Complete Blind Subspace Deconvolution

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1. Introduction

Cocktail-party Problems (increasing generality):

- Independent component analysis (ICA) [1, 2]: *one-dimensional* sound sources.
- Independent subspace analysis (ISA) [3]: *independent groups* of people.
- Blind source deconvolution (BSD) [4]: *one-dimensional* sound sources and *echoic* room.
- Blind subspace deconvolution (BSSD) [5]: *independent source groups* and *echoes*.

Separation Theorem:

- ISA ([3], proof for certain distribution types [5]):

  ISA = ICA + clustering.

  It forms the basis of the state-of-the-art ISA algorithms.
- Undercomplete BSSD (uBSSD):
  - uBSSD = temporal concatenation + ISA [5]: but 'high dimensional' associated ISA problem.
  - uBSSD = LPA (linear predictive approximation) + ISA [6]. Based on: an *undercomplete* polynomial matrix has a polynomial matrix left inverse (with prob. one).

Here: *complete BSSD problem* using linear predictive approximation. It is asymptotically consistent.

2. The BSSD Model

BSSD Equations [5]:

- Observation $\mathbf{x}(t)\in\mathbb{R}^{D_x}$ is convolutive mixture of hidden, independent, multidimensional *components* (random variables; $\mathbf{s}^m\in\mathbb{R}^{d_m}$)

$$\mathbf{x}(t)=\sum_{l=0}^{L}\mathbf{H}_l\mathbf{s}(t-l). \quad (1)$$

Here, $\mathbf{s}(t)=[\mathbf{s}^1(t);\ldots;\mathbf{s}^M(t)]\in\mathbb{R}^{D_s}$. Compactly:

$$\mathbf{x}=\mathbf{H}[z]\mathbf{s} \quad (2)$$

- Goal of BSSD: estimate the original source $\mathbf{s}(t)$ by using observations $\mathbf{x}(t)$ only.
- Specially: ISA ($L=0$), ICA ($L=0,\forall d_m=1$), BSD ($\forall d_m=1$).
- $D_x>D_s$ ($D_x=D_s$): *undercomplete* (*complete*) case.

BSSD Assumptions: Components $\mathbf{s}^m$ are

- independent: $I(\mathbf{s}^1,\ldots,\mathbf{s}^M)=0$,
- i.i.d. (independent identically distributed) in t,
- there is at most one Gaussian component among $\mathbf{s}^m$s.

3. Method

Our Scenario: complete task ($D=D_x=D_s$), and $\mathbf{H}[z]$ is *invertible*, that is

$$\det(\mathbf{H}[z])\neq 0,\forall z\in\mathbb{C},|z|\leq 1. \quad (3)$$

Without loss of generality [$\mathbf{x}=(\mathbf{H}[z]\mathbf{B}^{-1})(\mathbf{B}\mathbf{s})$] $\mathbf{s}$ is white: $E(\mathbf{s})=\mathbf{0}$, $cov(\mathbf{s})=\mathbf{I}$.

Separation: invertibility of $\mathbf{H}[z]$ $\Rightarrow$ observation process $\mathbf{x}$ has AR(∞) representation [7]:

$$\mathbf{x}(t)=\sum_{j=1}^{\infty}\mathbf{F}_j\mathbf{x}(t-j)+\mathbf{F}_0\mathbf{s}(t). \quad (4)$$

Steps:

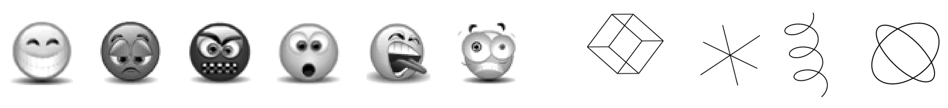
1. AR(p)-fit to $\mathbf{x}$: estimation for innovation $\mathbf{F}_0\mathbf{s}(t)$,
2. ISA on the estimated innovation (components of $\mathbf{s}$ are independent) $\Rightarrow\hat{\mathbf{s}}^m$.

If $p=o(T^{\frac{1}{3}})\xrightarrow{T\to\infty}\infty$ (T: sample number) $\Rightarrow$ the AR approx. for the MA model is asymptotically consistent [8].

4. Illustrations

Databases:

- *smiley*: density functions correspond to the 6 basic facial expressions [$d_m=2$, $M=6$, $D=12$; Fig. 1(a)].
- *3D-geom*: $\mathbf{s}^m$s were random variables uniformly distributed on 3-dimensional geometric forms [$d_m=3$, $M=4$, $D=12$; Fig. 1(b)].
- *Beatles* [5]: non-i.i.d., stereo Beatles songs ($d_m=2$, $M=2$, $D=4$).



