Probability Theory II (G6106)

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http://stat.columbia.edu/∼porbanz/G6106S15.html

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Homework 7

Due: 8 April 2015

Homework submission: We will collect your homework **at the beginning of class** on the due date. If you cannot attend class that day, you can leave your solution in my postbox in the Department of Statistics, 10th floor SSW, at any time before then.

Problem 1

Prove Proposition 3.21 (the chain rule for conditional independence).

Problem 2

For random variables X, X' and Y, show that

$$(X,Y) \stackrel{\mathrm{d}}{=} (X',Y) \qquad \Leftrightarrow \qquad \mathbb{P}[X \in A|Y] =_{\text{a.s.}} \mathbb{P}[X' \in A|Y] \text{ for any measurable set } A\,.$$

Problem 3

Assume $(X,Y) \stackrel{\mathrm{d}}{=} (X',Y')$, where X is integrable. Show that $\mathbb{E}[X|Y] \stackrel{\mathrm{d}}{=} \mathbb{E}[X'|Y']$.

Hint: Show first that the assumption and $\mathbb{E}[X|Y] =_{\text{a.s.}} f(Y)$, for some measurable mapping f, imply $\mathbb{E}[X'|Y'] =_{\text{a.s.}} f(Y')$.

Problem 4

Suppose $X \perp\!\!\!\perp_Y Z$ and $T \perp\!\!\!\perp (X,Y,Z)$. Show

$$X \perp\!\!\!\perp_{Y,T} Z \qquad \text{and} \qquad X \perp\!\!\!\perp_Y (Z,T)\,.$$

Problem 5*

Let X, Y and Z be random variables, and assume that Y is $\sigma(Z)$-measurable. Show that

$$(X,Y) \stackrel{\mathrm{d}}{=} (X,Z) \qquad \text{implies} \qquad X \perp\!\!\!\perp_Y Z\,.$$

Hint: Show first that $\mathbb{P}[X \in A|Y] \stackrel{\mathrm{d}}{=} \mathbb{P}[X \in A|Z]$. Then turn the equality in distribution into an almost sure equality.