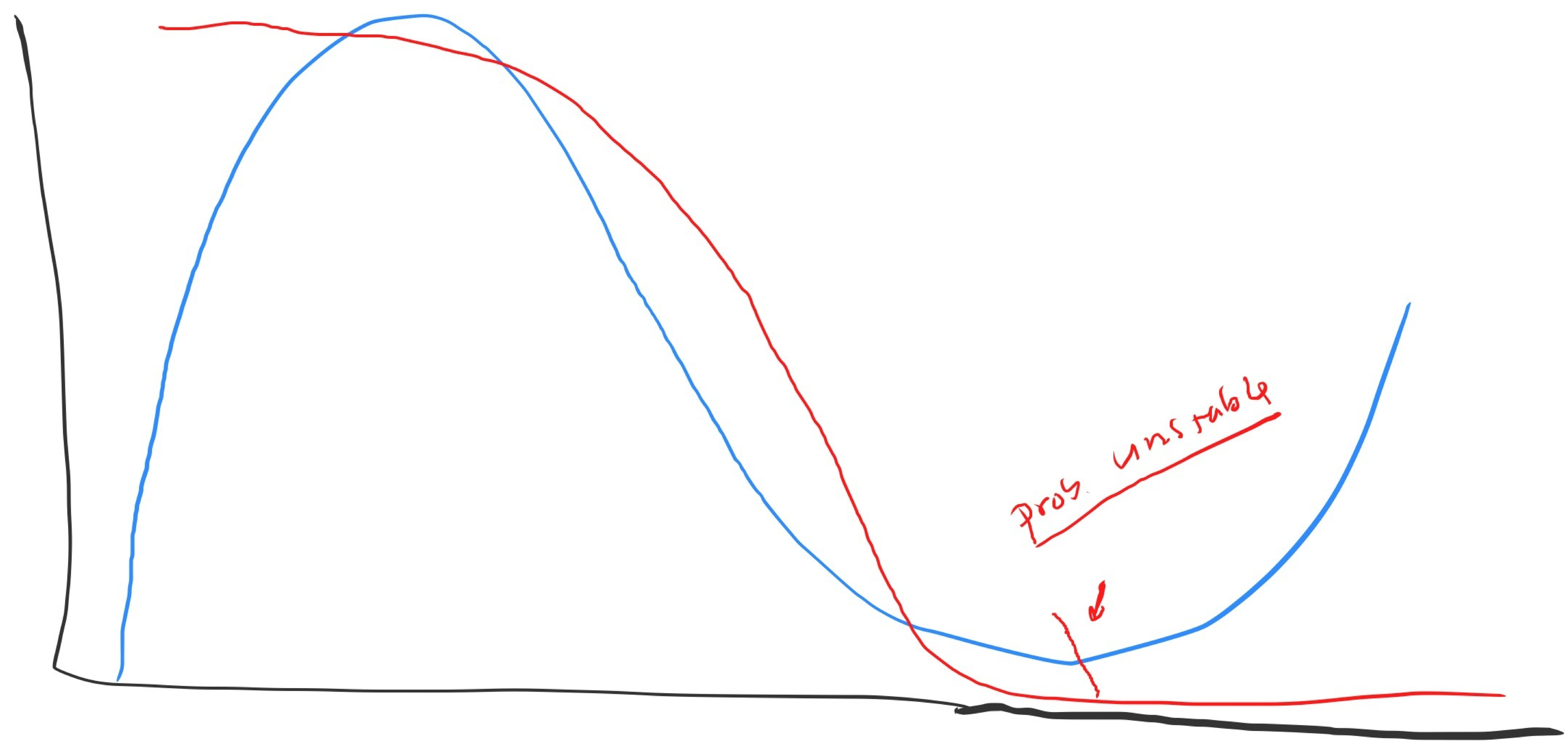
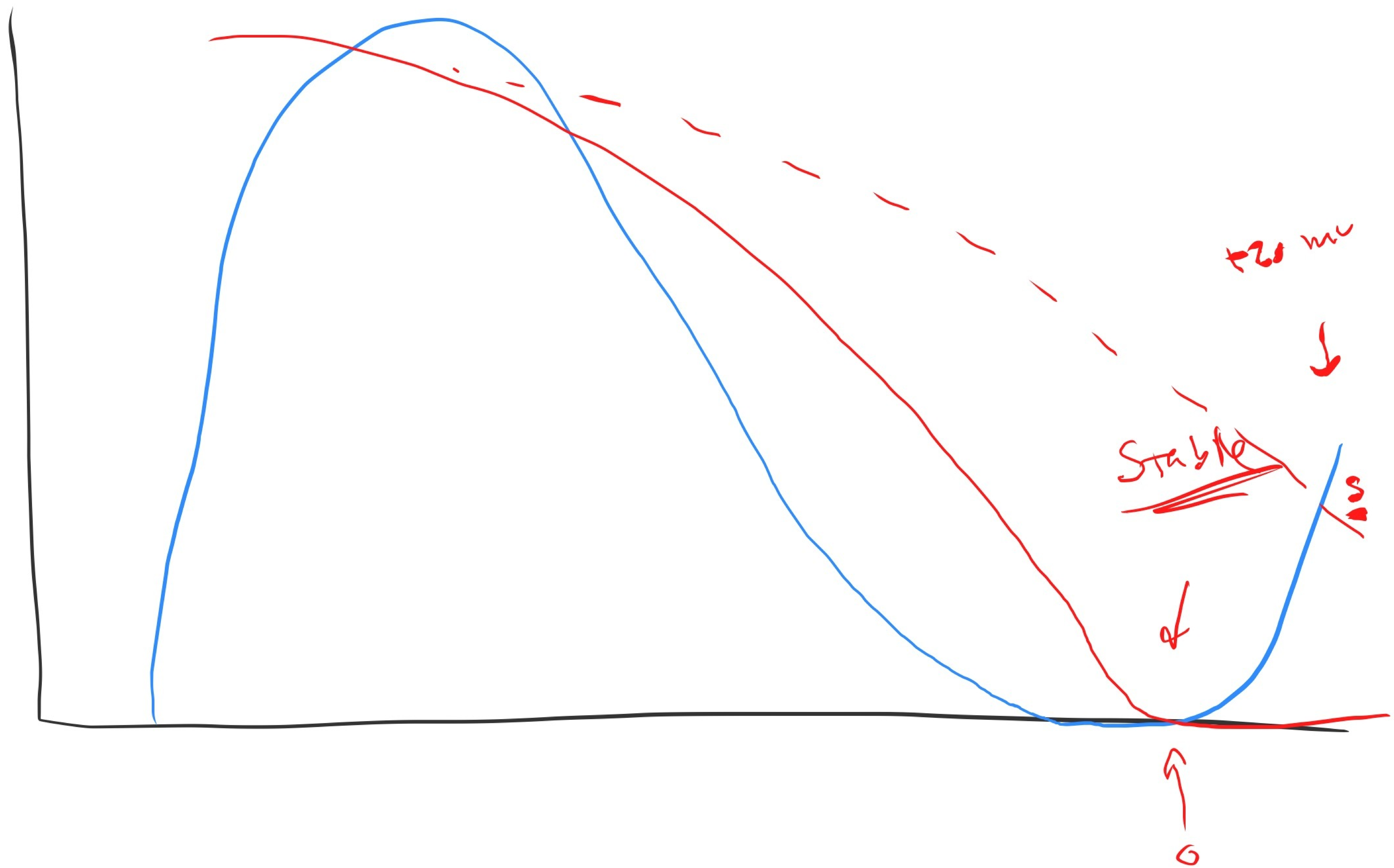
