Online Appendix 1. Finite Batch Sizes for GP-AUCB

In the absence of an explicitly specified maximum batch size, it is interesting to consider the scaling of batch sizes produced by GP-AUCB for large T. We are concerned with the case where actions are chosen when much is known about the structure of the reward function: many actions could be selected with little "danger" of choosing poorly, but also little information gain. In such a case, a great deal of regret could be accumulated between observations if the posterior mean fails to correctly order the available actions in D with respect to their reward.

The set of size T which gains the least information with respect to f, conditioned on observations $\boldsymbol{y}(\mathcal{S})$, is one which queries $x_* = \text{argmin}_{x \in D} \sigma^2(x|\boldsymbol{y}(\mathcal{S}))$ T times. These samples gain information $1/2 \log(1 + T\sigma_n^{-2}\sigma^2_{\mathcal{S}}(x_*))$, where $\sigma^2(x|\boldsymbol{y}_{\mathcal{S}})) = \sigma^2_{\mathcal{S}}(x)$ is the posterior variance, conditioned on the observations $\boldsymbol{y}_{\mathcal{S}}$. Using this observation, if a batch is terminated when a threshold C for hallucinated conditional information with respect to f is exceeded, as in the GP-AUCB algorithm, the maximum possible length of a batch, B_{max}, can be bounded as follows:

$$\begin{aligned} C \geq & 1/2 \log(1 + (B_{max} - 1)\sigma_n^{-2}\sigma^2_{\mathbf{S}}(x_*)) \\ \implies & \left[\frac{\sigma_n^2}{\sigma^2_{\mathbf{S}}(x_*)}[\exp(2C) - 1]\right] + 1 \geq B_{max}. \end{aligned} \tag{21}$$

Thus, if there does not exist any $\boldsymbol{x} \in D$ such that $\sigma^2(\boldsymbol{x}) = 0$, which is the case under the GP model if $\nexists \boldsymbol{x} \in D$ such that $k(\boldsymbol{x}, \boldsymbol{x}) = 0$ and $\sigma_n^2 \neq 0$, this upper limit on B_{max} is finite for any finite C and any previous sampling history; the batch sizes of GP-AUCB do not diverge to infinity in a finite number of rounds.

Bounding the rate at which the batch length B_{max} can grow is of interest, however. Consider cases where time is indexed by action number t or by batch number N. In the case of action number, by rearrangement of Equation (9) and using Inequalities (11) and (13), we have

$$\sigma_t^2(\boldsymbol{x}) \geq \sigma_0^2(\boldsymbol{x}) \exp\left(-2I(f; \boldsymbol{y}_{1:t-1})\right) \geq \sigma_0^2(\boldsymbol{x}) \exp\left(-2\gamma_{t-1}\right) \; \forall t \in \mathbb{N}.$$

At time t, using this result and Inequality (21), the maximum length of the batch which can be constructed under GP-AUCB (or any sampling procedure such that the batch terminates when the information gain threshold C is exceeded) is bounded as

$$B_{max} \leq \left[\frac{\sigma_n^2}{\min_{x \in D}(\sigma_0^2(x))}[\exp(2C) - 1][\exp\left(2\gamma_{t-1}\right)]\right] + 1.$$

This bound is no more than $O(\exp(t + C))$, since γ_t is no more than linear in t.

A similar bound may be obtained for the Nth batch. After $N-1$ batches, the posterior variance of $f(\boldsymbol{x})$, $\sigma^2_{N-1}(\boldsymbol{x})$, may be bounded as follows, for any $\boldsymbol{x} \in D$, via Equation (9) and Inequalities (11) and (13):

$$\sigma^2_{N-1}(\boldsymbol{x}) \geq \sigma_0^2(\boldsymbol{x}) \exp\left(-2(N-1)C_B\right) \forall t \in \mathbb{N}.$$

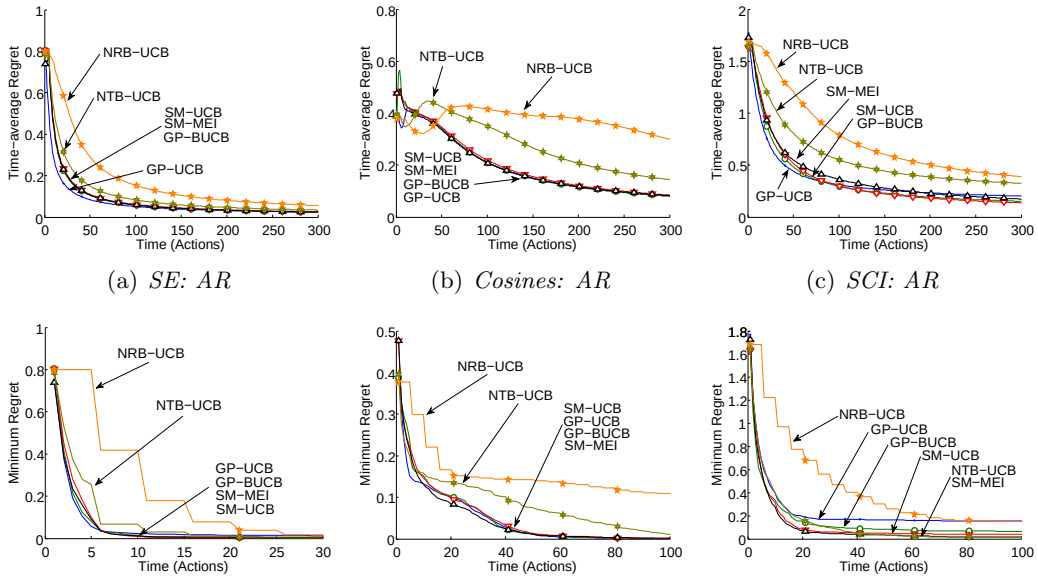
Here, C_B is an upper bound on the information which is obtained when the observations corresponding to the batch are made. C_B is greater than C, since the batch terminates only

when the information which would be hallucinated in order to select the next action exceeds the threshold C. One useful bound is $C_B \leq C + 1/2 \log\left(1 + \sigma_n^{-2} \max_{\boldsymbol{x} \in D} \sigma_0^2(\boldsymbol{x})\right)$, which follows because the termination condition is checked every round and mutual information is submodular. Using Inequality (21), the length of the Nth batch is thus bounded as

$$B_{max} \leq \left[\frac{\sigma_n^2}{\min_{x \in D}(\sigma_0^2(x))} [\exp(2C) - 1][\exp\left(2(N-1)C_B\right)] \right] + 1,$$

which is no more than $O(\exp(NC))$, but is bounded for finite batch number.

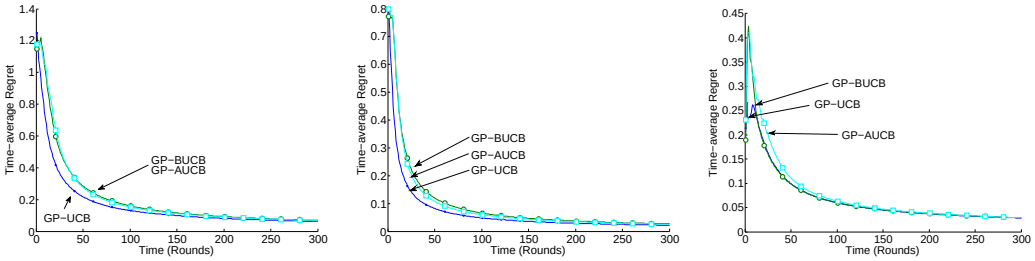
Online Appendix 2. Additional Figures



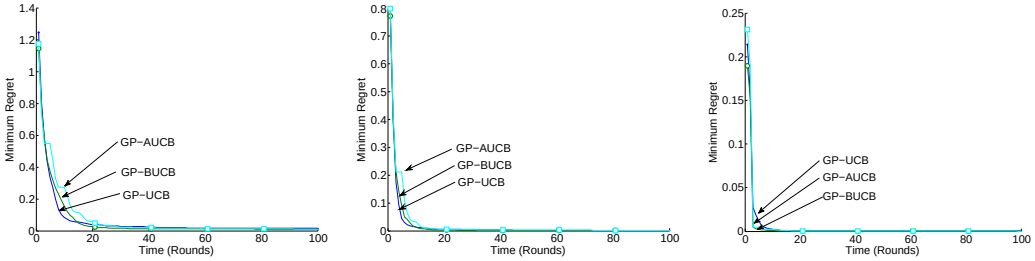
(a) *SE: AR* (b) *Cosines: AR* (c) *SCI: AR*

