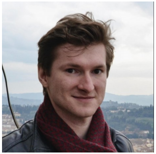
GANs with integral probability metrics: some results and conjectures

Arthur Gretton

Gatsby Computational Neuroscience Unit,
University College London

MILA, Montreal, 2019

A motivation: comparing two samples

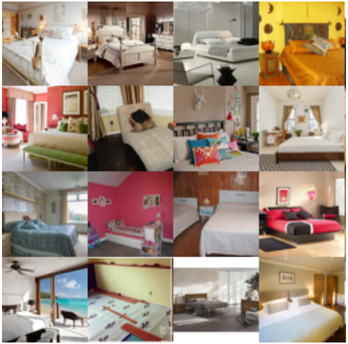
- **Given:** Samples from unknown distributions P and Q.
- **Goal:** do P and Q differ?

$$\sim P$$

$$\sim Q$$

Training implicit generative models

- **Have:** One collection of samples X from unknown distribution P.
- **Goal:** generate samples Q that look like P



LSUN bedroom samples P

Generated Q, MMD GAN

Using a critic $D(P, Q)$ to train a GAN

(Binkowski, Sutherland, Arbel, G., ICLR 2018),
(Arbel, Sutherland, Binkowski, G., NeurIPS 2018)

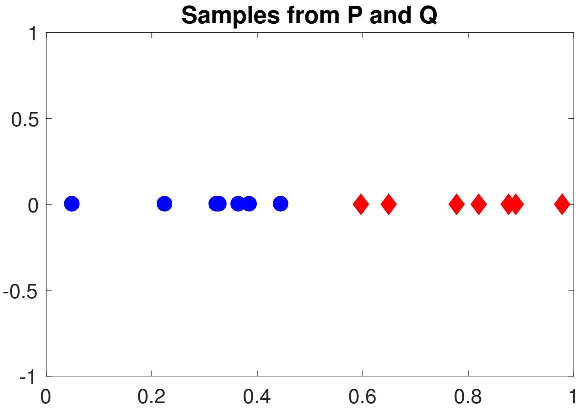
Outline

- Measures of distance between distributions
  - The MMD: an integral probability metric
  - f-divergences vs integral probability metrics
- Gradient penalties for GAN critics
  - The optimisation viewpoint
  - The regularisation viewpoint
- Theory
  - Relation of MMD critic and Wasserstein
  - Gradient bias
- Evaluating GAN performance, experiments

The Maximum Mean Discrepancy: An Integral Probability Metric

Integral probability metrics

Are P and Q different?



Integral probability metrics

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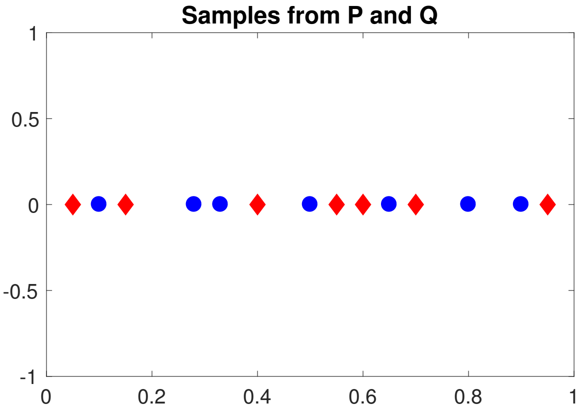
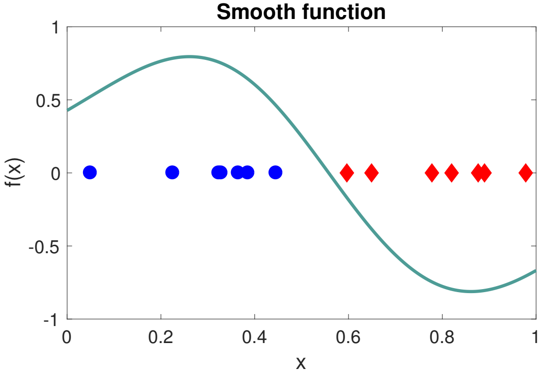


Integral probability metrics

Integral probability metric:

Find a "well behaved function" $f(x)$ to maximize

$$\mathbf{E}_P f(X) - \mathbf{E}_Q f(Y)$$



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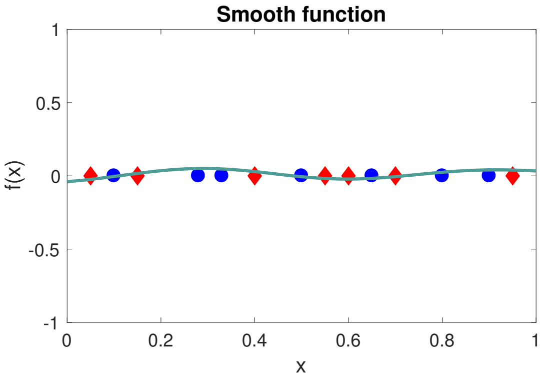
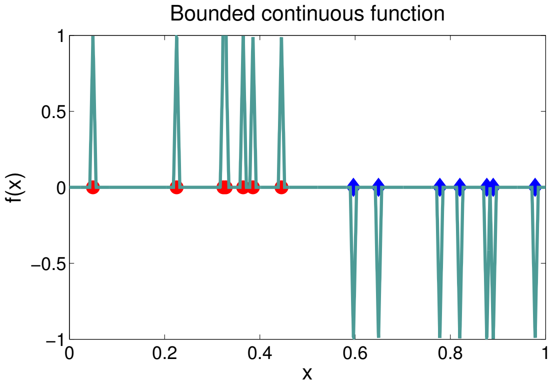


Integral probability metrics

What if the function is not well behaved?

$$\mathbf{E}_P f(X) - \mathbf{E}_Q f(Y)$$



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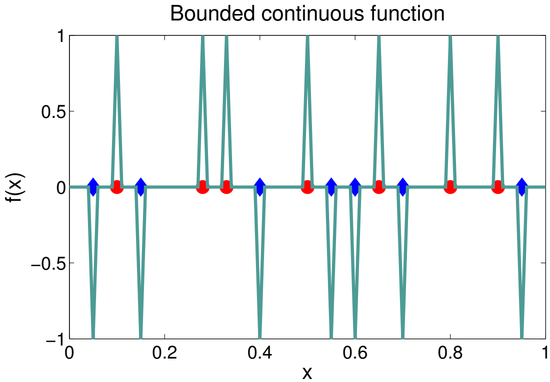

The MMD: an integral probability metric

Maximum mean discrepancy: smooth function for P vs Q

$$MMD(P, Q; F) := \sup_{\|f\| \leq 1} \left[\mathbf{E}_P f(X) - \mathbf{E}_Q f(Y) \right]$$

$$(F = \text{unit ball in RKHS } \mathcal{F})$$

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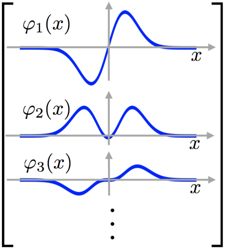
Functions are linear combinations of features:

$$f(x) = \langle f, \varphi(x) \rangle_{\mathcal{F}} = \sum_{\ell=1}^{\infty} f_\ell \varphi_\ell(x) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \end{bmatrix}^\top$$



$$\|f\|_{\mathcal{F}}^2 := \sum_{i=1}^{\infty} f_i^2 \leq 1$$

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For characteristic RKHS $\mathcal{F}$, $MMD(P, Q; F) = 0$ iff $P = Q$