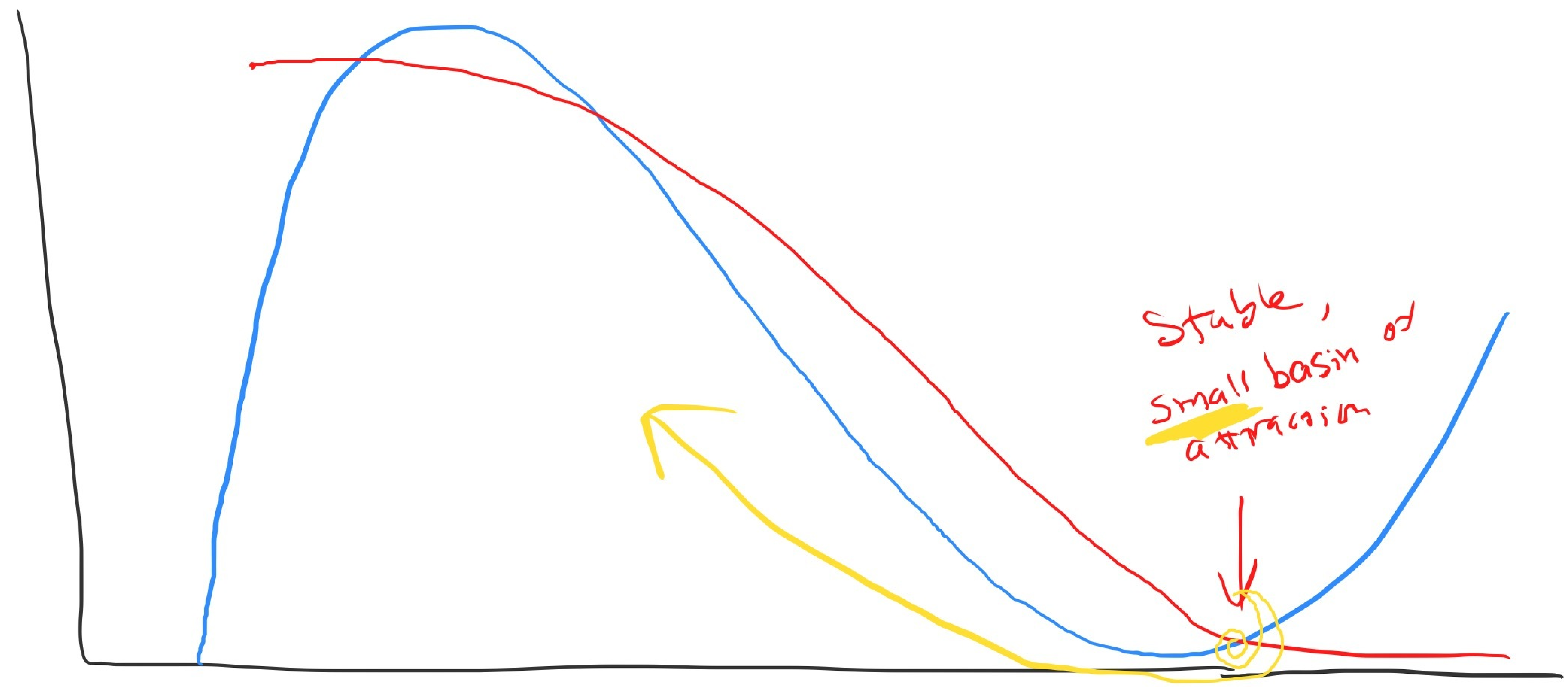
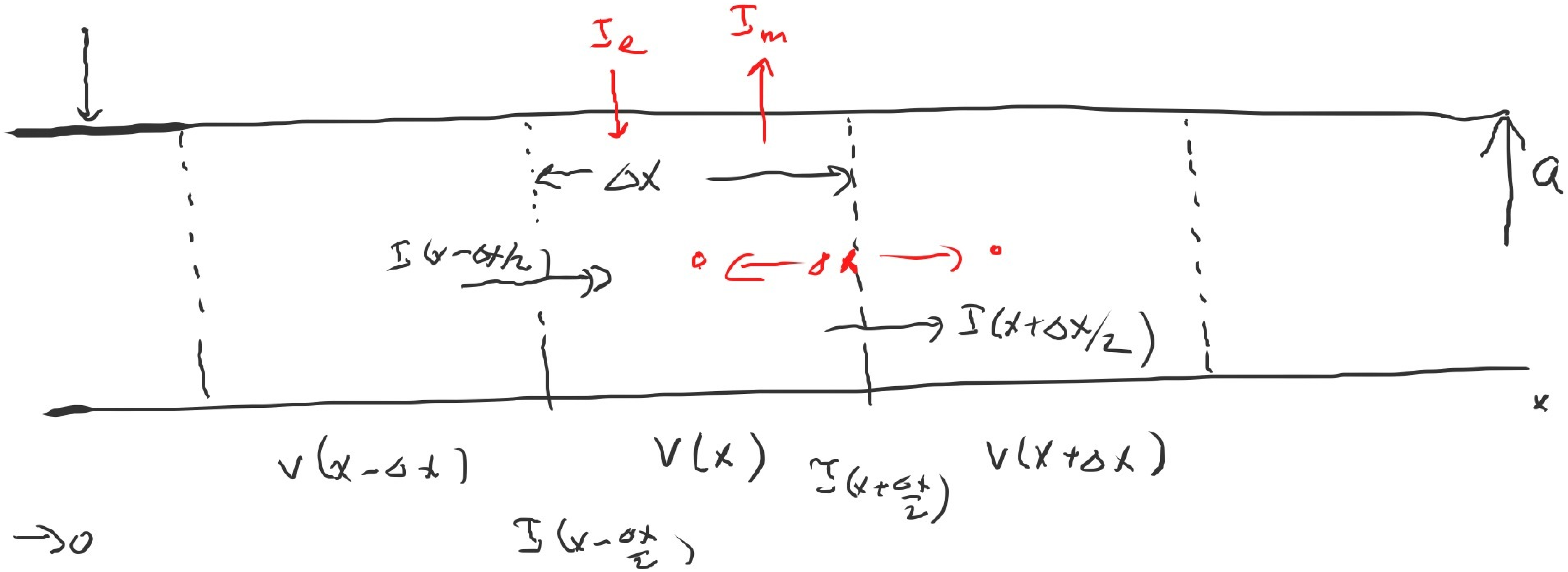