(d) *SE: MR* (e) *Cosines: MR* (f) *SCI: MR*

Figure 9: Time-average (AR) and minimum (MR) regret plots, batch setting, $B = 5$. Note that these plots replicate Figures 2(b), (g), and (i) (top row), and (e), (j), and (l) (bottom row), with the addition of the naïve algorithms discussed in Section 3.4; these are here denoted NRB-UCB (orange) for the algorithm which repeats the same action B times and NTB-UCB (gold) for the algorithm which chooses the top B elements of D under the decision rule. Note that while the performance of NRB-UCB is universially poor, NTB-UCB performs well in terms of minimum regret in the SCI setting. This last result may be because $\boldsymbol{x}^*$ is unique and hard to find in the SCI setting, but NTB-UCB's enforced query diversity makes finding it easier.

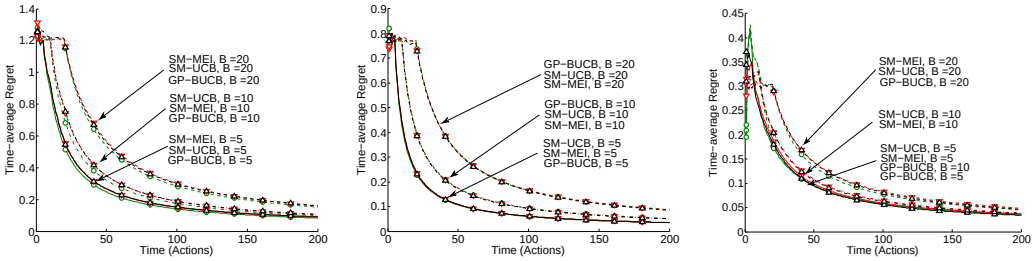


(a) *Matérn: AR* (b) *SE: AR* (c) *Rosenbrock: AR*

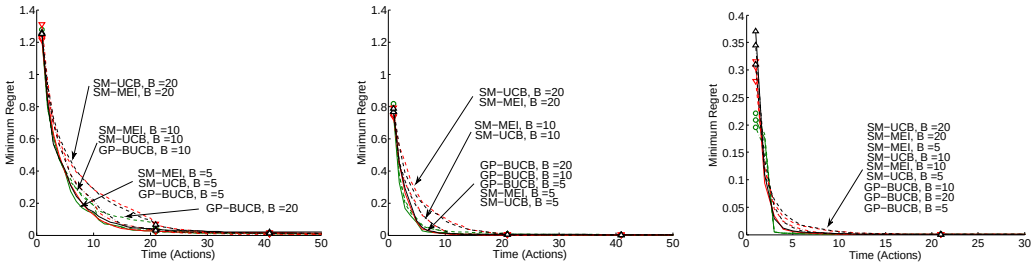


(d) *Matérn: MR* (e) *SE: MR* (f) *Rosenbrock: MR*

Figure 10: Additional time-average (AR) and minimum (MR) regret plots, delay setting, with a delay length of 5 rounds between action and observation. See Figure 3 in the main text for other settings.

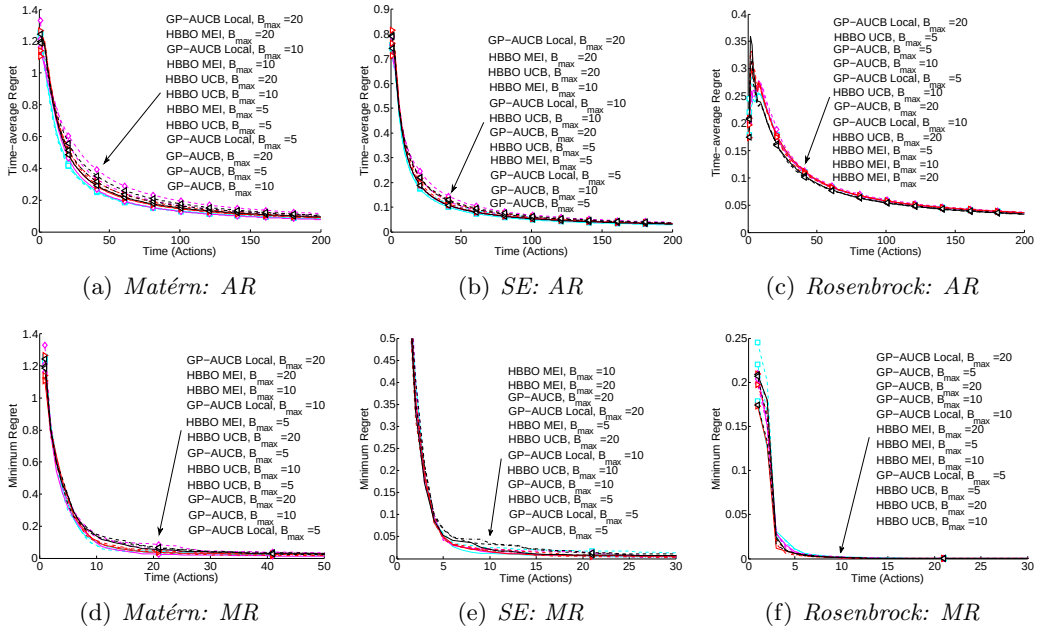


(a) *Matérn: AR* (b) *SE: AR* (c) *Rosenbrock: AR*



(d) *Matérn: MR* (e) *SE: MR* (f) *Rosenbrock: MR*

Figure 11: Additional time-average (AR) and minimum (MR) regret plots, non-adaptive batch algorithms, batch sizes 5 (solid), 10 (dash-dot), and 20 (dashed). Figure 4 presents other settings.



(a) *Matérn: AR* (b) *SE: AR* (c) *Rosenbrock: AR*

(d) *Matérn: MR* (e) *SE: MR* (f) *Rosenbrock: MR*

Figure 12: Additional time-average (AR) and minimum (MR) regret plots, adaptive batch algorithms, maximum batch sizes 5, 10, and 20. See Figure 5 in the main text for other settings.

