Inferring Mouse Behavior in Naturalistic Settings With the Kalman Filter and Smoother

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1. Introduction
A central aim of current neuroscience is to understand the relation between neural circuits and behavior. Many important behaviors are only observed in naturalistic settings. Therefore, signal processing methods to accurately infer behavior in these settings are essential.

Here we describe and evaluate the use of the Kalman Filter and Smoother to perform inference in the Discrete Time Wiener Process Acceleration (DWPA) model, a linear dynamical system model used for tracking. We use these inference methods to track the position of mice while freely exploring a large arena.

2. Mice Foraging in Naturalistic Setting
Videos were recorded of mice foraging in a well-lit circular arena. Foraging behaviors are only observed in naturalistic settings. Therefore, these settings are essential.

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3. Computer Vision Functionality
We used the frame differencing technique to detect moving objects in video frames. This involved calculating the absolute difference between every eight frames and the background model. To get the background model, we took the median of fifty randomly selected frames. We then extracted the mouse objects in video frames. This involved calculating the absolute difference between every eight frames and the background model. To get the background model, we took the median of fifty randomly selected frames. We then extracted the mouse objects in video frames.

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4. The Linear Dynamical Systems Model

\[
\begin{align*}
\dot{x}_n &= A x_{n-1} + B n + r(t)_n \\
\sigma^2 &= \text{Var}(r(t))
\end{align*}
\]

\[
\begin{align*}
\dot{y}_n &= C x_n + \xi'_n + \zeta_n \\
\sigma^2 &= \text{Var}(\xi'_n) + \text{Var}(\zeta_n)
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\]

5. Kalman Filter

\[
\begin{align*}
\hat{x}_n|n-1 &= \text{Predicted mean} = A \hat{x}_{n-1} | \text{Predicted covariance} = A P_{n-1|n-1} A^\top + Q \\
K_n &= \text{Kalman gain} = P_{n|n-1} C^\top (C P_{n|n-1} C^\top + R)^{-1} \\
\hat{x}_n &= \text{Smoothed mean} = \hat{x}_n | n + \hat{x}_{n-1} | n-1 \\
\hat{x}_{n|n-1} &= \hat{x}_{n|n-1} + K_n (y_n - C \hat{x}_{n|n-1}) \\
P_{n|n-1} &= \text{Smoothed covariance} = (I - K_n C) P_{n|n-1}
\end{align*}
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\end{align*}
\]

6. Kalman Smoother

\[
\begin{align*}
\hat{x}_n &= \text{Smoothed mean} = \hat{x}_n | n + \hat{x}_{n-1} | n-1 \\
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\end{align*}
\]

7. Discrete Time Wiener Process Acceleration (DWPA) Model

\[
\begin{align*}
\mathbf{x}_n &= \mathbf{x}_{n-1} A \mathbf{x}_{n-1} + \mathbf{B} n + r(t)_n, \quad \mathbf{P}_{n|n-1} = \mathbf{P}_{n-1|n-1} A \mathbf{P}_{n-1|n-1} A^\top + \mathbf{Q} \\
\mathbf{y}_n &= \mathbf{C} \mathbf{x}_n + \mathbf{D} n + \mathbf{r}_n + \mathbf{w}_n, \quad \mathbf{R} = \mathbf{D} \mathbf{Q} \mathbf{D}^\top + \mathbf{R}_n
\end{align*}
\]

\[
\begin{align*}
\mathbf{x}_n &= \mathbf{x}_{n-1} A \mathbf{x}_{n-1} + \mathbf{B} n + r(t)_n, \quad \mathbf{P}_{n|n-1} = \mathbf{P}_{n-1|n-1} A \mathbf{P}_{n-1|n-1} A^\top + \mathbf{Q} \\
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\end{align*}
\]

8. Simulated Mouse

\[
\begin{align*}
\mathbf{x}_n &= \mathbf{x}_{n-1} A \mathbf{x}_{n-1} + \mathbf{B} n + r(t)_n, \quad \mathbf{P}_{n|n-1} = \mathbf{P}_{n-1|n-1} A \mathbf{P}_{n-1|n-1} A^\top + \mathbf{Q} \\
\mathbf{y}_n &= \mathbf{C} \mathbf{x}_n + \mathbf{D} n + \mathbf{r}_n + \mathbf{w}_n, \quad \mathbf{R} = \mathbf{D} \mathbf{Q} \mathbf{D}^\top + \mathbf{R}_n
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\mathbf{y}_n &= \mathbf{C} \mathbf{x}_n + \mathbf{D} n + \mathbf{r}_n + \mathbf{w}_n, \quad \mathbf{R} = \mathbf{D} \mathbf{Q} \mathbf{D}^\top + \mathbf{R}_n
\end{align*}
\]

9. Real Mouse

\[
\begin{align*}
\mathbf{x}_n &= \mathbf{x}_{n-1} A \mathbf{x}_{n-1} + \mathbf{B} n + r(t)_n, \quad \mathbf{P}_{n|n-1} = \mathbf{P}_{n-1|n-1} A \mathbf{P}_{n-1|n-1} A^\top + \mathbf{Q} \\
\mathbf{y}_n &= \mathbf{C} \mathbf{x}_n + \mathbf{D} n + \mathbf{r}_n + \mathbf{w}_n, \quad \mathbf{R} = \mathbf{D} \mathbf{Q} \mathbf{D}^\top + \mathbf{R}_n
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\end{align*}
\]

10. Conclusions
We used computer vision techniques to acquire good estimates of positions from a high-quality video of a mouse foraging in an arena. For simulated data, velocities and accelerations of mice, even at times where mice were not visible (Figure 9a). For simulated data, velocities and accelerations of mice, even at times where mice were not visible (Figure 9a). For simulated data, velocities and accelerations of mice, even at times where mice were not visible (Figure 9a). For simulated data, velocities and accelerations of mice, even at times where mice were not visible (Figure 9a). For simulated data, velocities and accelerations of mice, even at times where mice were not visible (Figure 9a).

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References
[2] https://github.com/Asy30888/LDS_KF_KS.git