pros. unstable



+20 mV
Stable
0


Stable,
small basin of
attracation



$\lim \Delta x \to 0$

$$C\frac{\partial V(x,t)}{\partial t} = I\left(x-\frac{\Delta x}{2}\right) - I(x+\Delta x/2) - I_m + I_e \quad \leftarrow \quad \underline{c\frac{dv}{dt} = \text{current}}$$

$$I\left(x+\frac{\Delta x}{2}\right) = \frac{V(x)-V(x+\Delta x)}{R} \quad \leftarrow \quad \underline{I = \frac{V}{R}}$$

$$C\frac{\partial V(x,t)}{dt} = \frac{V(x-\Delta x)-V(x)}{R} - \frac{V(x)-V(x+\Delta x)}{R} - I_m + I_e$$

$$= \frac{V(x-\Delta x)-2V(x)+V(x+\Delta x)}{R} - I_m + I_e$$

$$c\frac{\partial V(x,t)}{\partial t} = \frac{V(x-\Delta x) - 2V(x) + V(x+\Delta x)}{\Omega} - I_m + I_e$$

Taylor expand →

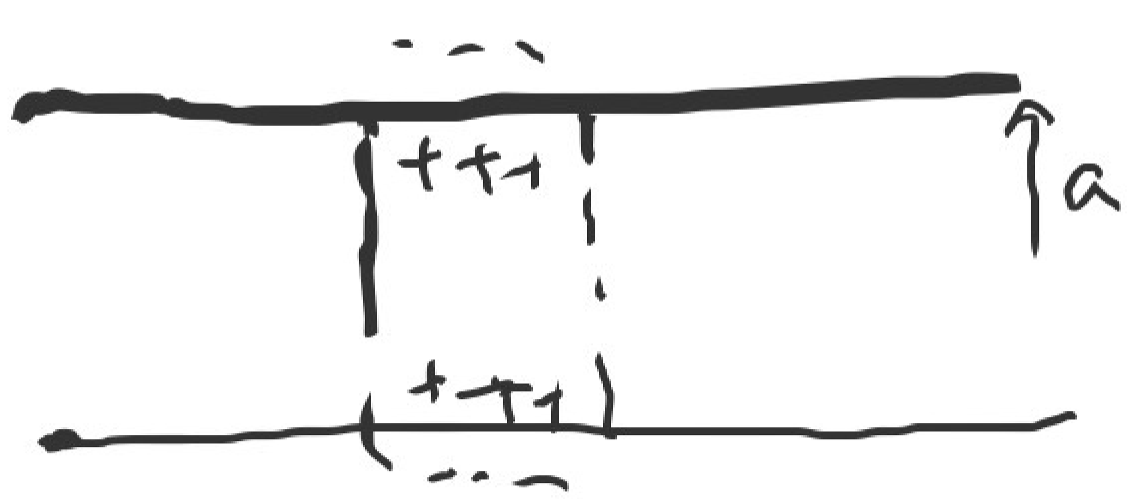
$$\cong \frac{\cancel{V(x)} - \cancel{\Delta x \frac{\partial V}{\partial x}} + \frac{\Delta x^2}{2}\frac{\partial^2 V}{\partial x^2} \cancel{-2V(x)} + \cancel{V(x)} + \cancel{\Delta x\frac{\partial V}{\partial x}} + \frac{\Delta x^2}{2}\frac{\partial^2 V}{\partial x^2}}{R}$$

$-I_m + I_e$

$C_m 2\pi a \Delta x$

$$c\frac{\partial V}{\partial t} = \frac{\Delta x^2 \frac{\partial^2 V}{\partial x^2}}{R} - I_m + I_e$$