Online Appendix 3. Tables of Results

Data Set	Algorithm	AR, Query 100	MR, Query 100	AR, Query 200	MR, Query 200
Matérn GP	GP-UCB	0.1268 ± 0.0076	0.0285 ± 0.0075	0.0845 ± 0.0073	0.0243 ± 0.0069
	GP-BUCB	0.1434 ± 0.0040	0.0113 ± 0.0032	0.0855 ± 0.0035	0.0107 ± 0.0032
	SM-UCB	0.1479 ± 0.0055	0.0089 ± 0.0052	0.0849 ± 0.0037	0.0035 ± 0.0011
	SM-MEI	0.1549 ± 0.0048	0.0147 ± 0.0031	0.0937 ± 0.0036	0.0099 ± 0.0026
SE GP	GP-UCB	0.0513 ± 0.0038	0.0054 ± 0.0033	0.0322 ± 0.0033	0.0021 ± 0.0012
	GP-BUCB	0.0577 ± 0.0014	0.0005 ± 0.0002	0.0329 ± 0.0008	0.0003 ± 0.0001
	SM-UCB	0.0612 ± 0.0018	0.0016 ± 0.0011	0.0349 ± 0.0011	0.0004 ± 0.0002
	SM-MEI	0.0593 ± 0.0017	0.0016 ± 0.0007	0.0338 ± 0.0011	0.0006 ± 0.0002
Rosenbrock	GP-UCB	0.0571 ± 0.0005	0.0000 ± 0.0000	0.0353 ± 0.0003	0.0000 ± 0.0000
	GP-BUCB	0.0579 ± 0.0005	0.0000 ± 0.0000	0.0359 ± 0.0003	0.0000 ± 0.0000
	SM-UCB	0.0598 ± 0.0004	0.0000 ± 0.0000	0.0366 ± 0.0003	0.0000 ± 0.0000
	SM-MEI	0.0560 ± 0.0005	0.0000 ± 0.0000	0.0340 ± 0.0003	0.0000 ± 0.0000
Cosines	GP-UCB	0.2109 ± 0.0013	0.0009 ± 0.0002	0.1152 ± 0.0007	0.0001 ± 0.0000
	GP-BUCB	0.2110 ± 0.0013	0.0010 ± 0.0002	0.1158 ± 0.0008	0.0002 ± 0.0001
	SM-UCB	0.2195 ± 0.0012	0.0010 ± 0.0002	0.1213 ± 0.0008	0.0003 ± 0.0001
	SM-MEI	0.2092 ± 0.0013	0.0019 ± 0.0004	0.1173 ± 0.0011	0.0010 ± 0.0003
Vaccine Design	GP-UCB	0.8147 ± 0.0402	0.3465 ± 0.0346	0.6009 ± 0.0354	0.2987 ± 0.0304
	GP-BUCB	0.8605 ± 0.0374	0.2834 ± 0.0291	0.6013 ± 0.0314	0.2326 ± 0.0269
	SM-UCB	0.8149 ± 0.0321	0.1521 ± 0.0212	0.5261 ± 0.0264	0.1446 ± 0.0207
	SM-MEI	0.7750 ± 0.0337	0.1525 ± 0.0214	0.5125 ± 0.0266	0.1066 ± 0.0171
SCI	GP-UCB	0.3099 ± 0.0142	0.1540 ± 0.0129	0.2329 ± 0.0127	0.1345 ± 0.0121
	GP-BUCB	0.2965 ± 0.0102	0.0666 ± 0.0085	0.1920 ± 0.0085	0.0544 ± 0.0076
	SM-UCB	0.3016 ± 0.0092	0.0398 ± 0.0061	0.1813 ± 0.0069	0.0303 ± 0.0052
	SM-MEI	0.3622 ± 0.0085	0.0146 ± 0.0019	0.2280 ± 0.0049	0.0096 ± 0.0008

Table 3: Average (AR) and Minimum regret (MR) for fixed batch size B = 5.

Data Set	Algorithm	AR, Round 100	MR, Round 100	AR, Round 200	MR, Round 200
Matérn GP	GP-UCB	0.1307 ± 0.0054	0.0167 ± 0.0052	0.0811 ± 0.0050	0.0095 ± 0.0030
	GP-BUCB	0.1601 ± 0.0046	0.0096 ± 0.0039	0.0925 ± 0.0041	0.0079 ± 0.0038
	GP-AUCB	0.1527 ± 0.0052	0.0105 ± 0.0041	0.0898 ± 0.0047	0.0101 ± 0.0041
SE GP	GP-UCB	0.0482 ± 0.0012	0.0013 ± 0.0005	0.0290 ± 0.0010	0.0009 ± 0.0005
	GP-BUCB	0.0656 ± 0.0014	0.0003 ± 0.0001	0.0369 ± 0.0008	0.0002 ± 0.0001
	GP-AUCB	0.0597 ± 0.0015	0.0013 ± 0.0005	0.0343 ± 0.0010	0.0009 ± 0.0005
Rosenbrock	GP-UCB	0.0598 ± 0.0004	0.0000 ± 0.0000	0.0369 ± 0.0003	0.0000 ± 0.0000
	GP-BUCB	0.0601 ± 0.0004	0.0000 ± 0.0000	0.0376 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB	0.0635 ± 0.0005	0.0000 ± 0.0000	0.0382 ± 0.0003	0.0000 ± 0.0000
Cosines	GP-UCB	0.2224 ± 0.0013	0.0013 ± 0.0003	0.1224 ± 0.0008	0.0004 ± 0.0002
	GP-BUCB	0.2199 ± 0.0013	0.0012 ± 0.0003	0.1214 ± 0.0009	0.0001 ± 0.0000
	GP-AUCB	0.2693 ± 0.0013	0.0024 ± 0.0005	0.1352 ± 0.0010	0.0002 ± 0.0000
Vaccine Design	GP-UCB	0.8217 ± 0.0371	0.3058 ± 0.0317	0.5834 ± 0.0316	0.2555 ± 0.0286
	GP-BUCB	0.9650 ± 0.0337	0.2501 ± 0.0279	0.6453 ± 0.0277	0.2100 ± 0.0248
	GP-AUCB	0.9653 ± 0.0355	0.2031 ± 0.0252	0.6153 ± 0.0281	0.1783 ± 0.0243
SCI	GP-UCB	0.3092 ± 0.0131	0.0920 ± 0.0100	0.2108 ± 0.0106	0.0718 ± 0.0091
	GP-BUCB	0.3558 ± 0.0095	0.0339 ± 0.0060	0.2068 ± 0.0067	0.0237 ± 0.0050
	GP-AUCB	0.3155 ± 0.0122	0.0455 ± 0.0072	0.1880 ± 0.0084	0.0317 ± 0.0058

Table 4: Average (AR) and Minimum regret (MR) for fixed delay length B = 5.