Other choices for witness function class:

- Bounded continuous [Dudley, 2002]
- Bounded varation 1 (Kolmogorov metric) [Müller, 1997]
- Lipschitz (Wasserstein distances) [Dudley, 2002]

The MMD: an integral probability metric

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Expectations of functions are linear combinations of expected features

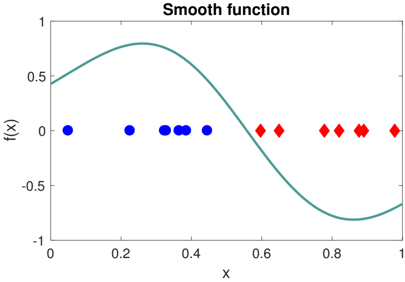
$$\mathbf{E}_P(f(X)) = \langle f, \mathbf{E}_P \varphi(X) \rangle_{\mathcal{F}} = \langle f, \mu_P \rangle_{\mathcal{F}}$$

(always true if kernel is bounded)

Integral prob. metric vs feature mean difference

The MMD:

$$MMD(P, Q; F) = \sup_{f \in F} \left[\mathbf{E}_P f(X) - \mathbf{E}_Q f(Y) \right]$$



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$$
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&= \sup_{f \in F} \langle f, \mu_P - \mu_Q \rangle_{\mathcal{F}}
\end{aligned}
$$

use

$$\mathbf{E}_P f(X) = \langle \mu_P, f \rangle_{\mathcal{F}}$$

Integral prob. metric vs feature mean difference

The MMD:

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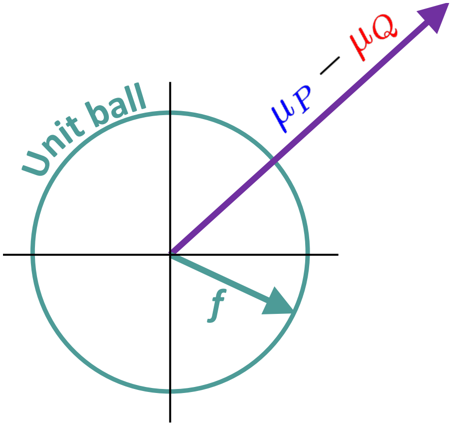

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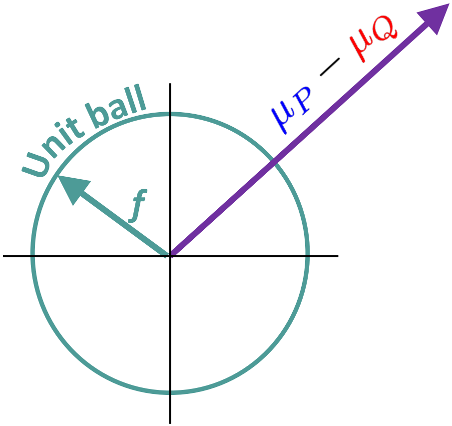

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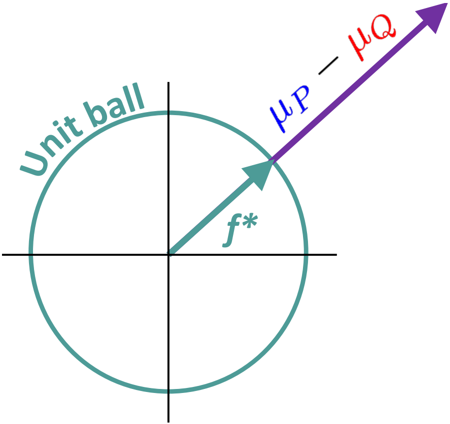

$$f^* = \frac{\mu_P - \mu_Q}{\|\mu_P - \mu_Q\|}$$

Integral prob. metric vs feature mean difference

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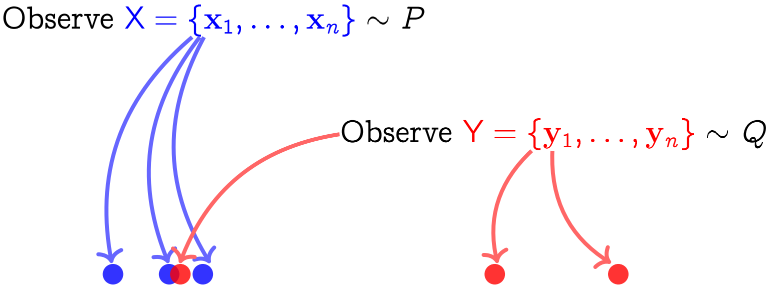
&= \sup_{f \in F} \langle f, \mu_P - \mu_Q \rangle_{\mathcal{F}} \\
&= \| \mu_P - \mu_Q \|
\end{aligned}
$$

IPM view equivalent to feature mean difference (kernel case only)

Construction of MMD witness

Construction of empirical witness function (proof: next slide!)



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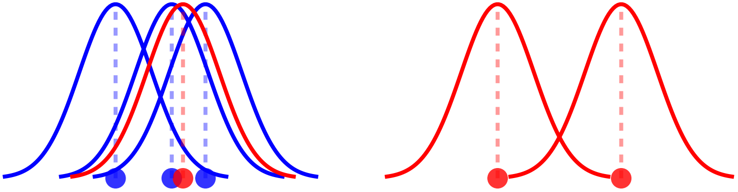

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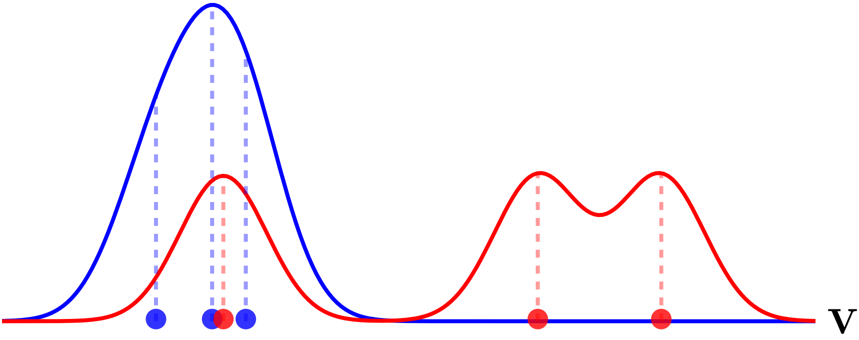