$R \leftarrow r_L \, \Delta x / \pi a^2$



$C \propto$ area

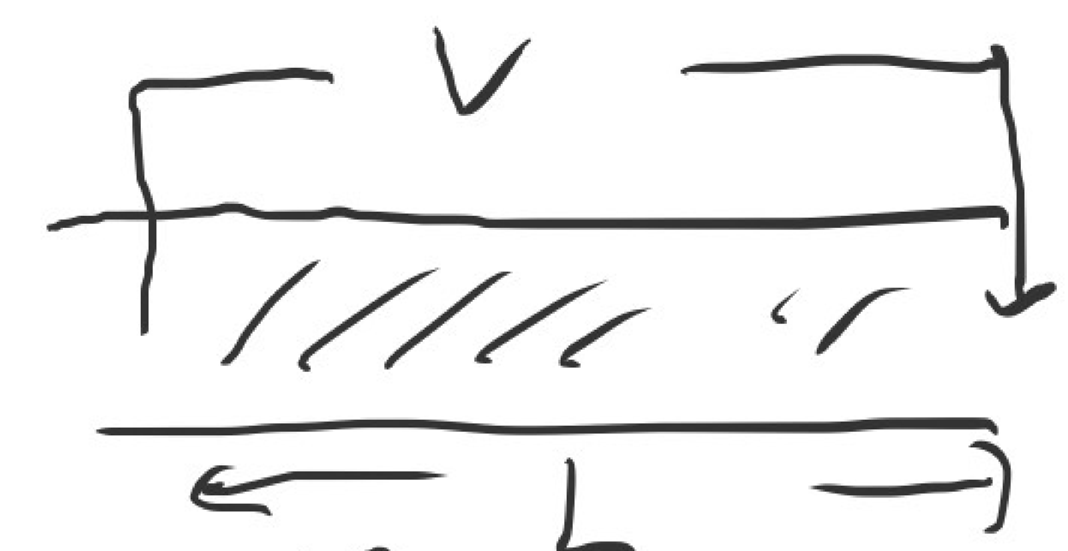
$Q = CV$

$\propto$ area

$\frac{\Delta x}{\pi a^2}$

$R = r_m \frac{L}{A} \Leftarrow I \propto \frac{A}{L} \propto \frac{1}{R}$

$I = \frac{V}{R}$



$$C_m 2\pi a \Delta x \frac{\partial v}{\partial t} = \frac{1}{r_L \Delta x / \pi a^2} \frac{\Delta x^2 \partial^2 v}{\partial x^2} - I_m + I_e$$

$\tau \cong 10$ ms

i_m = current density

i_e = current density

$$r_m C_m \frac{\partial v}{\partial t} = \frac{a r_m}{2 r_L} \frac{\partial^2 v}{\partial x^2} - \frac{I_m r_m}{2\pi a \Delta x} + \frac{I_e r_m}{2\pi a \Delta x}$$

λ^2

$R \propto r_L \frac{\ell}{A}$

ℓ

$R = \frac{r_m}{A}$ ← function of membrane

λ = electrotonic length

$$r_m c_m = AR \frac{C}{A} = RC = \tau \simeq 10 \text{ ms}$$

$$C \frac{dv}{dt} = -\frac{1}{R}(v - E_L) + \ldots \text{HH}$$

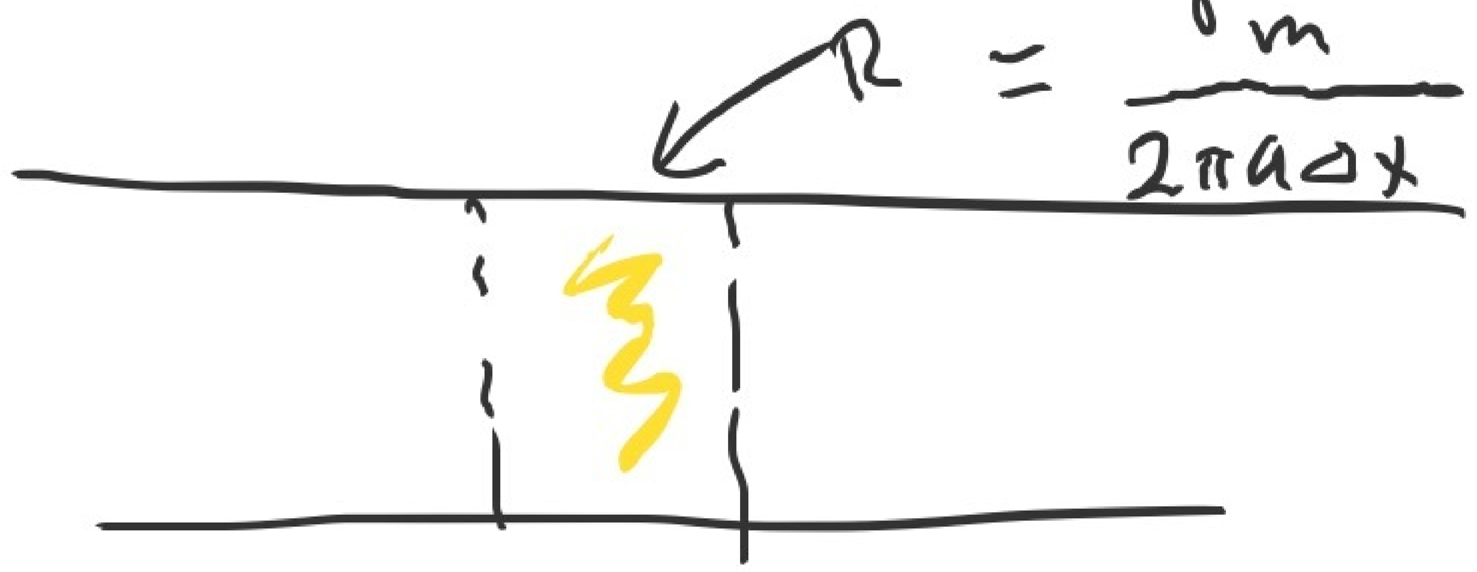
$RC = \tau$

$$\tau \frac{\partial v}{\partial t} = \lambda^2 \frac{\partial^2 v}{\partial x^2} - i_m r_m + i_e r_m$$ Cable equation

passive

$$i_m = \frac{I_m}{2\pi a \Delta x} = \frac{(V - E_L)}{R} \frac{1}{2\pi a \Delta x} = \frac{(V - E_L)}{\frac{r_m}{2\pi a \Delta x} \cdot 2\pi a \Delta x} = \frac{(V - E_L)}{r_m}$$

$$R = \frac{r_m}{2\pi a \Delta x}$$



$$i_m r_m = V - E_L$$

$$\tau \frac{\partial V}{\partial t} = \lambda^2 \frac{\partial^2 V}{\partial x^2} - (V - E_L) + i_e r_m$$

$$u = V - E_L$$

$$\tau \frac{\partial u}{\partial t} = \lambda^2 \frac{\partial^2 u}{\partial x^2} - u + i_e r_m$$