Data Set	Algorithm	AR, Query 100	MR, Query 100	AR, Query 200	MR, Query 200
Matérn GP	GP-BUCB, $B = 5$	0.1405 ± 0.0033	0.0080 ± 0.0024	0.0827 ± 0.0028	0.0076 ± 0.0024
	GP-BUCB, $B = 10$	0.1751 ± 0.0029	0.0068 ± 0.0016	0.0980 ± 0.0020	0.0060 ± 0.0016
	GP-BUCB, $B = 20$	0.2843 ± 0.0047	0.0038 ± 0.0009	0.1513 ± 0.0024	0.0029 ± 0.0008
	SM-UCB, $B = 5$	0.1509 ± 0.0048	0.0117 ± 0.0043	0.0889 ± 0.0045	0.0110 ± 0.0043
	SM-UCB, $B = 10$	0.1891 ± 0.0028	0.0029 ± 0.0009	0.1036 ± 0.0017	0.0025 ± 0.0009
	SM-UCB, $B = 20$	0.3022 ± 0.0051	0.0025 ± 0.0008	0.1597 ± 0.0026	0.0005 ± 0.0002
	SM-MEI, $B = 5$	0.1524 ± 0.0047	0.0141 ± 0.0041	0.0905 ± 0.0040	0.0099 ± 0.0036
	SM-MEI, $B = 10$	0.1897 ± 0.0037	0.0076 ± 0.0025	0.1064 ± 0.0028	0.0068 ± 0.0023
	SM-MEI, $B = 20$	0.2978 ± 0.0047	0.0081 ± 0.0019	0.1609 ± 0.0029	0.0063 ± 0.0015
SE GP	GP-BUCB, $B = 5$	0.0600 ± 0.0014	0.0005 ± 0.0001	0.0344 ± 0.0008	0.0002 ± 0.0001
	GP-BUCB, $B = 10$	0.0937 ± 0.0024	0.0005 ± 0.0001	0.0515 ± 0.0014	0.0004 ± 0.0001
	GP-BUCB, $B = 20$	0.1653 ± 0.0045	0.0010 ± 0.0002	0.0864 ± 0.0023	0.0004 ± 0.0002
	SM-UCB, $B = 5$	0.0607 ± 0.0016	0.0006 ± 0.0002	0.0349 ± 0.0011	0.0004 ± 0.0002
	SM-UCB, $B = 10$	0.0920 ± 0.0024	0.0004 ± 0.0002	0.0501 ± 0.0013	0.0001 ± 0.0000
	SM-UCB, $B = 20$	0.1660 ± 0.0048	0.0006 ± 0.0001	0.0869 ± 0.0024	0.0003 ± 0.0001
	SM-MEI, $B = 5$	0.0606 ± 0.0017	0.0014 ± 0.0004	0.0349 ± 0.0011	0.0011 ± 0.0003
	SM-MEI, $B = 10$	0.0920 ± 0.0025	0.0019 ± 0.0006	0.0501 ± 0.0014	0.0013 ± 0.0005
	SM-MEI, $B = 20$	0.1639 ± 0.0049	0.0013 ± 0.0002	0.0853 ± 0.0024	0.0009 ± 0.0002
Rosenbrock	GP-BUCB, $B = 5$	0.0576 ± 0.0004	0.0000 ± 0.0000	0.0356 ± 0.0003	0.0000 ± 0.0000
	GP-BUCB, $B = 10$	0.0573 ± 0.0004	0.0000 ± 0.0000	0.0353 ± 0.0003	0.0000 ± 0.0000
	GP-BUCB, $B = 20$	0.0771 ± 0.0004	0.0000 ± 0.0000	0.0453 ± 0.0003	0.0000 ± 0.0000
	SM-UCB, $B = 5$	0.0590 ± 0.0005	0.0000 ± 0.0000	0.0368 ± 0.0003	0.0000 ± 0.0000
	SM-UCB, $B = 10$	0.0639 ± 0.0005	0.0000 ± 0.0000	0.0386 ± 0.0003	0.0000 ± 0.0000
	SM-UCB, $B = 20$	0.0828 ± 0.0008	0.0000 ± 0.0000	0.0483 ± 0.0004	0.0000 ± 0.0000
	SM-MEI, $B = 5$	0.0558 ± 0.0005	0.0000 ± 0.0000	0.0340 ± 0.0003	0.0000 ± 0.0000
	SM-MEI, $B = 10$	0.0626 ± 0.0006	0.0000 ± 0.0000	0.0374 ± 0.0004	0.0000 ± 0.0000
	SM-MEI, $B = 20$	0.0806 ± 0.0007	0.0000 ± 0.0000	0.0464 ± 0.0004	0.0000 ± 0.0000
Cosines	GP-BUCB, $B = 5$	0.2107 ± 0.0013	0.0014 ± 0.0003	0.1158 ± 0.0008	0.0003 ± 0.0001
	GP-BUCB, $B = 10$	0.2066 ± 0.0013	0.0009 ± 0.0002	0.1131 ± 0.0009	0.0002 ± 0.0001
	GP-BUCB, $B = 20$	0.2136 ± 0.0017	0.0023 ± 0.0007	0.1186 ± 0.0011	0.0006 ± 0.0004
	SM-UCB, $B = 5$	0.2211 ± 0.0014	0.0012 ± 0.0003	0.1210 ± 0.0009	0.0003 ± 0.0001
	SM-UCB, $B = 10$	0.2330 ± 0.0014	0.0013 ± 0.0004	0.1278 ± 0.0008	0.0003 ± 0.0001
	SM-UCB, $B = 20$	0.2729 ± 0.0015	0.0019 ± 0.0004	0.1505 ± 0.0011	0.0001 ± 0.0000
	SM-MEI, $B = 5$	0.2106 ± 0.0016	0.0033 ± 0.0006	0.1184 ± 0.0012	0.0015 ± 0.0005
	SM-MEI, $B = 10$	0.2253 ± 0.0016	0.0027 ± 0.0005	0.1257 ± 0.0010	0.0011 ± 0.0002
	SM-MEI, $B = 20$	0.2631 ± 0.0016	0.0041 ± 0.0006	0.1454 ± 0.0010	0.0011 ± 0.0003
Vaccine Design	GP-BUCB, $B = 5$	0.9413 ± 0.0406	0.3302 ± 0.0340	0.6615 ± 0.0348	0.2775 ± 0.0293
	GP-BUCB, $B = 10$	1.0379 ± 0.0349	0.1839 ± 0.0278	0.6540 ± 0.0299	0.1711 ± 0.0265
	GP-BUCB, $B = 20$	1.4637 ± 0.0323	0.1024 ± 0.0176	0.8327 ± 0.0247	0.0951 ± 0.0169
	SM-UCB, $B = 5$	0.8531 ± 0.0366	0.1790 ± 0.0245	0.5428 ± 0.0279	0.1444 ± 0.0215
	SM-UCB, $B = 10$	1.0513 ± 0.0275	0.0906 ± 0.0174	0.6170 ± 0.0219	0.0866 ± 0.0168
	SM-UCB, $B = 20$	1.5212 ± 0.0278	0.0349 ± 0.0113	0.8341 ± 0.0190	0.0349 ± 0.0113
	SM-MEI, $B = 5$	0.8239 ± 0.0325	0.1667 ± 0.0229	0.5418 ± 0.0256	0.1383 ± 0.0214
	SM-MEI, $B = 10$	1.0751 ± 0.0330	0.1202 ± 0.0231	0.6557 ± 0.0249	0.0801 ± 0.0158
	SM-MEI, $B = 20$	1.5440 ± 0.0270	0.0277 ± 0.0098	0.8590 ± 0.0195	0.0271 ± 0.0098
SCI	GP-BUCB, $B = 5$	0.2748 ± 0.0103	0.0492 ± 0.0076	0.1728 ± 0.0082	0.0433 ± 0.0071
	GP-BUCB, $B = 10$	0.3884 ± 0.0091	0.0440 ± 0.0065	0.2275 ± 0.0069	0.0391 ± 0.0060
	GP-BUCB, $B = 20$	0.6031 ± 0.0093	0.0427 ± 0.0063	0.3349 ± 0.0070	0.0298 ± 0.0052
	SM-UCB, $B = 5$	0.3075 ± 0.0094	0.0445 ± 0.0066	0.1894 ± 0.0072	0.0392 ± 0.0063
	SM-UCB, $B = 10$	0.4162 ± 0.0078	0.0190 ± 0.0030	0.2290 ± 0.0049	0.0152 ± 0.0025
	SM-UCB, $B = 20$	0.6608 ± 0.0120	0.0236 ± 0.0045	0.3571 ± 0.0072	0.0213 ± 0.0043
	SM-MEI, $B = 5$	0.3734 ± 0.0089	0.0170 ± 0.0026	0.2379 ± 0.0050	0.0109 ± 0.0013
	SM-MEI, $B = 10$	0.4838 ± 0.0078	0.0132 ± 0.0022	0.2981 ± 0.0048	0.0087 ± 0.0015
	SM-MEI, $B = 20$	0.7177 ± 0.0099	0.0124 ± 0.0022	0.4086 ± 0.0060	0.0072 ± 0.0013

Table 5: Average (AR) and Minimum regret (MR) for batch sizes $B = 5$, 10, and 20, non-adaptive algorithms.