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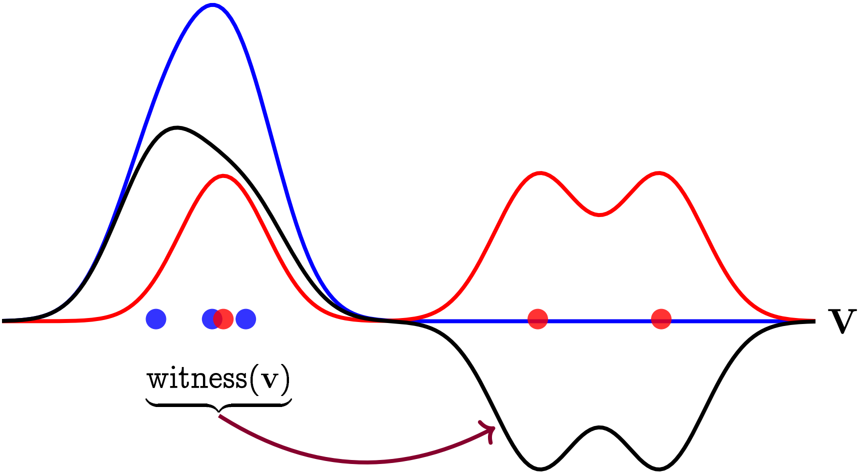

Derivation of empirical witness function

Recall the witness function expression

$$f^* \propto \mu_P - \mu_Q$$

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The empirical feature mean for P

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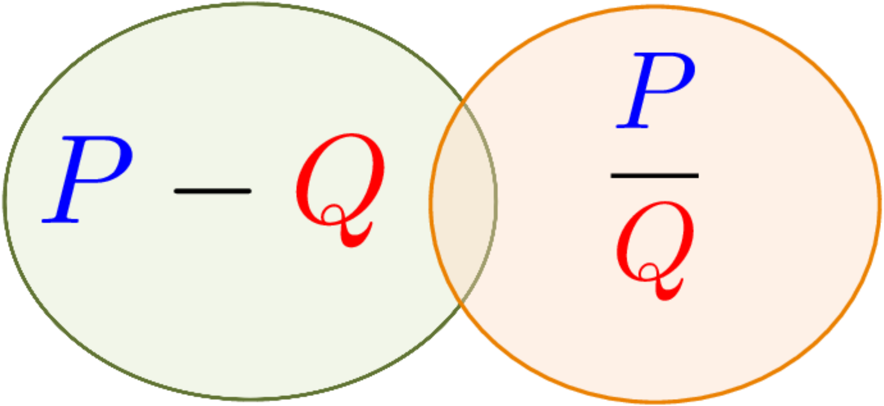
The empirical witness function at v

$$\begin{aligned} f^*(v) &= \langle f^*, \varphi(v)\rangle_{\mathcal{F}} \\ &\propto \langle \widehat{\mu}_P - \widehat{\mu}_Q, \varphi(v)\rangle_{\mathcal{F}} \\ &= \frac{1}{n}\sum_{i=1}^{n} k(x_i, v) - \frac{1}{n}\sum_{i=1}^{n} k(\mathrm{y}_i, v) \end{aligned}$$

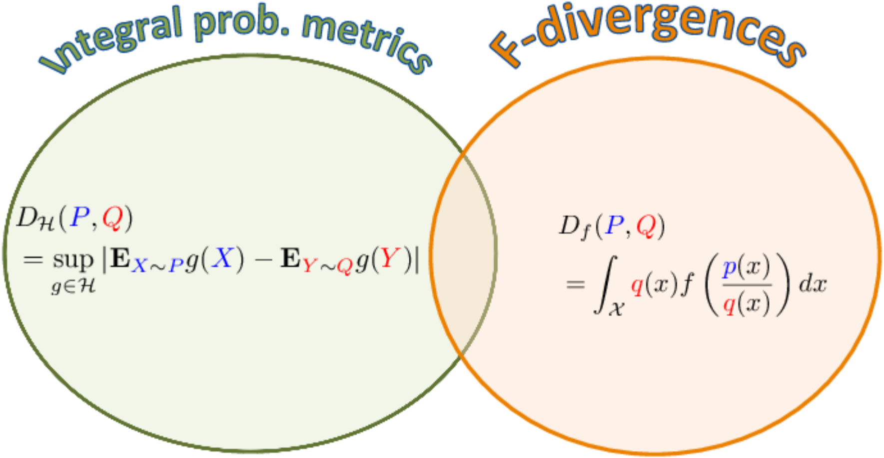
Don't need explicit feature coefficients $f^* := \begin{bmatrix} f_1^* & f_2^* & \dots \end{bmatrix}$

Interlude: divergence measures

Divergences



Divergences



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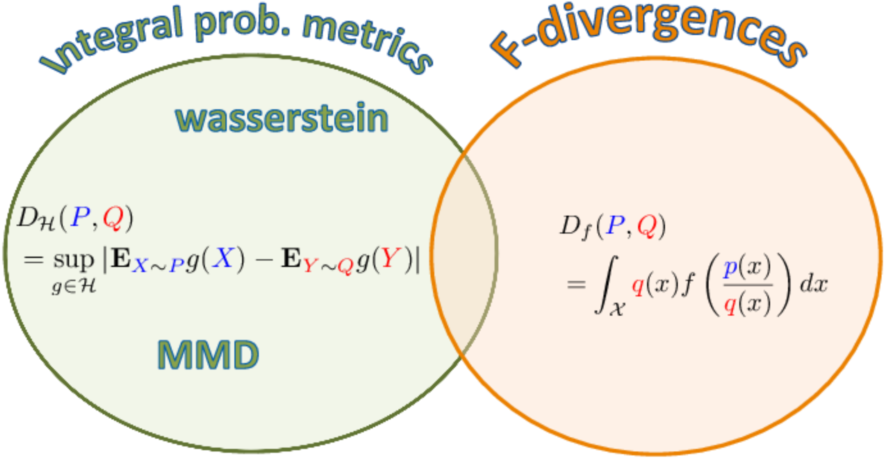

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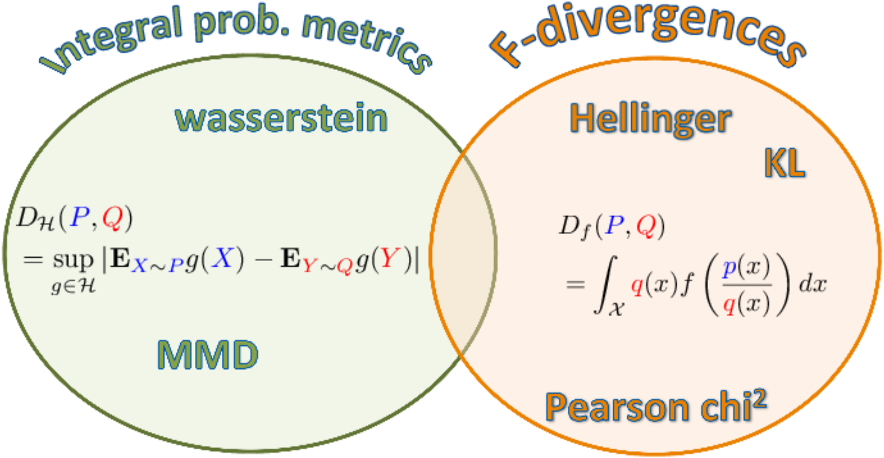