(a) (b)

Figure 1: *Illustration:* smiley *(a),* 3D-geom *dataset (b).*

Performance Measure, the Amari-index:

- Recovery of components $\mathbf{s}^m$: subject to ISA ambiguities [9]
  - permutation (components of equal dimension),
  - invertible transformation within the subspaces.
- $\Rightarrow$In the ideal case: $\mathbf{G}$ optimally approximating $\mathbf{s}\mapsto\hat{\mathbf{s}}$ resides also within the subspaces, a *block-permutation matrix*.
- Its measure: *Amari-index* ($r=r(\mathbf{G})\in[0,1]$)
  - ICA: Amari-error [10] $\xrightarrow{[11]}$ ISA $\xrightarrow{[12]}$ ISA, $\in[0,1]$,
  - $r=0\leftrightarrow$ perfect estimation, $r=1\leftrightarrow$ worst possible.

Simulation Parameters:

- performance measure: Amari-index over 20 random ($\mathbf{H}[z]$, $\mathbf{e}$) runs.
- studied parameters: sample number (T), convolution length ($L+1$); invertibility of $\mathbf{H}[z]$ ($\lambda\to 1$)

$$\mathbf{H}[z]=[\prod_{l=0}^{L}(\mathbf{I}-\lambda\mathbf{O}_i z)]\mathbf{H}_0 \quad (|\lambda|<1,\lambda\in\mathbb{R}). \quad (5)$$

Here, $\mathbf{H}_0$ and $\mathbf{O}_i\in\mathbb{R}^{D\times D}$: random orthogonal.
- ARfit: [13].
- Upper limit for the AR order (+SBC): $p_{max}(T)=2\lfloor T^{\frac{1}{3}-\frac{1}{1000}}\rfloor\Rightarrow p_{opt}\in[1,p_{max}(T)]$.
- ISA subtask: joint f-decorrelation method [14].

Illustrations:

- *smiley*, *3D-geom* tests: $1,000\leq T\leq 20,000$, $L\in\{1,2,5,10\}$, $\lambda\in\{0.4,0.6,0.7,0.8,0.85,0.9\}$. Results in Fig. 2(a)-(b):
  - $L=10,\lambda=0.85$: estimation is still efficient (Fig. 3).
  - Power law decrease of the Amari-indices: $r(T)\propto T^{-c}$ ($c>0$).
  - Numerical values of the estimation errors: Table 1.
  - Estimated optimal AR order: Fig. 2(c), as $\lambda\to 1$ $p_{max}(T)$ is more and more exploited.
- *Beatles*: $\lambda=0.9$, $1,000\leq T\leq 100,000$, Fig. 2(d).
  - $L=1,2,5$: error of estimation drops for sample number $T=10,000-20,000$.
  - $L=10$: 'power law' decline appears.
  - Numerical values of the estimation errors: Table 1,
  - Estimated optimal AR order: $p_{max}(T)$ fully exploited.

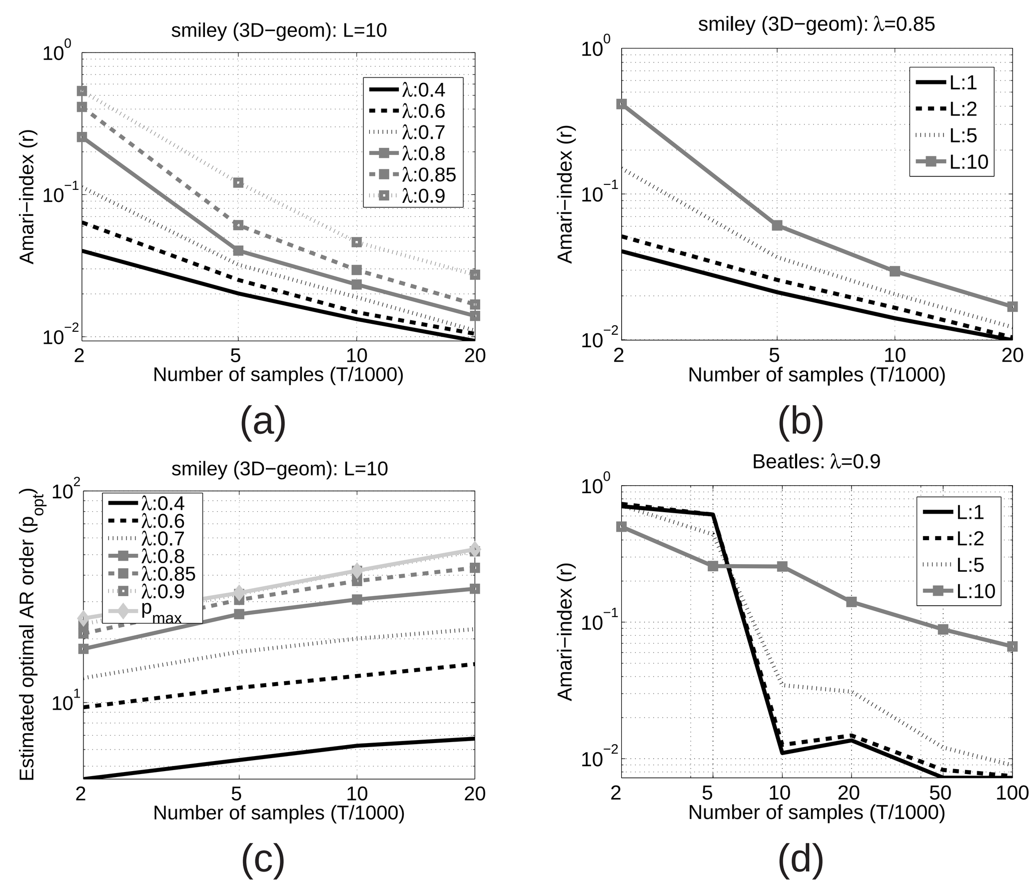


Figure 2: *Precision, estimated optimal AR orders, on log-log scale. (a) [$L=10$, $\lambda\to 1$], (b) [$\lambda=0.85$]: Amari-index,* smiley *(*3D-geom*). (c): estimated AR order, $L=10$, different λ values. (d): as (b),* Beatles*, $\lambda=0.9$.*