Data Set	Algorithm	AR, Query 100	MR, Query 100	AR, Query 200	MR, Query 200
Matérn GP	GP-AUCB, $B_{max} = 5$	0.1303 ± 0.0057	0.0182 ± 0.0053	0.0819 ± 0.0053	0.0166 ± 0.0052
	GP-AUCB, $B_{max} = 10$	0.1293 ± 0.0060	0.0185 ± 0.0058	0.0816 ± 0.0057	0.0138 ± 0.0042
	GP-AUCB, $B_{max} = 20$	0.1326 ± 0.0061	0.0197 ± 0.0059	0.0835 ± 0.0059	0.0187 ± 0.0059
	GP-AUCB Local, $B_{max} = 5$	0.1290 ± 0.0042	0.0112 ± 0.0036	0.0774 ± 0.0038	0.0108 ± 0.0036
	GP-AUCB Local, $B_{max} = 10$	0.1667 ± 0.0071	0.0238 ± 0.0069	0.1020 ± 0.0068	0.0190 ± 0.0057
	GP-AUCB Local, $B_{max} = 20$	0.1952 ± 0.0081	0.0173 ± 0.0055	0.1148 ± 0.0062	0.0153 ± 0.0055
	HBBO UCB, $B_{max} = 5$	0.1455 ± 0.0070	0.0179 ± 0.0064	0.0895 ± 0.0065	0.0163 ± 0.0063
	HBBO UCB, $B_{max} = 10$	0.1459 ± 0.0047	0.0136 ± 0.0038	0.0873 ± 0.0040	0.0121 ± 0.0037
	HBBO UCB, $B_{max} = 20$	0.1587 ± 0.0055	0.0150 ± 0.0044	0.0950 ± 0.0047	0.0137 ± 0.0044
	HBBO MEI, $B_{max} = 5$	0.1515 ± 0.0059	0.0197 ± 0.0051	0.0935 ± 0.0052	0.0169 ± 0.0050
	HBBO MEI, $B_{max} = 10$	0.1644 ± 0.0063	0.0224 ± 0.0053	0.1017 ± 0.0054	0.0162 ± 0.0047
	HBBO MEI, $B_{max} = 20$	0.1771 ± 0.0069	0.0131 ± 0.0040	0.1049 ± 0.0050	0.0111 ± 0.0040
SE GP	GP-AUCB, $B_{max} = 5$	0.0489 ± 0.0015	0.0005 ± 0.0002	0.0286 ± 0.0009	0.0004 ± 0.0002
	GP-AUCB, $B_{max} = 10$	0.0505 ± 0.0016	0.0017 ± 0.0006	0.0296 ± 0.0011	0.0010 ± 0.0003
	GP-AUCB, $B_{max} = 20$	0.0590 ± 0.0041	0.0061 ± 0.0035	0.0362 ± 0.0035	0.0026 ± 0.0012
	GP-AUCB Local, $B_{max} = 5$	0.0540 ± 0.0035	0.0044 ± 0.0033	0.0323 ± 0.0026	0.0007 ± 0.0005
	GP-AUCB Local, $B_{max} = 10$	0.0591 ± 0.0021	0.0022 ± 0.0012	0.0340 ± 0.0015	0.0020 ± 0.0012
	GP-AUCB Local, $B_{max} = 20$	0.0683 ± 0.0027	0.0012 ± 0.0005	0.0382 ± 0.0015	0.0008 ± 0.0005
	HBBO UCB, $B_{max} = 5$	0.0547 ± 0.0020	0.0040 ± 0.0016	0.0331 ± 0.0017	0.0021 ± 0.0013
	HBBO UCB, $B_{max} = 10$	0.0554 ± 0.0017	0.0010 ± 0.0004	0.0326 ± 0.0011	0.0003 ± 0.0001
	HBBO UCB, $B_{max} = 20$	0.0610 ± 0.0023	0.0015 ± 0.0005	0.0343 ± 0.0013	0.0006 ± 0.0003
	HBBO MEI, $B_{max} = 5$	0.0533 ± 0.0022	0.0017 ± 0.0006	0.0315 ± 0.0014	0.0013 ± 0.0005
	HBBO MEI, $B_{max} = 10$	0.0601 ± 0.0023	0.0021 ± 0.0006	0.0346 ± 0.0014	0.0014 ± 0.0005
	HBBO MEI, $B_{max} = 20$	0.0640 ± 0.0036	0.0032 ± 0.0017	0.0361 ± 0.0021	0.0010 ± 0.0002
Rosenbrock	GP-AUCB, $B_{max} = 5$	0.0572 ± 0.0005	0.0000 ± 0.0000	0.0353 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB, $B_{max} = 10$	0.0580 ± 0.0005	0.0000 ± 0.0000	0.0359 ± 0.0004	0.0000 ± 0.0000
	GP-AUCB, $B_{max} = 20$	0.0577 ± 0.0005	0.0000 ± 0.0000	0.0359 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB Local, $B_{max} = 5$	0.0574 ± 0.0005	0.0000 ± 0.0000	0.0356 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB Local, $B_{max} = 10$	0.0579 ± 0.0005	0.0000 ± 0.0000	0.0360 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB Local, $B_{max} = 20$	0.0602 ± 0.0006	0.0000 ± 0.0000	0.0368 ± 0.0004	0.0000 ± 0.0000
	HBBO UCB, $B_{max} = 5$	0.0579 ± 0.0005	0.0000 ± 0.0000	0.0362 ± 0.0003	0.0000 ± 0.0000
	HBBO UCB, $B_{max} = 10$	0.0578 ± 0.0005	0.0000 ± 0.0000	0.0356 ± 0.0003	0.0000 ± 0.0000
	HBBO UCB, $B_{max} = 20$	0.0580 ± 0.0006	0.0000 ± 0.0000	0.0362 ± 0.0004	0.0000 ± 0.0000
	HBBO MEI, $B_{max} = 5$	0.0540 ± 0.0005	0.0000 ± 0.0000	0.0332 ± 0.0003	0.0000 ± 0.0000
	HBBO MEI, $B_{max} = 10$	0.0545 ± 0.0005	0.0000 ± 0.0000	0.0334 ± 0.0004	0.0000 ± 0.0000