passive cable equation

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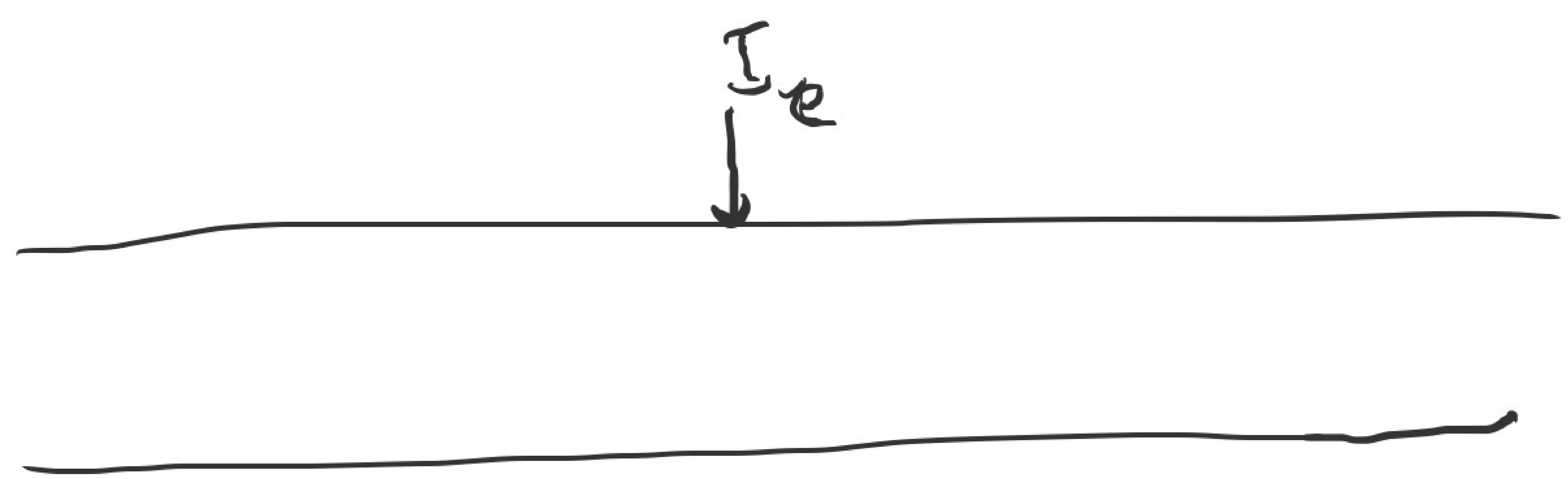
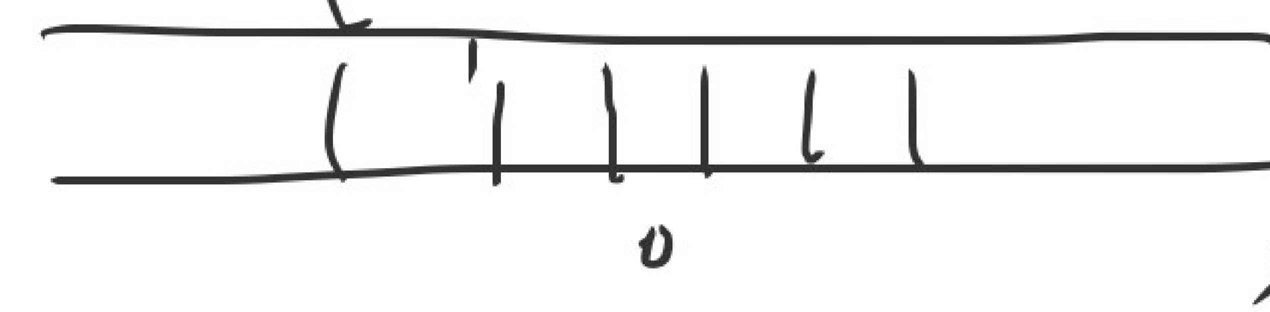
1. Steady state: $\frac{\partial u}{\partial t} = 0$

2. non " " :



$$\lambda^2 \frac{\partial^2 u}{\partial x^2} = u - i_e r_m$$

$$i_e r_m = \begin{cases} \dfrac{I_e r_m}{2\pi a \Delta x} & \text{at } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

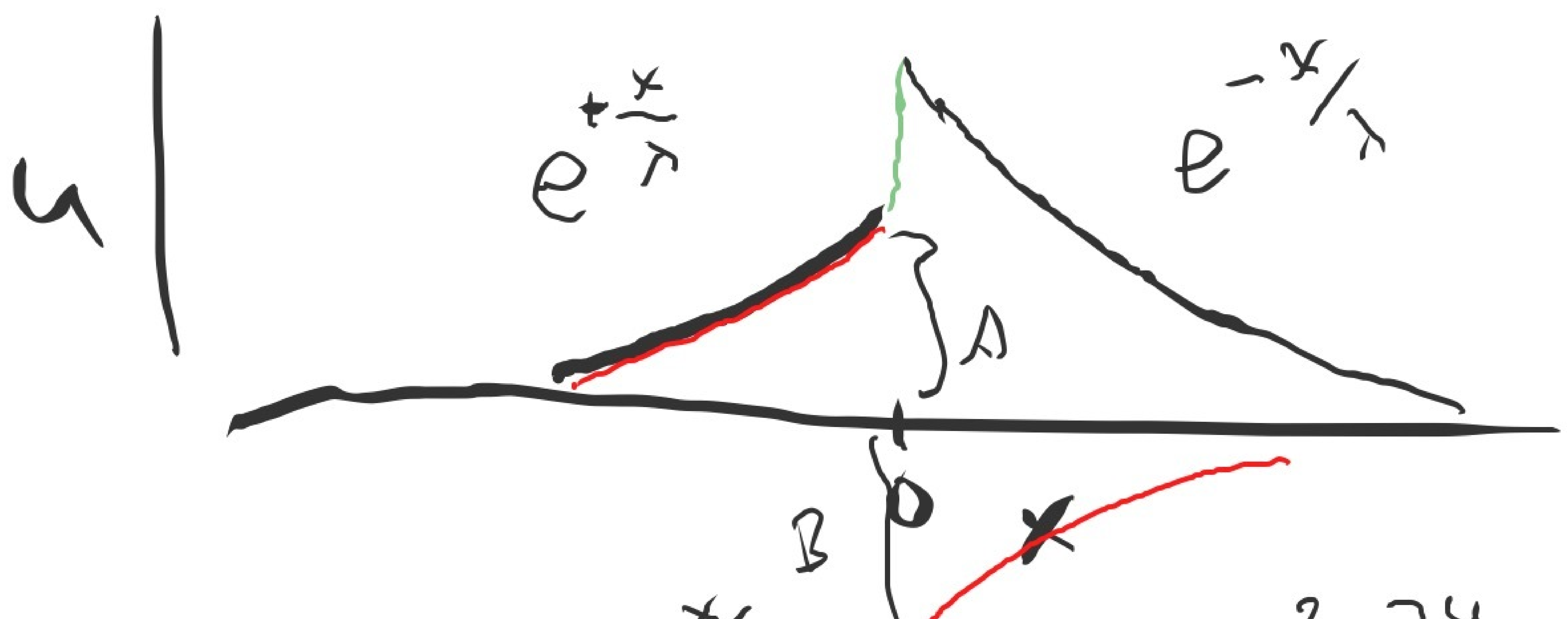


$$\lambda^2 \frac{\partial^2 u}{\partial x^2} = u - \frac{I_e r_m}{2\pi a} \delta(x)$$