	$L=1$	$L=5$	$L=10$
smiley	0.99% $(\pm0.11\%)$	1.22% $(\pm0.15\%)$	1.69% $(\pm0.26\%)$
3D-geom	0.42% $(\pm0.06\%)$	0.88% $(\pm0.14\%)$	1.15% $(\pm0.24\%)$
Beatles	0.72% $(\pm0.12\%)$	0.90% $(\pm0.23\%)$	6.64% $(\pm7.49\%)$

Table 1: *Amari-index in percentages, mean$\pm$ standard deviation, for different L values:* smiley, 3D-geom *($\lambda=0.85,T=20,000$), Beatles dataset ($\lambda=0.9,T=100,000$).*

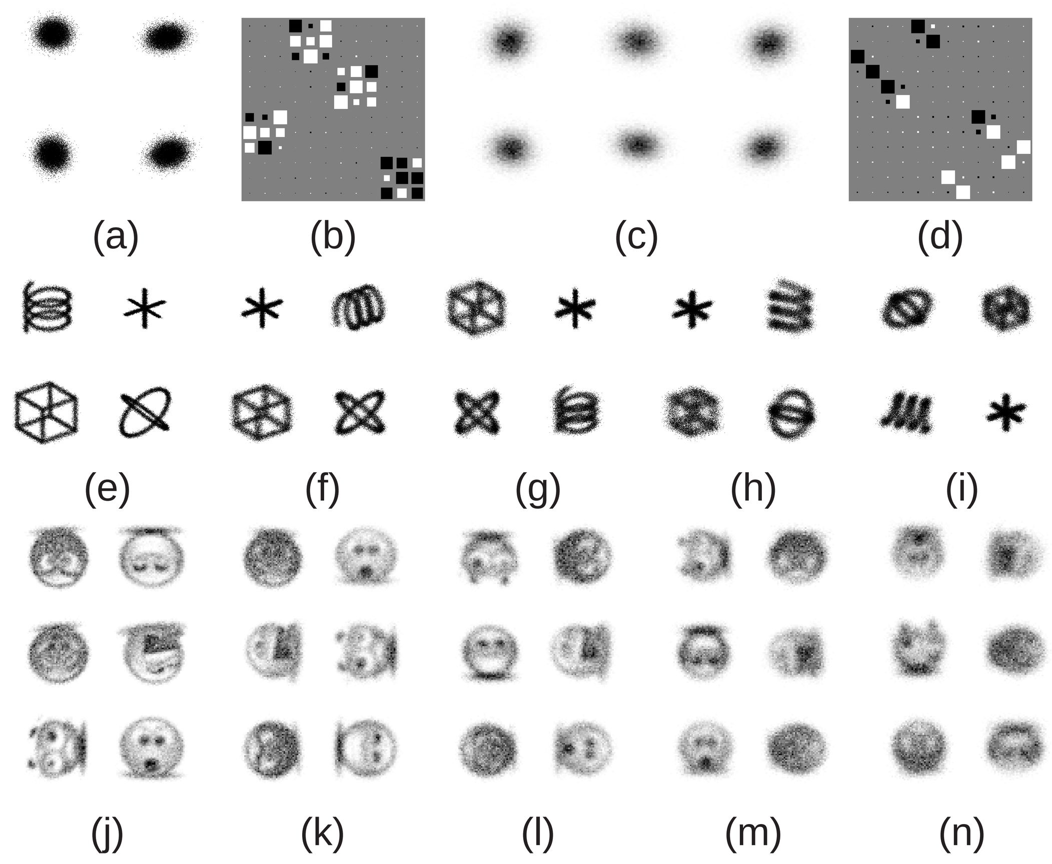


Figure 3: *Illustration of the estimations ($T=20,000$, $L=10$):* 3D-geom *[(a),(b),(e)-(i)],* smiley *[(c),(d),(j)-(n)]. First row: $\lambda=0.4$. (a), (c): observed convolved signal $\mathbf{x}(t)$. (b), (d): Hinton-diagram of $\mathbf{G}$, ideally a block-permutation matrix with 2×2 and 3×3 blocks, respectively. (e)-(i), (j)-(n): estimated components $\hat{\mathbf{s}}^m$, recovered up to the ISA ambiguities from left to right for $\lambda=0.4,0.6,0.7,0.8,0.85$.*

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