	HBBO MEI, $B_{max} = 20$	0.0550 ± 0.0005	0.0000 ± 0.0000	0.0343 ± 0.0004	0.0000 ± 0.0000
Cosines	GP-AUCB, $B_{max} = 5$	0.2168 ± 0.0012	0.0009 ± 0.0002	0.1191 ± 0.0009	0.0002 ± 0.0001
	GP-AUCB, $B_{max} = 10$	0.2183 ± 0.0015	0.0014 ± 0.0003	0.1182 ± 0.0009	0.0002 ± 0.0001
	GP-AUCB, $B_{max} = 20$	0.2156 ± 0.0014	0.0020 ± 0.0004	0.1186 ± 0.0010	0.0005 ± 0.0001
	GP-AUCB Local, $B_{max} = 5$	0.2118 ± 0.0014	0.0009 ± 0.0002	0.1162 ± 0.0008	0.0002 ± 0.0000
	GP-AUCB Local, $B_{max} = 10$	0.2108 ± 0.0015	0.0016 ± 0.0004	0.1160 ± 0.0011	0.0005 ± 0.0002
	GP-AUCB Local, $B_{max} = 20$	0.2187 ± 0.0015	0.0016 ± 0.0003	0.1218 ± 0.0011	0.0005 ± 0.0002
	HBBO UCB, $B_{max} = 5$	0.2122 ± 0.0012	0.0010 ± 0.0002	0.1160 ± 0.0009	0.0002 ± 0.0001
	HBBO UCB, $B_{max} = 10$	0.2129 ± 0.0013	0.0009 ± 0.0003	0.1155 ± 0.0008	0.0001 ± 0.0000
	HBBO UCB, $B_{max} = 20$	0.2168 ± 0.0017	0.0019 ± 0.0004	0.1204 ± 0.0012	0.0004 ± 0.0001
	HBBO MEI, $B_{max} = 5$	0.2018 ± 0.0014	0.0031 ± 0.0008	0.1126 ± 0.0010	0.0017 ± 0.0006
	HBBO MEI, $B_{max} = 10$	0.2031 ± 0.0014	0.0030 ± 0.0006	0.1138 ± 0.0010	0.0008 ± 0.0002
	HBBO MEI, $B_{max} = 20$	0.2074 ± 0.0016	0.0033 ± 0.0005	0.1177 ± 0.0012	0.0009 ± 0.0002
Vaccine Design	GP-AUCB, $B_{max} = 5$	0.8811 ± 0.0451	0.3048 ± 0.0341	0.6217 ± 0.0373	0.2714 ± 0.0316
	GP-AUCB, $B_{max} = 10$	0.8410 ± 0.0402	0.3016 ± 0.0325	0.6048 ± 0.0345	0.2710 ± 0.0305
	GP-AUCB, $B_{max} = 20$	0.8455 ± 0.0387	0.2963 ± 0.0315	0.6136 ± 0.0353	0.2725 ± 0.0301
	GP-AUCB Local, $B_{max} = 5$	0.9044 ± 0.0367	0.2729 ± 0.0293	0.6174 ± 0.0314	0.2580 ± 0.0287
	GP-AUCB Local, $B_{max} = 10$	0.9564 ± 0.0375	0.2197 ± 0.0265	0.6202 ± 0.0301	0.1911 ± 0.0248
	GP-AUCB Local, $B_{max} = 20$	1.0741 ± 0.0367	0.1562 ± 0.0228	0.6656 ± 0.0277	0.1436 ± 0.0211
	HBBO UCB, $B_{max} = 5$	0.8472 ± 0.0413	0.3141 ± 0.0326	0.6021 ± 0.0346	0.2767 ± 0.0315
	HBBO UCB, $B_{max} = 10$	0.8628 ± 0.0381	0.3456 ± 0.0324	0.6343 ± 0.0326	0.3153 ± 0.0305
	HBBO UCB, $B_{max} = 20$	0.8606 ± 0.0420	0.3188 ± 0.0344	0.6154 ± 0.0368	0.2862 ± 0.0327
	HBBO MEI, $B_{max} = 5$	0.8574 ± 0.0403	0.3030 ± 0.0310	0.6020 ± 0.0316	0.2134 ± 0.0248
	HBBO MEI, $B_{max} = 10$	0.8712 ± 0.0378	0.3135 ± 0.0324	0.6299 ± 0.0325	0.2366 ± 0.0290
	HBBO MEI, $B_{max} = 20$	0.8675 ± 0.0370	0.2934 ± 0.0316	0.6105 ± 0.0308	0.2168 ± 0.0275
SCI	GP-AUCB, $B_{max} = 5$	0.3152 ± 0.0132	0.1444 ± 0.0121	0.2339 ± 0.0120	0.1287 ± 0.0115
	GP-AUCB, $B_{max} = 10$	0.3242 ± 0.0138	0.1252 ± 0.0110	0.2315 ± 0.0118	0.1124 ± 0.0107
	GP-AUCB, $B_{max} = 20$	0.3226 ± 0.0125	0.1403 ± 0.0111	0.2354 ± 0.0112	0.1326 ± 0.0111
	GP-AUCB Local, $B_{max} = 5$	0.3114 ± 0.0134	0.0961 ± 0.0106	0.2128 ± 0.0113	0.0810 ± 0.0100
	GP-AUCB Local, $B_{max} = 10$	0.3084 ± 0.0128	0.0727 ± 0.0088	0.1962 ± 0.0099	0.0606 ± 0.0080
	GP-AUCB Local, $B_{max} = 20$	0.3766 ± 0.0128	0.0739 ± 0.0089	0.2306 ± 0.0095	0.0627 ± 0.0081
	HBBO UCB, $B_{max} = 5$	0.2751 ± 0.0100	0.0557 ± 0.0081	0.1745 ± 0.0087	0.0491 ± 0.0078
	HBBO UCB, $B_{max} = 10$	0.3060 ± 0.0093	0.0516 ± 0.0078	0.1862 ± 0.0081	0.0448 ± 0.0073
	HBBO UCB, $B_{max} = 20$	0.3186 ± 0.0106	0.0613 ± 0.0090	0.1966 ± 0.0094	0.0559 ± 0.0087
	HBBO MEI, $B_{max} = 5$	0.3206 ± 0.0099	0.0292 ± 0.0055	0.1989 ± 0.0058	0.0104 ± 0.0008
	HBBO MEI, $B_{max} = 10$	0.3362 ± 0.0086	0.0201 ± 0.0038	0.2080 ± 0.0053	0.0087 ± 0.0008
	HBBO MEI, $B_{max} = 20$	0.3527 ± 0.0087	0.0274 ± 0.0049	0.2221 ± 0.0057	0.0093 ± 0.0008