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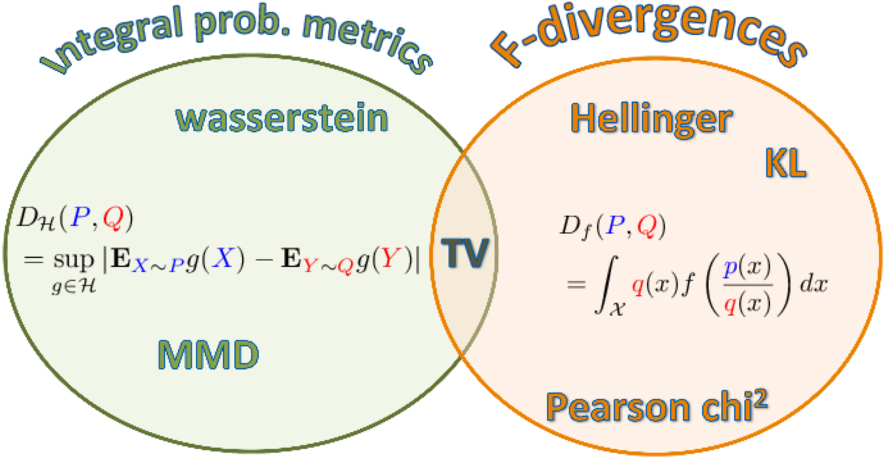
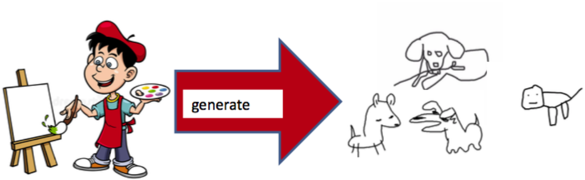


Sriperumbudur, Fukumizu, G, Schoelkopf, Lanckriet (2012)

Training Generative Adversarial Networks: Critics and Gradient Penalties

Visual notation: GAN setting



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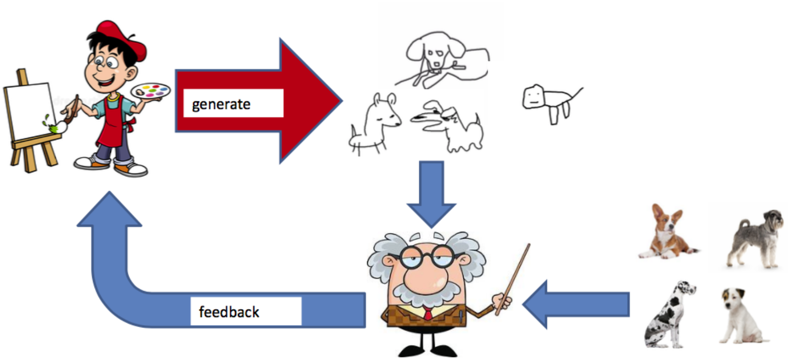
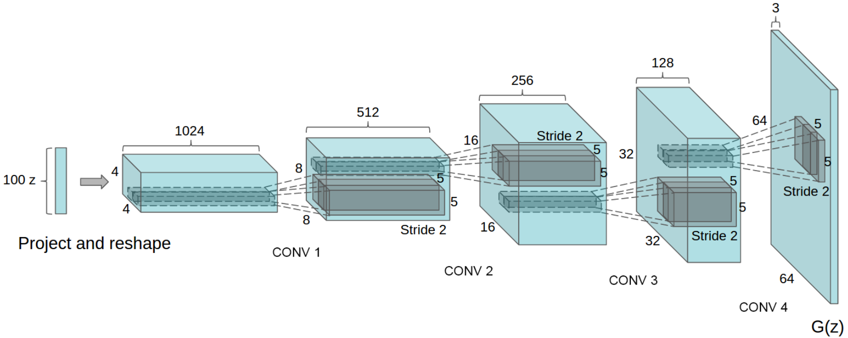


What I won't cover: the generator



Radford, Metz, Chintala, ICLR 2016

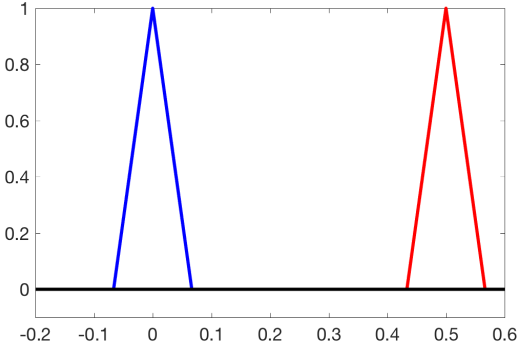
F-divergence as critic

An unhelpful critic? Jensen-Shannon,

Goodfellow et al. (NeurIPS 2014), Arjovsky and Bottou [ICLR 2017]

$$D_{JS}(P, Q) = \frac{1}{2} D_{KL}\left(p, \frac{p+q}{2}\right) + \frac{1}{2} D_{KL}\left(q, \frac{p+q}{2}\right)$$

$$D_{JS}(P, Q) = \log 2$$



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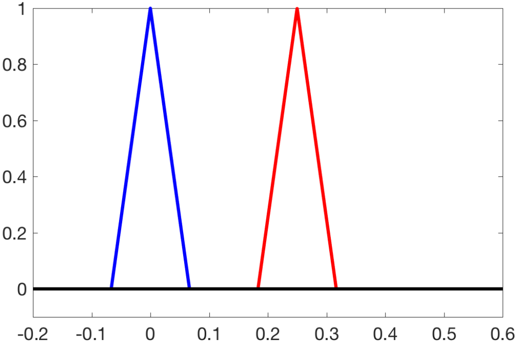

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What is done in practice?

- Use a variational approximation to the critic, alternate generator and critic training (we will return to this!) Goodfellow et al. [NeurIPS 2014], Nowozin et al. [NeurIPS 2016]

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- Add "instance noise" to the reference and generator observations Sonderby et al. [arXiv 2016], Arjovsky and Bottou [ICLR 2017]
  - ...or (approx. equivalently) a data-dependent gradient penalty for the variational critic (we will return to this!) Roth et al [NeurIPS 2017], Nagarajan and Kolter [NeurIPS 2017], Mescheder et al. [ICML 2018]

