Probability Theory II (G6106)

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http://stat.columbia.edu/∼porbanz/G6106S16.html

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Homework 8

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Problem 1

Prove Proposition 3.20 (the chain rule for conditional independence).

Problem 2

For random variables X, X' and Y, show that

$$(X,Y) \stackrel{\mathrm{d}}{=} (X',Y) \qquad \Leftrightarrow \qquad \mathbb{P}[X \in A|Y] =_{\text{a.s.}} \mathbb{P}[X' \in A|Y] \text{ for any measurable set } A\,.$$

Problem 3

Assume $(X,Y) \stackrel{\mathrm{d}}{=} (X',Y')$, where X is integrable. Show that $\mathbb{E}[X|Y] \stackrel{\mathrm{d}}{=} \mathbb{E}[X'|Y']$.

Hint: Show first that the assumption and $\mathbb{E}[X|Y] =_{\text{a.s.}} f(Y)$, for some measurable mapping f, imply $\mathbb{E}[X'|Y'] =_{\text{a.s.}} f(Y')$.

Problem 4

Suppose $X \perp\!\!\!\perp_Y Z$ and $T \perp\!\!\!\perp (X,Y,Z)$. Show

$$X \perp\!\!\!\perp_{Y,T} Z \qquad \text{and} \qquad X \perp\!\!\!\perp_Y (Z,T)\,.$$

Problem 5*

Let X, Y and Z be random variables, and assume that Y is $\sigma(Z)$-measurable. Show that

$$(X,Y) \stackrel{\mathrm{d}}{=} (X,Z) \qquad \text{implies} \qquad X \perp\!\!\!\perp_Y Z\,.$$

Hint: Show first that $\mathbb{P}[X \in A|Y] \stackrel{\mathrm{d}}{=} \mathbb{P}[X \in A|Z]$. Then turn the equality in distribution into an almost sure equality.