$$i_e r_m = \frac{I_e r_m}{2\pi a} \delta(x)$$

$x \neq 0$ $\quad \lambda^2 \frac{\partial^2 u}{\partial x^2} = u$

$$u(x) \propto e^{\pm \frac{x}{\lambda}}$$



$$u(x) = \begin{cases} A e^{x/\lambda} & x < 0 \\ B e^{-x/\lambda} & x > 0 \end{cases}$$

$$\lambda \frac{\partial u}{\partial x} = \begin{matrix} A e^{x/\lambda} & x < 0 \\ -B e^{x/\lambda} & x > 0 \end{matrix}$$

$$+ \lambda (A - B) \delta(x) \Rightarrow A = B$$

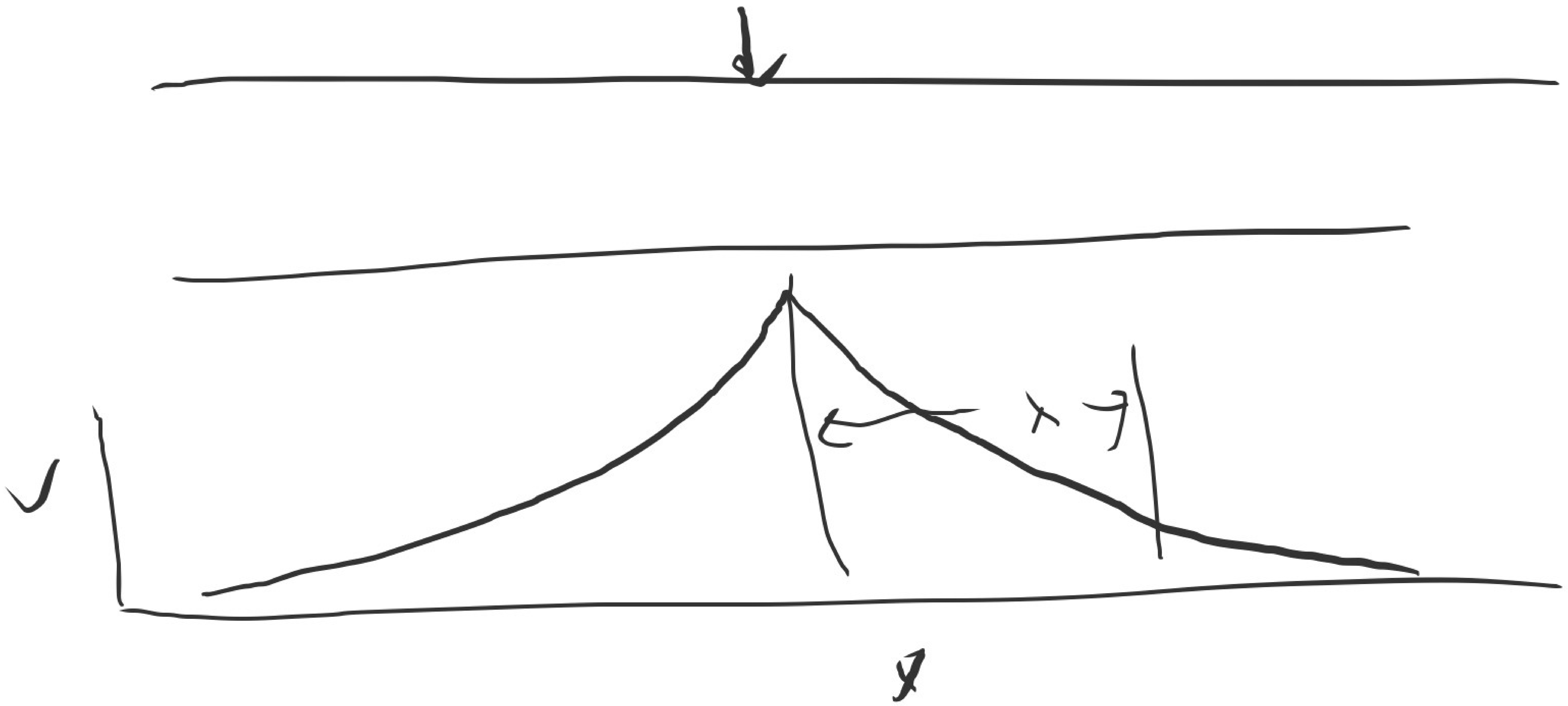
$$\lambda^2 \frac{\partial u}{\partial x} = \begin{matrix} A e^{x/\lambda} & x < 0 \\ +B e^{x/\lambda} & x > 0 \end{matrix}$$