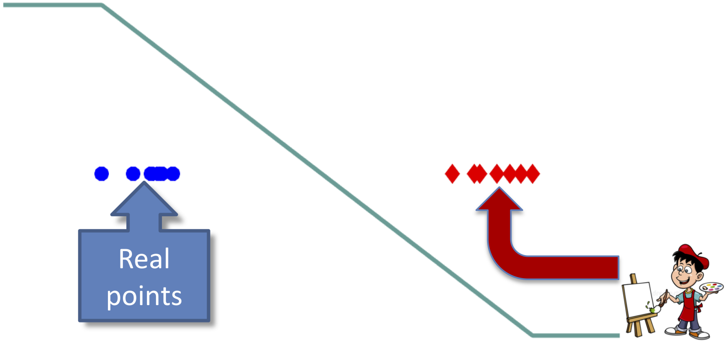
Wasserstein distance as critic

A helpful critic witness:

$$W_1(P, Q) = \sup_{\|f\|_L \leq 1} E_P f(X) - E_Q f(Y).$$

$$\|f\|_L := \sup_{x \neq y} |f(x) - f(y)| / \|x - y\|$$

W_1=0.88



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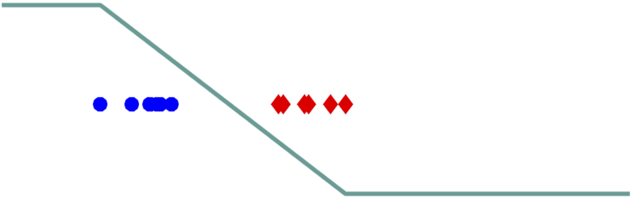
$W_1 = 0.65$



MMD as critic

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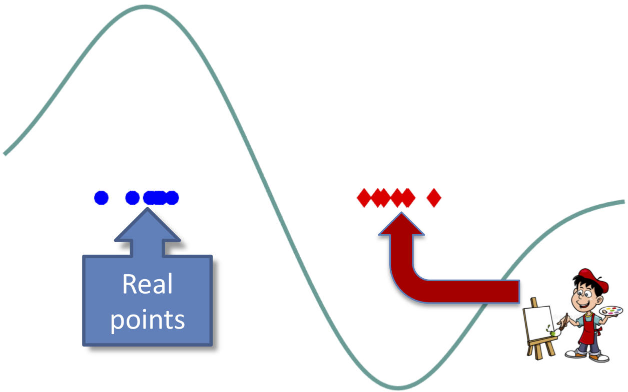
MMD=1.8



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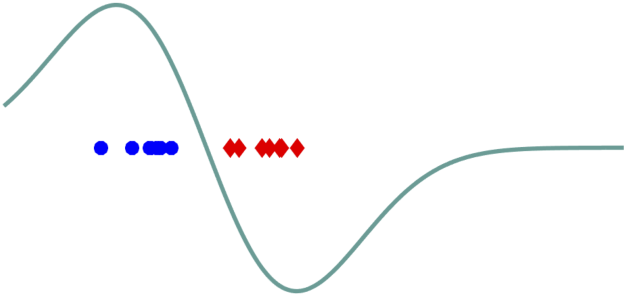
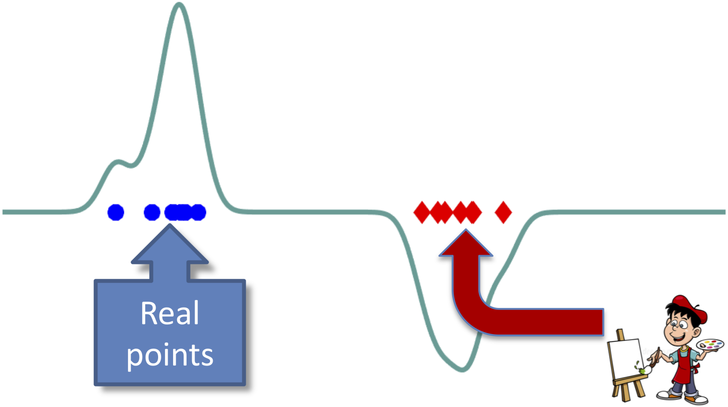
MMD=1.1



MMD as critic

An unhelpful critic witness:
$MMD(P, Q)$ with a narrow kernel.

MMD=0.64



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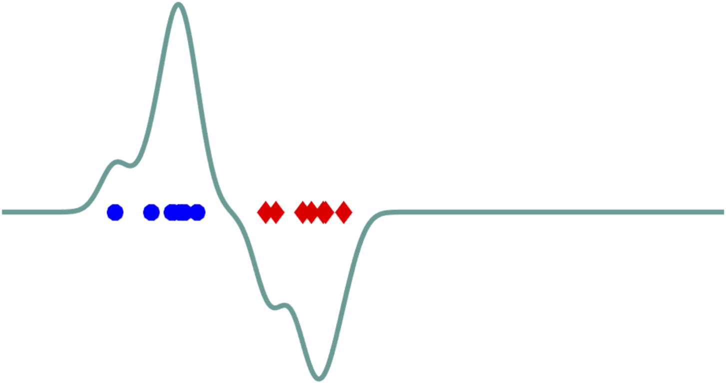


MMD for GAN critic

Can you use MMD as a critic to train GANs?

From ICML 2015:

Generative Moment Matching Networks

Yujia Li[1] YUJIALI@CS.TORONTO.EDU
Kevin Swersky[1] KSWERSKY@CS.TORONTO.EDU
Richard Zemel[1,2] ZEMEL@CS.TORONTO.EDU
[1]Department of Computer Science, University of Toronto, Toronto, ON, CANADA
[2]Canadian Institute for Advanced Research, Toronto, ON, CANADA

From UAI 2015:

Training generative neural networks via Maximum Mean Discrepancy optimization

Gintare Karolina Dziugaite
University of Cambridge

Daniel M. Roy
University of Toronto

Zoubin Ghahramani
University of Cambridge

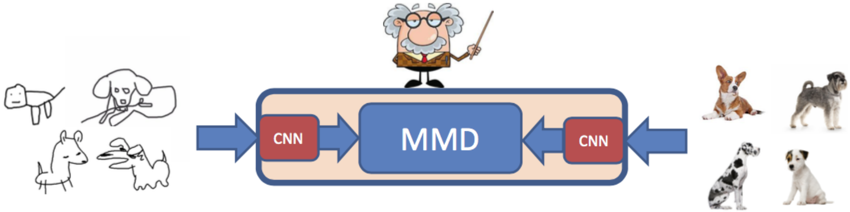
MMD for GAN critic

Can you use MMD as a critic to train GANs?

Need better image features.

CNN features for an MMD witness

- Add convolutional features!
- The critic (teacher) also needs to be trained.



$\mathfrak{K}(x,y) = h_\psi{}^\top(x) h_\psi(y)$
where $h_\psi(x)$ is a CNN map:

- Wasserstein GAN Arjovsky et al. [ICML 2017]
- WGAN-GP Gulrajani et al. [NeurIPS 2017]

$\mathfrak{K}(x,y) = k(h_\psi(x), h_\psi(y))$
where $h_\psi(x)$ is a CNN map,
k is e.g. an exponentiated quadratic kernel

MMD Li et al., [NeurIPS 2017]
Cramer Bellemare et al. [2017]
Coulomb Unterthiner et al., [ICLR 2018]
Demystifying MMD GANs Binkowski, Sutherland, Arbel, G., [ICLR 2018]

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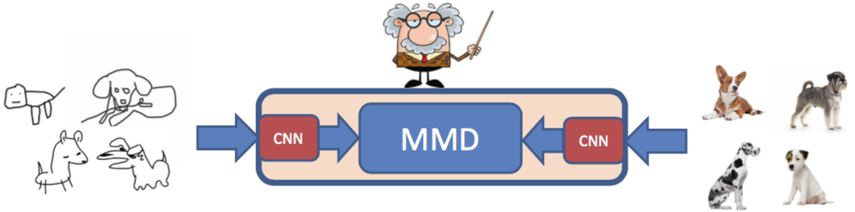

