Assignment 9
Theoretical Neuroscience

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1. **Binary weights**

   Assume a large network of interconnected neurones with weights that take two values:

$$\nu_i = \phi(\overline{\nu} + \sum_{j=1}^{N} W_{ij}\nu_j) \tag{1}$$

$$W_{ij} = \frac{\sigma}{\sqrt{Np(1-p)}} \begin{cases} 1-p & \text{with probability } p \\ -p & \text{with probability } 1-p \end{cases} \tag{2}$$

   where $\overline{\nu} \equiv N^{-1}\sum_i \nu_i$, and below you will also need $\overline{\nu^2} \equiv N^{-1}\sum_i \nu_i^2$.

   1a Write down the mean field equations for $\overline{\nu}$ and $\overline{\nu^2}$.

   1b Let $\phi(x) = \nu_0\Theta(x - \nu_0\theta_0)$ where $\Theta(x)$ is the Heaviside function. Solve for $\overline{\nu^2}$ in terms of $\overline{\nu}$. You will get an equation

$$\overline{\nu} = f(\overline{\nu})$$

   Sketch $f(\overline{\nu})$ for $\theta_0 > 0$ and $\theta_0 < 0$.

   1c Is it possible to have three equilibria when $\theta_0 < 0$?

   1d Find $p(\nu)$ when $\theta_0 > 0$. Choose any θ_0 you want, but be sure to let us know what it is.

2. **Threshold-linear activation function**

   In a Wilson-Cowan-like network, let

$$\dot{\nu}_E = g(W_{EE}\nu_E - W_{EI}\nu_I + \theta_E) - \nu_E \tag{3}$$

$$\dot{\nu}_I = g(-W_{II}\nu_I + W_{IE}\nu_E + \theta_I) - \nu_I \tag{4}$$

   where all the Ws are positive, and let the activation function be

$$g(\nu) = \min[\nu_0, \max[0, \nu]]$$

   2a Sketch the nullclines. Under what conditions is there a a line attractor?

   2b Find the conditions that guarantee a stable equilibrium on the unstable branch of the excitatory nullcline.