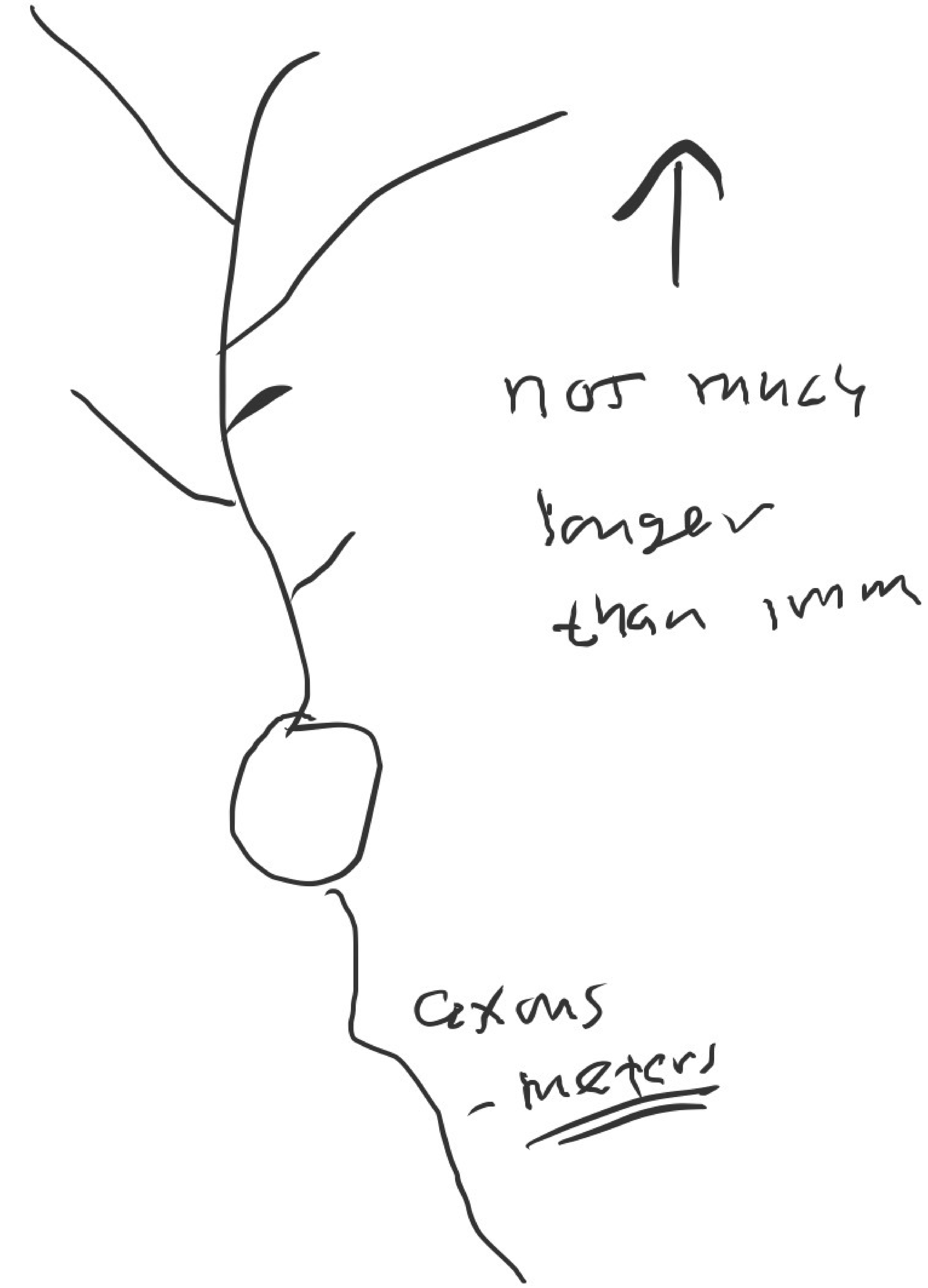
$$-(A+B)\lambda \delta(x)$$

$$\left.\begin{matrix} A = B \\ 2A = \dfrac{I_e r_m}{2\pi a} \end{matrix}\right\}$$