$\mathfrak{K}(x, y) = h_\psi{}^\top(x) h_\psi(y)$
where $h_\psi(x)$ is a CNN map:

- Wasserstein GAN Arjovsky et al. [ICML 2017]
- WGAN-GP Gulrajani et al. [NeurIPS 2017]

$\mathfrak{K}(x, y) = k(h_\psi(x), h_\psi(y))$
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MMD Li et al., [NeurIPS 2017]
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Demystifying MMD GANs Binkowski, Sutherland, Arbel, G., [ICLR 2018]

Gradient penalty: the optimisation viewpoint

WGAN-GP

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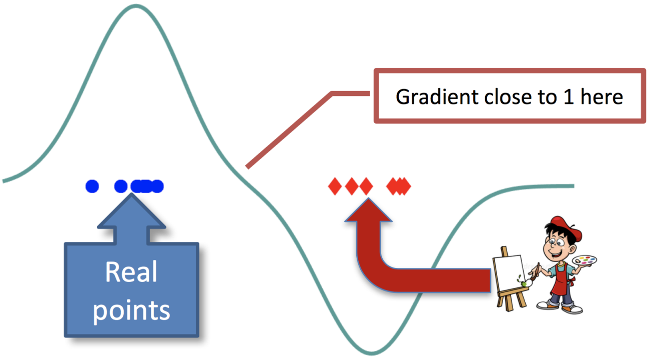

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- Given a generator G_θ with parameters θ to be trained. Samples $Y \sim G_\theta(Z)$ where $Z \sim R$

- Given critic features h_ψ with parameters ψ to be trained. f_ψ a linear function, $\mathfrak{K}(x, y) = {h_\psi}^\top(x) h_\psi(y)$.

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WGAN-GP gradient penalty:

$$\max_\psi \mathbf{E}_{X \sim P} f_\psi(X) - \mathbf{E}_{Z \sim R} f_\psi(G_\theta(Z)) + \lambda \mathbf{E}_{\widetilde{X}} \left(\left\| \nabla_{\widetilde{X}} f_\psi(\widetilde{X}) \right\| - 1 \right)^2$$

where

$$\widetilde{X} = \gamma x_i + (1 - \gamma) G_\theta(z_j)$$

$$\gamma \sim \mathcal{U}([0, 1]) \quad x_i \in \{x_\ell\}_{\ell=1}^m \quad z_j \in \{z_\ell\}_{\ell=1}^n$$

DiracGAN toy example

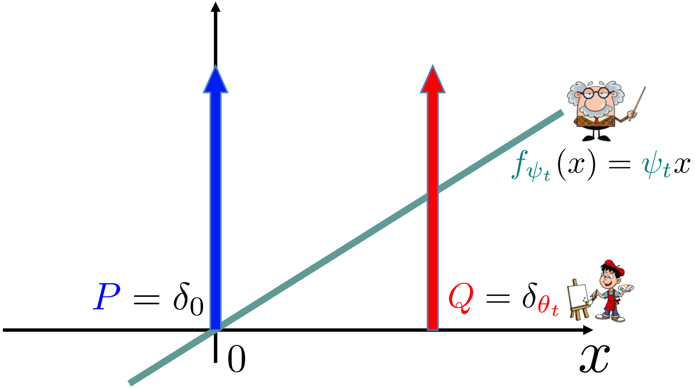
From ICML 2018:

Which Training Methods for GANs do actually Converge?

Lars Mescheder [1] **Andreas Geiger** [1,2] **Sebastian Nowozin** [3]

Gives an optimisation viewpoint on gradient regularisation.

DiracGAN toy example

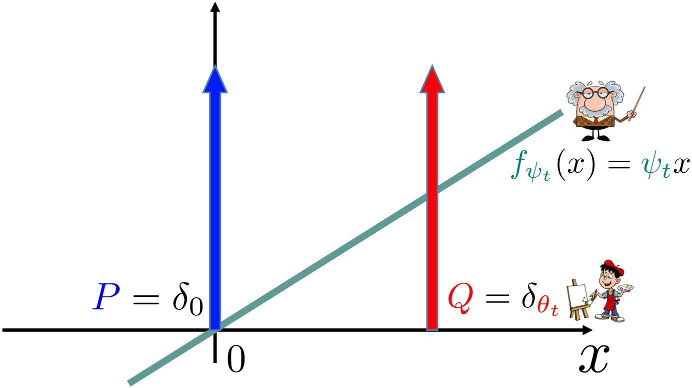


$$D(P, Q; \psi_t) = \mathbf{E}_Q f_{\psi_t}(Y) - \mathbf{E}_P f_{\psi_t}(X)$$
$$= \psi_t \theta_t$$

Mescheder et al. [ICML 2018]

DiracGAN toy example

Gradient descent on generator:



$$\frac{\partial}{\partial \theta} D(P, Q; \psi_t) = \frac{\partial}{\partial \theta} \psi_t \theta_t = \psi_t$$

DiracGAN toy example

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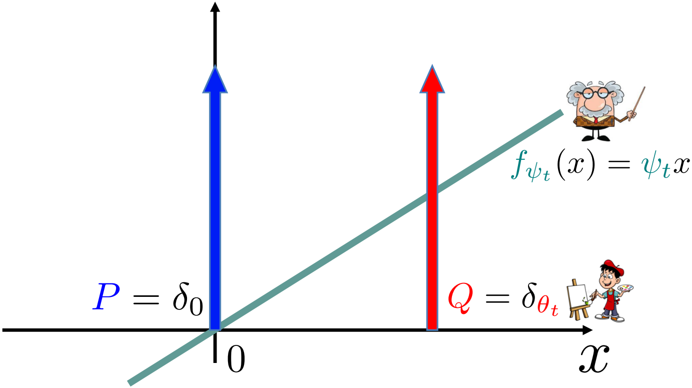

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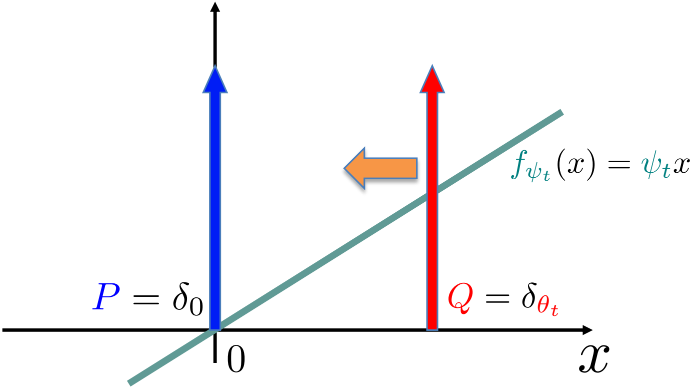
for stepsize γ

DiracGAN toy example

Gradient **descent** on generator:



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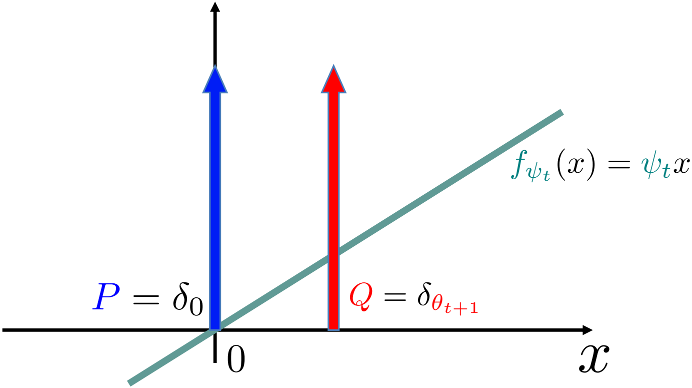

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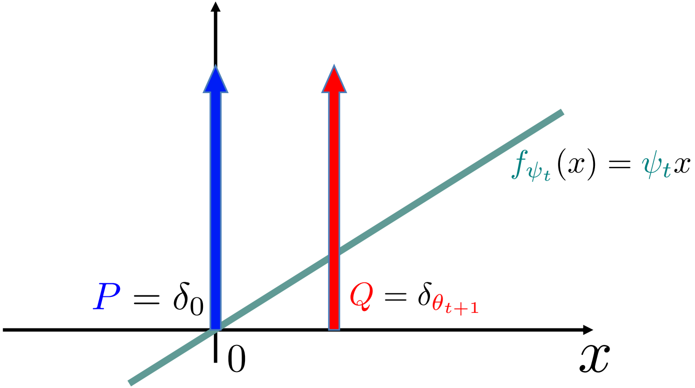
DiracGAN toy example

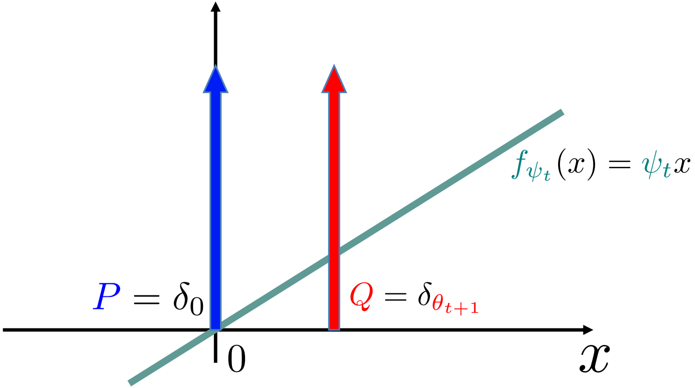
Gradient **ascent** on critic:



$$\frac{\partial}{\partial \psi} D(P, Q; \psi_t) = \theta_{t+1}$$

DiracGAN toy example

Gradient **ascent** on critic:



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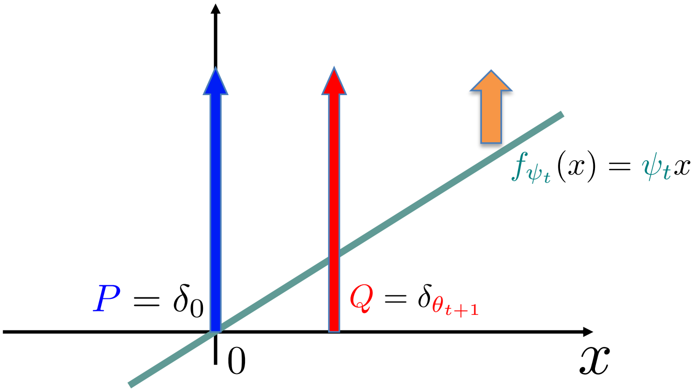
for stepsize ζ

DiracGAN toy example

Gradient ascent on critic:



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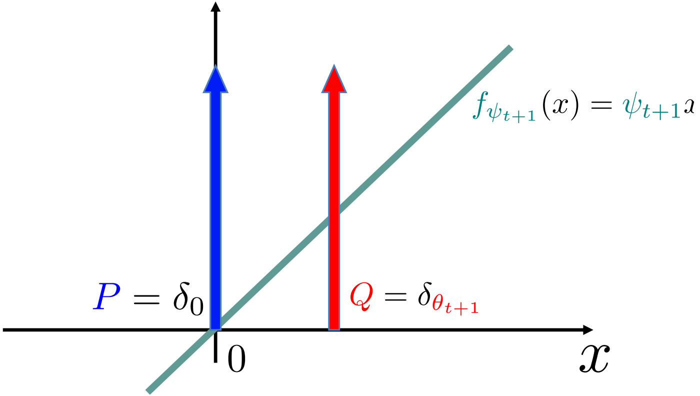

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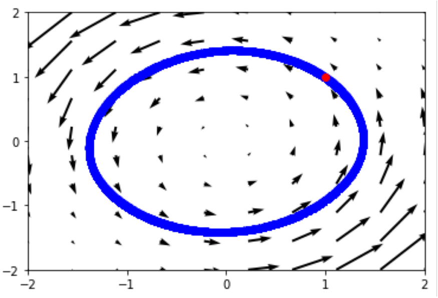
DiracGAN toy example

Idealised continuous system (infinitely small learning rate)

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\nabla_{\psi} D(P, Q; \psi) \\ \nabla_{\theta} D(P, Q; \psi) \end{bmatrix}$$

Every integral curve $(\psi(t), \theta(t))$ of the gradient vector field satisfies $\psi^2(t) + \theta^2(t) = c$ for all $t \in [0, \infty)$.



Mescheder et al. [ICML 2018, Lemma 2.3]

WGAN toy example

WGAN-GP style gradient penalty may not converge near solution

Nagarajan and Kolter [NeurIPS 2017], Mescheder et al. [ICML 2018], Balduzzi et al. [ICML 2018]

Recall the WGAN-GP penalisation

$$\max_{\psi} \mathbf{E}_{X\sim P} f_\psi(X) - \mathbf{E}_{Z\sim R} f_\psi(G_\theta(Z)) + \lambda \mathbf{E}_{\widetilde{X}} \left(\left\| \nabla_{\widetilde{X}} f_\psi(\widetilde{X}) \right\| - 1 \right)^2$$

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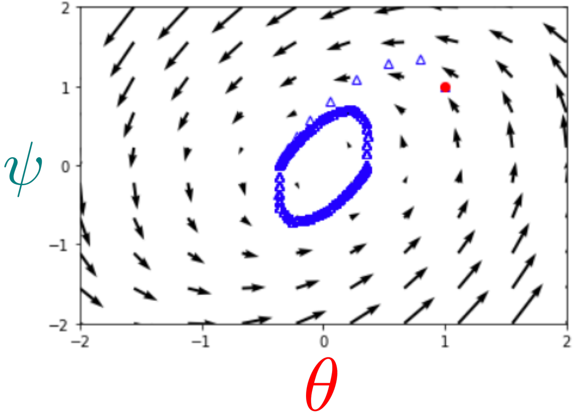