Table 6: Average (AR) and Minimum regret (MR) for maximum adaptive batch sizes $B_{max} = 5$, 10, and 20.

Data Set	Algorithm	AR, Round 100	MR, Round 100	AR, Round 200	MR, Round 200
Matérn GP	GP-BUCB, $B = 5$	0.1530 ± 0.0029	0.0037 ± 0.0013	0.0857 ± 0.0020	0.0033 ± 0.0013
	GP-BUCB, $B = 10$	0.2089 ± 0.0032	0.0036 ± 0.0014	0.1138 ± 0.0019	0.0033 ± 0.0014
	GP-BUCB, $B = 20$	0.3314 ± 0.0053	0.0033 ± 0.0012	0.1758 ± 0.0028	0.0022 ± 0.0012
	GP-AUCB, $B_{max} = 5$	0.1501 ± 0.0056	0.0130 ± 0.0052	0.0883 ± 0.0053	0.0120 ± 0.0052
	GP-AUCB, $B_{max} = 10$	0.1742 ± 0.0038	0.0062 ± 0.0018	0.0904 ± 0.0022	0.0029 ± 0.0013
	GP-AUCB, $B_{max} = 20$	0.3144 ± 0.0095	0.0217 ± 0.0040	0.1220 ± 0.0042	0.0087 ± 0.0029
	GP-AUCB Local, $B_{max} = 5$	0.1578 ± 0.0028	0.0057 ± 0.0017	0.0891 ± 0.0020	0.0050 ± 0.0016
	GP-AUCB Local, $B_{max} = 10$	0.2089 ± 0.0035	0.0022 ± 0.0007	0.1138 ± 0.0021	0.0014 ± 0.0007
	GP-AUCB Local, $B_{max} = 20$	0.3287 ± 0.0052	0.0017 ± 0.0007	0.1746 ± 0.0027	0.0013 ± 0.0007
SE GP	GP-BUCB, $B = 5$	0.0663 ± 0.0015	0.0007 ± 0.0002	0.0369 ± 0.0008	0.0004 ± 0.0002
	GP-BUCB, $B = 10$	0.1027 ± 0.0024	0.0005 ± 0.0002	0.0553 ± 0.0012	0.0004 ± 0.0002
	GP-BUCB, $B = 20$	0.1784 ± 0.0047	0.0008 ± 0.0005	0.0931 ± 0.0024	0.0002 ± 0.0001
	GP-AUCB, $B_{max} = 5$	0.0591 ± 0.0015	0.0015 ± 0.0009	0.0338 ± 0.0011	0.0013 ± 0.0009
	GP-AUCB, $B_{max} = 10$	0.0673 ± 0.0027	0.0009 ± 0.0004	0.0361 ± 0.0015	0.0003 ± 0.0001
	GP-AUCB, $B_{max} = 20$	0.0885 ± 0.0025	0.0024 ± 0.0012	0.0422 ± 0.0013	0.0010 ± 0.0007
	GP-AUCB Local, $B_{max} = 5$	0.0683 ± 0.0015	0.0011 ± 0.0003	0.0387 ± 0.0010	0.0006 ± 0.0003
	GP-AUCB Local, $B_{max} = 10$	0.0941 ± 0.0022	0.0004 ± 0.0001	0.0506 ± 0.0011	0.0002 ± 0.0001
	GP-AUCB Local, $B_{max} = 20$	0.1074 ± 0.0028	0.0004 ± 0.0001	0.0549 ± 0.0014	0.0001 ± 0.0001
Rosenbrock	GP-BUCB, $B = 5$	0.0594 ± 0.0005	0.0000 ± 0.0000	0.0371 ± 0.0003	0.0000 ± 0.0000
	GP-BUCB, $B = 10$	0.0602 ± 0.0005	0.0000 ± 0.0000	0.0375 ± 0.0003	0.0000 ± 0.0000
	GP-BUCB, $B = 20$	0.0794 ± 0.0005	0.0000 ± 0.0000	0.0468 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB, $B_{max} = 5$	0.0638 ± 0.0005	0.0000 ± 0.0000	0.0379 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB, $B_{max} = 10$	0.0809 ± 0.0007	0.0000 ± 0.0000	0.0423 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB, $B_{max} = 20$	0.2001 ± 0.0019	0.0004 ± 0.0001	0.0570 ± 0.0005	0.0000 ± 0.0000
	GP-AUCB Local, $B_{max} = 5$	0.0598 ± 0.0004	0.0000 ± 0.0000	0.0369 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB Local, $B_{max} = 10$	0.0602 ± 0.0004	0.0000 ± 0.0000	0.0373 ± 0.0003	0.0000 ± 0.0000
	GP-AUCB Local, $B_{max} = 20$	0.0806 ± 0.0004	0.0000 ± 0.0000	0.0474 ± 0.0003	0.0000 ± 0.0000
Cosines	GP-BUCB, $B = 5$	0.2199 ± 0.0012	0.0010 ± 0.0003	0.1216 ± 0.0008	0.0002 ± 0.0001
	GP-BUCB, $B = 10$	0.2265 ± 0.0015	0.0019 ± 0.0005	0.1255 ± 0.0010	0.0003 ± 0.0001
	GP-BUCB, $B = 20$	0.2401 ± 0.0015	0.0030 ± 0.0005	0.1358 ± 0.0012	0.0003 ± 0.0001
	GP-AUCB, $B_{max} = 5$	0.2719 ± 0.0014	0.0023 ± 0.0004	0.1356 ± 0.0010	0.0003 ± 0.0001
	GP-AUCB, $B_{max} = 10$	0.3930 ± 0.0007	0.0696 ± 0.0032	0.2034 ± 0.0014	0.0006 ± 0.0002
	GP-AUCB, $B_{max} = 20$	0.4205 ± 0.0015	0.1032 ± 0.0037	0.3933 ± 0.0008	0.0726 ± 0.0032
	GP-AUCB Local, $B_{max} = 5$	0.2220 ± 0.0014	0.0013 ± 0.0004	0.1217 ± 0.0010	0.0003 ± 0.0001
	GP-AUCB Local, $B_{max} = 10$	0.2251 ± 0.0015	0.0020 ± 0.0007	0.1247 ± 0.0010	0.0004 ± 0.0002
	GP-AUCB Local, $B_{max} = 20$	0.2383 ± 0.0014	0.0020 ± 0.0004	0.1340 ± 0.0010	0.0002 ± 0.0001
Vaccine Design	GP-BUCB, $B = 5$	0.9260 ± 0.0380	0.1995 ± 0.0235	0.5953 ± 0.0289	0.1796 ± 0.0230
	GP-BUCB, $B = 10$	1.2659 ± 0.0345	0.1252 ± 0.0215	0.7446 ± 0.0266	0.1200 ± 0.0214
	GP-BUCB, $B = 20$	1.8475 ± 0.0307	0.0490 ± 0.0124	1.0281 ± 0.0208	0.0345 ± 0.0097
	GP-AUCB, $B_{max} = 5$	0.9702 ± 0.0394	0.2391 ± 0.0292	0.6358 ± 0.0325	0.2149 ± 0.0280
	GP-AUCB, $B_{max} = 10$	1.1655 ± 0.0446	0.2131 ± 0.0288	0.6466 ± 0.0293	0.1515 ± 0.0222
	GP-AUCB, $B_{max} = 20$	2.1901 ± 0.0789	0.6204 ± 0.0627	1.0715 ± 0.0521	0.1958 ± 0.0258
	GP-AUCB Local, $B_{max} = 5$	1.0020 ± 0.0342	0.2335 ± 0.0287	0.6645 ± 0.0301	0.2074 ± 0.0280
	GP-AUCB Local, $B_{max} = 10$	1.1721 ± 0.0278	0.1076 ± 0.0163	0.6812 ± 0.0201	0.0825 ± 0.0138
	GP-AUCB Local, $B_{max} = 20$	1.8291 ± 0.0306	0.0226 ± 0.0059	1.0094 ± 0.0201	0.0189 ± 0.0050
SCI	GP-BUCB, $B = 5$	0.3614 ± 0.0100	0.0386 ± 0.0065	0.2140 ± 0.0071	0.0201 ± 0.0040
	GP-BUCB, $B = 10$	0.5019 ± 0.0086	0.0200 ± 0.0037	0.2757 ± 0.0052	0.0094 ± 0.0008
	GP-BUCB, $B = 20$	0.7114 ± 0.0075	0.0045 ± 0.0013	0.3775 ± 0.0041	0.0033 ± 0.0006
	GP-AUCB, $B_{max} = 5$	0.3641 ± 0.0136	0.0648 ± 0.0091	0.2203 ± 0.0100	0.0455 ± 0.0076
	GP-AUCB, $B_{max} = 10$	0.4735 ± 0.0215	0.0747 ± 0.0093	0.2548 ± 0.0114	0.0434 ± 0.0073
	GP-AUCB, $B_{max} = 20$	0.7793 ± 0.0353	0.1353 ± 0.0173	0.3831 ± 0.0197	0.0543 ± 0.0073
	GP-AUCB Local, $B_{max} = 5$	0.3701 ± 0.0109	0.0434 ± 0.0069	0.2192 ± 0.0076	0.0235 ± 0.0049
	GP-AUCB Local, $B_{max} = 10$	0.4893 ± 0.0083	0.0199 ± 0.0040	0.2674 ± 0.0050	0.0103 ± 0.0021
	GP-AUCB Local, $B_{max} = 20$	0.7197 ± 0.0070	0.0032 ± 0.0010	0.3849 ± 0.0042	0.0023 ± 0.0005

Table 7: Average (AR) and Minimum regret (MR) for delay lengths $B = 5$, 10, and 20.