Steady state

$$V(x) = \frac{I_e r_m}{4\pi a} e^{-\frac{|x|}{\lambda}}$$

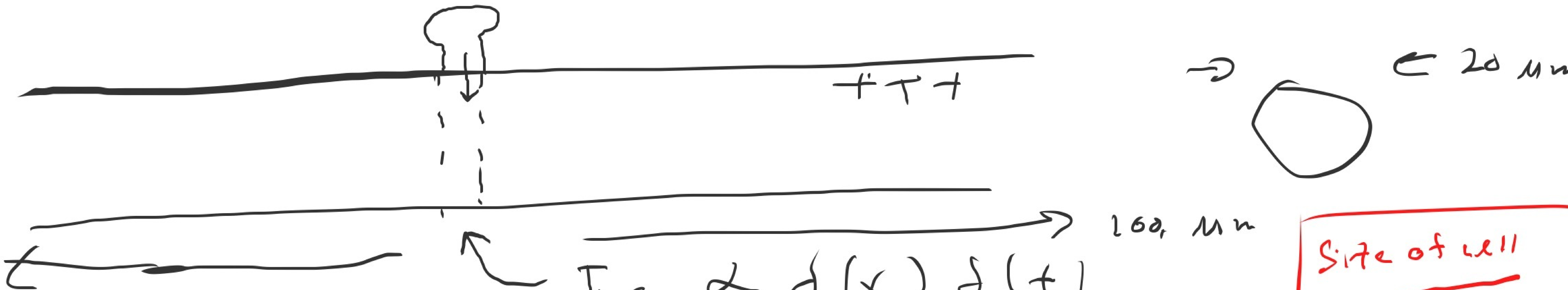




$$\lambda = \sqrt{\frac{a r_m}{2 r_L}}$$

$a \simeq 1$ micron

$\lambda \simeq 1$ mm



$I_e \propto \delta(x)\,\delta(t)$

Size of cell

speed

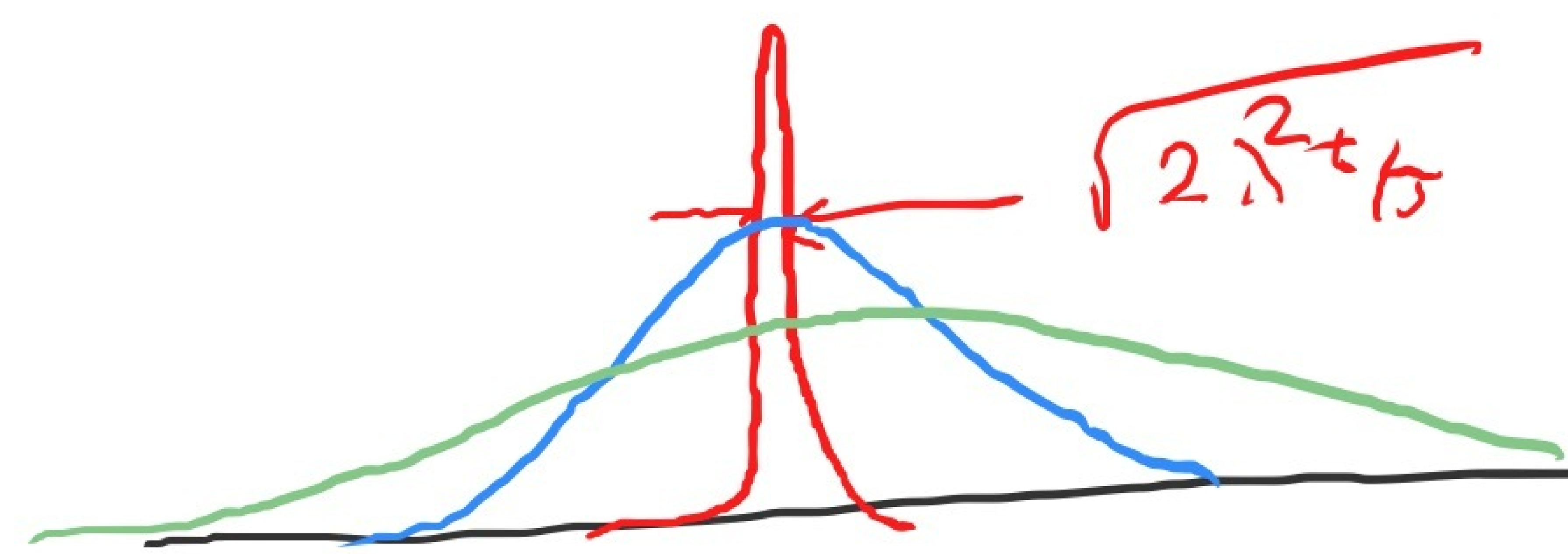
$\uparrow \frac{\lambda}{\tau}$

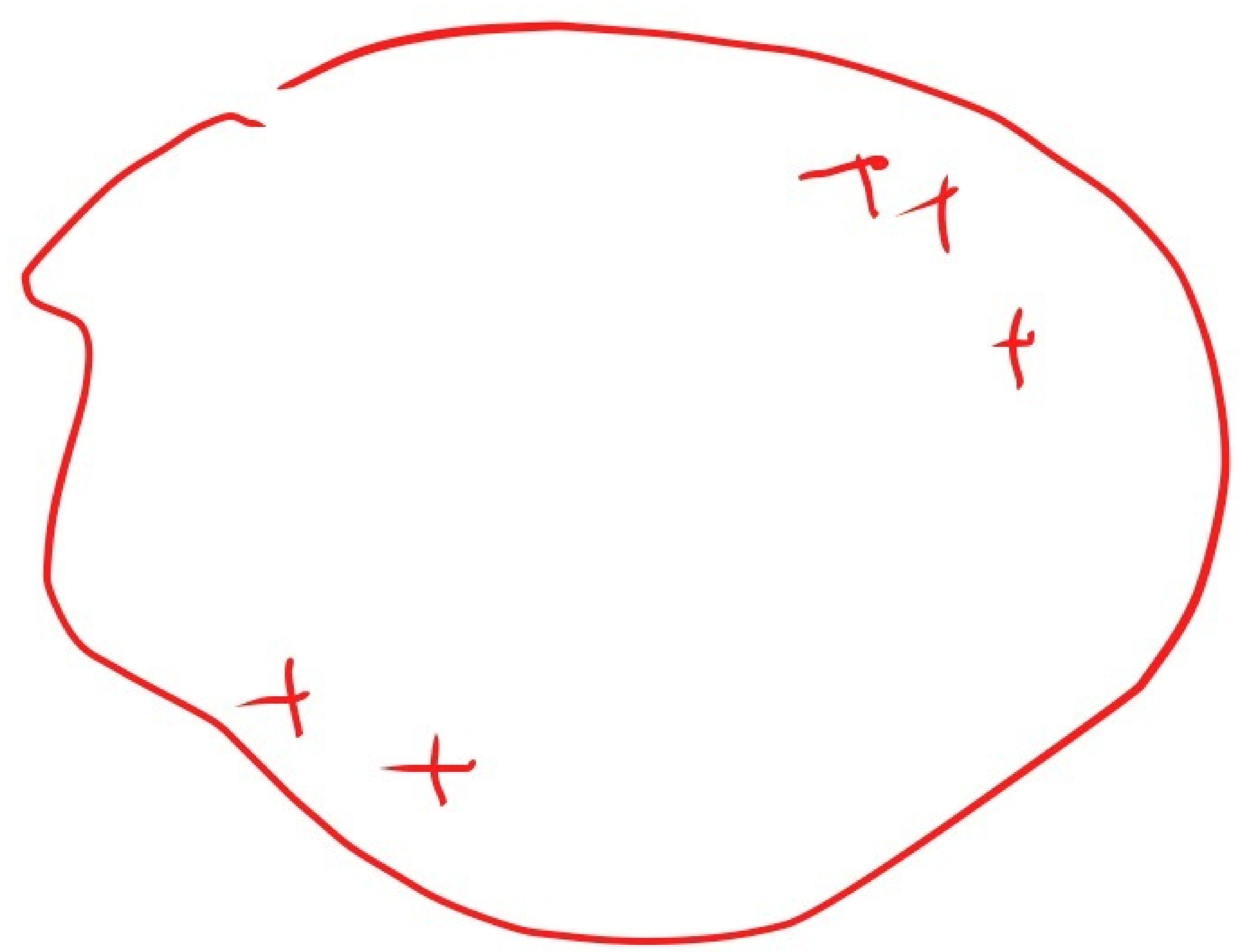
$$\tau \frac{\partial u}{\partial t} = \lambda^2 \frac{\partial^2 u}{\partial x^2} - u + \delta(x)\,\delta(t)$$

- fourier transform in space $\quad u(k,t) = \int \frac{dx\, e^{ikx}}{\sqrt{2\pi}}\, u(x,t)$
- ODE in time

$$\tau \frac{\partial u}{\partial t} = -(\lambda^2 k^2 + 1)\,u + \delta(t)$$

$$u(x,t) \propto \frac{e^{-t/\tau}\; e^{-\frac{x^2}{4\lambda^2 t/\tau}}}{\sqrt{4\pi\lambda^2 t/\tau}}$$





$$\text{timescale for equilibration} = \frac{\text{Size of cell}}{\text{speed}}$$

$$= \frac{S_{cell}}{(\lambda/\tau)} \simeq \tau \left(\frac{S_{cell}}{\lambda}\right)$$

cell $\frac{20}{1000} \ll 1$

dendrite $\frac{1000}{1000} \sim 1$