Data Set	Algorithm	Query 40	Query 100	Query 200
Matern GP	GP-UCB	0.5992 ± 0.0010	1.9532 ± 0.0037	6.5840 ± 0.0040
	GP-UCB Lazy	0.1764 ± 0.0026	0.2357 ± 0.0033	0.3824 ± 0.0044
	GP-BUCB	0.5947 ± 0.0006	1.9363 ± 0.0027	6.5302 ± 0.0053
	GP-BUCB Lazy	0.2957 ± 0.0018	0.3481 ± 0.0026	0.4592 ± 0.0035
	SM-UCB	2.9618 ± 0.0078	10.8404 ± 0.0123	44.5631 ± 0.0137
	SM-UCB Lazy	6.3990 ± 0.0186	16.0585 ± 0.0370	37.0588 ± 0.0703
	SM-MEI	3.0716 ± 0.0010	11.1101 ± 0.0035	45.1232 ± 0.0072
	SM-MEI Lazy	15.8325 ± 0.0273	38.2692 ± 0.0358	80.2965 ± 0.0446
	HBBO UCB	0.5594 ± 0.0016	1.8075 ± 0.0044	6.2063 ± 0.0064
	HBBO MEI	0.5658 ± 0.0007	1.8250 ± 0.0008	6.2486 ± 0.0009
	GP-AUCB	0.6699 ± 0.0001	1.8984 ± 0.0003	6.2417 ± 0.0020
	GP-AUCB Lazy	0.3527 ± 0.0021	0.4040 ± 0.0029	0.5125 ± 0.0039
	GP-AUCB Local	0.4708 ± 0.0008	1.5885 ± 0.0030	5.6950 ± 0.0071
	GP-AUCB Lazy Local	0.2823 ± 0.0022	0.3338 ± 0.0029	0.4420 ± 0.0040
SE GP	GP-UCB	0.5993 ± 0.0001	1.9520 ± 0.0004	6.5182 ± 0.0011
	GP-UCB Lazy	0.2891 ± 0.0061	0.4265 ± 0.0109	0.6406 ± 0.0170
	GP-BUCB	0.6011 ± 0.0001	1.9462 ± 0.0003	6.5005 ± 0.0008
	GP-BUCB Lazy	0.3703 ± 0.0053	0.4982 ± 0.0100	0.7039 ± 0.0165
	SM-UCB	2.9105 ± 0.0009	10.6819 ± 0.0028	44.0896 ± 0.0062
	SM-UCB Lazy	7.3562 ± 0.0410	17.7201 ± 0.0773	39.3720 ± 0.1229
	SM-MEI	3.0133 ± 0.0010	10.9425 ± 0.0028	44.6246 ± 0.0056
	SM-MEI Lazy	17.6639 ± 0.0782	41.0165 ± 0.1256	83.6221 ± 0.1674
	HBBO UCB	0.5549 ± 0.0006	1.7936 ± 0.0008	6.1503 ± 0.0013
	HBBO MEI	0.5630 ± 0.0006	1.8161 ± 0.0007	6.1973 ± 0.0011
	GP-AUCB	0.6749 ± 0.0001	1.8999 ± 0.0003	6.2147 ± 0.0007
	GP-AUCB Lazy	0.4348 ± 0.0057	0.5588 ± 0.0103	0.7555 ± 0.0162
	GP-AUCB Local	0.4684 ± 0.0001	1.5820 ± 0.0003	5.6564 ± 0.0010
	GP-AUCB Lazy Local	0.3206 ± 0.0050	0.4442 ± 0.0095	0.6404 ± 0.0155
Rosenbrock	GP-UCB	0.5916 ± 0.0030	1.9470 ± 0.0091	6.4193 ± 0.0172
	GP-UCB Lazy	0.3356 ± 0.0013	0.4521 ± 0.0020	0.6437 ± 0.0034
	GP-BUCB	0.5841 ± 0.0009	1.9074 ± 0.0032	6.3437 ± 0.0071
	GP-BUCB Lazy	0.3787 ± 0.0012	0.4900 ± 0.0020	0.6732 ± 0.0034
	SM-UCB	2.8155 ± 0.0010	10.4979 ± 0.0023	43.1365 ± 0.0113
	SM-UCB Lazy	6.4663 ± 0.0075	15.9830 ± 0.0109	36.1740 ± 0.0158
	SM-MEI	2.9311 ± 0.0077	10.7848 ± 0.0181	43.6449 ± 0.0199
	SM-MEI Lazy	15.5865 ± 0.0180	36.8226 ± 0.0204	76.0677 ± 0.0253
	HBBO UCB	0.6033 ± 0.0012	1.8202 ± 0.0017	6.0460 ± 0.0019
	HBBO MEI	0.6076 ± 0.0010	1.8366 ± 0.0011	6.0840 ± 0.0014
	GP-AUCB	0.6510 ± 0.0001	1.8495 ± 0.0004	6.0354 ± 0.0008
	GP-AUCB Lazy	0.4372 ± 0.0010	0.5463 ± 0.0018	0.7317 ± 0.0031
	GP-AUCB Local	0.4582 ± 0.0001	1.5449 ± 0.0002	5.4983 ± 0.0039
	GP-AUCB Lazy Local	0.3569 ± 0.0011	0.4609 ± 0.0016	0.6349 ± 0.0029
Cosines	GP-UCB	0.5829 ± 0.0001	1.9113 ± 0.0004	6.3865 ± 0.0013
	GP-UCB Lazy	0.2750 ± 0.0007	0.3654 ± 0.0010	0.4913 ± 0.0015
	GP-BUCB	0.5810 ± 0.0001	1.9094 ± 0.0004	6.3497 ± 0.0011
	GP-BUCB Lazy	0.3452 ± 0.0009	0.4267 ± 0.0012	0.5242 ± 0.0013
	SM-UCB	2.8365 ± 0.0014	10.6211 ± 0.0046	43.3563 ± 0.0134
	SM-UCB Lazy	5.9717 ± 0.0038	15.0289 ± 0.0059	34.3330 ± 0.0123
	SM-MEI	2.9458 ± 0.0010	10.9126 ± 0.0049	43.9570 ± 0.0134
	SM-MEI Lazy	14.8944 ± 0.0084	35.7111 ± 0.0094	74.2792 ± 0.0243
	HBBO UCB	0.6536 ± 0.0012	1.8741 ± 0.0013	6.1179 ± 0.0015
	HBBO MEI	0.6654 ± 0.0014	1.9006 ± 0.0015	6.1691 ± 0.0017
	GP-AUCB	0.6493 ± 0.0002	1.8570 ± 0.0003	6.0636 ± 0.0008
	GP-AUCB Lazy	0.3962 ± 0.0006	0.4772 ± 0.0010	0.5727 ± 0.0011
	GP-AUCB Local	0.4602 ± 0.0001	1.5595 ± 0.0004	5.5394 ± 0.0008
	GP-AUCB Lazy Local	0.4940 ± 0.0009	0.5946 ± 0.0022	0.6863 ± 0.0023
Vaccine Design	GP-UCB	1.8238 ± 0.0005	6.0469 ± 0.0019	20.3408 ± 0.0047
	GP-UCB Lazy	0.6347 ± 0.0094	0.7021 ± 0.0094	0.9267 ± 0.0094
	GP-BUCB	1.8252 ± 0.0004	5.9950 ± 0.0016	20.1145 ± 0.0039
	GP-BUCB Lazy	1.1121 ± 0.0024	1.1472 ± 0.0024	1.2584 ± 0.0024
	SM-UCB	8.3346 ± 0.0032	32.0995 ± 0.0167	134.3169 ± 0.0458
	SM-UCB Lazy	22.5270 ± 0.2438	46.4903 ± 0.7422	99.7192 ± 1.6901
	SM-MEI	8.5207 ± 0.0023	32.6054 ± 0.0135	135.4947 ± 0.0391
	SM-MEI Lazy	49.0936 ± 0.2575	115.2695 ± 0.7228	243.0290 ± 1.2733
	HBBO UCB	2.3149 ± 0.0128	6.2659 ± 0.0234	19.8061 ± 0.0377
	HBBO MEI	2.2665 ± 0.0121	6.2584 ± 0.0211	19.9313 ± 0.0376
	GP-AUCB	2.2982 ± 0.0003	6.1147 ± 0.0014	19.5122 ± 0.0041
	GP-AUCB Lazy	1.2533 ± 0.0049	1.2877 ± 0.0050	1.3968 ± 0.0049
	GP-AUCB Local	1.4302 ± 0.0003	4.8850 ± 0.0013	17.4855 ± 0.0039
	GP-AUCB Lazy Local	1.0196 ± 0.0083	1.0676 ± 0.0090	1.1843 ± 0.0095
Spinal Cord Therapy	GP-UCB	0.0721 ± 0.0001	0.2404 ± 0.0006	0.8161 ± 0.0021
	GP-UCB Lazy	0.0138 ± 0.0001	0.0323 ± 0.0002	0.0917 ± 0.0003
	GP-BUCB	0.0721 ± 0.0000	0.2395 ± 0.0003	0.8134 ± 0.0014
	GP-BUCB Lazy	0.0236 ± 0.0001	0.0421 ± 0.0001	0.1042 ± 0.0004
	SM-UCB	0.5920 ± 0.0005	1.9495 ± 0.0020	6.7668 ± 0.0024
	SM-UCB Lazy	0.7619 ± 0.0009	2.0412 ± 0.0039	4.8634 ± 0.0116
	SM-MEI	0.6547 ± 0.0002	2.1107 ± 0.0019	7.0953 ± 0.0037
	SM-MEI Lazy	1.8187 ± 0.0018	4.4822 ± 0.0096	9.3589 ± 0.0268
	HBBO UCB	0.0702 ± 0.0003	0.2333 ± 0.0008	0.8007 ± 0.0019
	HBBO MEI	0.0759 ± 0.0003	0.2482 ± 0.0009	0.8317 ± 0.0021
	GP-AUCB	0.0742 ± 0.0001	0.2268 ± 0.0004	0.7722 ± 0.0019
	GP-AUCB Lazy	0.0266 ± 0.0001	0.0442 ± 0.0002	0.1050 ± 0.0004
	GP-AUCB Local	0.0585 ± 0.0003	0.1996 ± 0.0007	0.7152 ± 0.0018
	GP-AUCB Lazy Local	0.0342 ± 0.0004	0.0533 ± 0.0006	0.1186 ± 0.0012

Table 8: Mean wall-clock execution times and standard deviations of estimate (S).