Figure from Mescheder et al. [ICML 2018]

WGAN toy example

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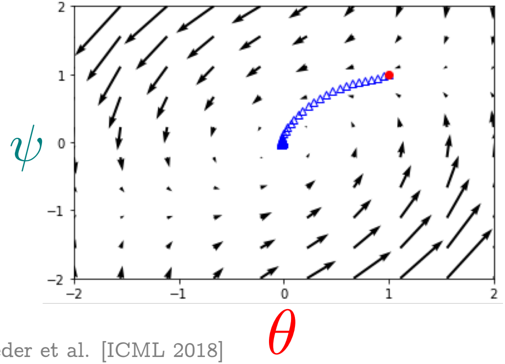
A solution? Modified control of witness gradient

$$\max_{\psi} \mathbf{E}_{X\sim P} f_\psi(X) - \mathbf{E}_{Z\sim R} f_\psi(G_\theta(Z)) + \underbrace{\lambda \mathbf{E}_{\widetilde{X}} \left\| \nabla_{\widetilde{X}} f_\psi(\widetilde{X}) \right\|^2}_{\text{new}}$$



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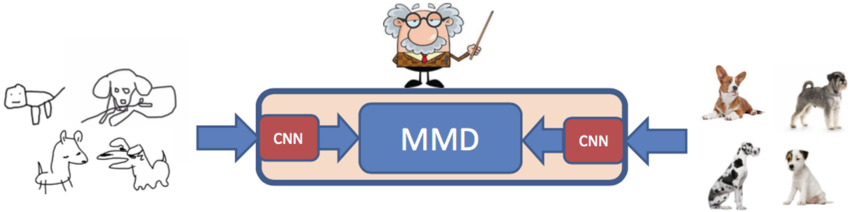
Gradient penalty: the regularisation viewpoint

CNN features for an MMD witness

- Add convolutional features!
- The critic (teacher) also needs to be trained.



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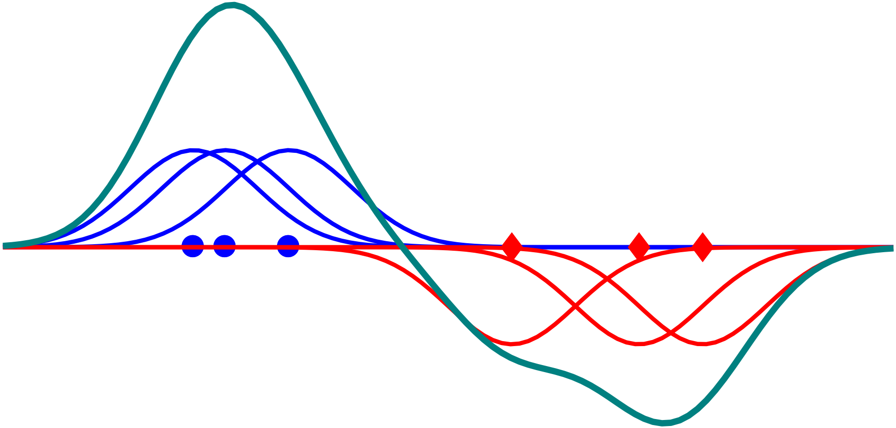
Witness function, kernels on deep features

Reminder: witness function,

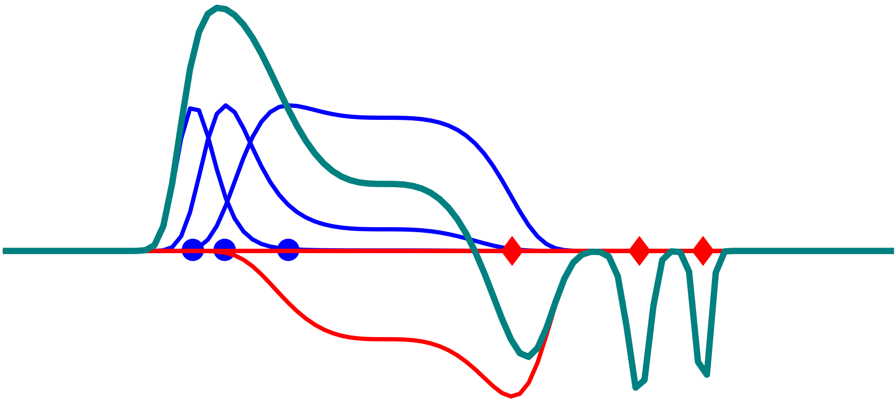
$k(x, y)$ is exponentiated quadratic



Witness function, kernels on deep features

Reminder: witness function,

$k(h_\psi(x), h_\psi(y))$ with nonlinear h_ψ and exp. quadratic k



Challenges for learned critic features

Learned critic features:

MMD with kernel $k(h_\psi(x), h_\psi(y))$ must give useful gradient to generator.

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Relation with test power?

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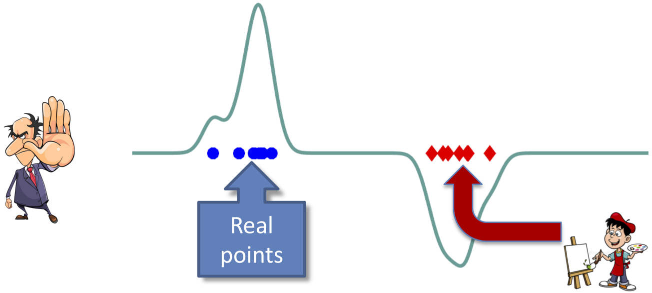
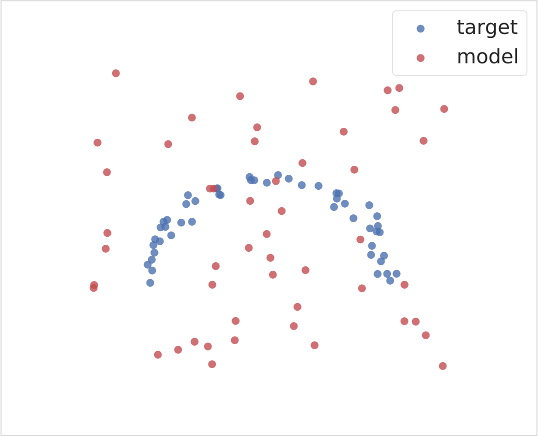


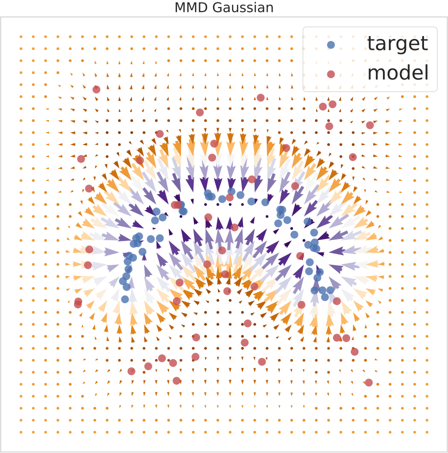
A simple 2-D example

Samples from target P and model Q



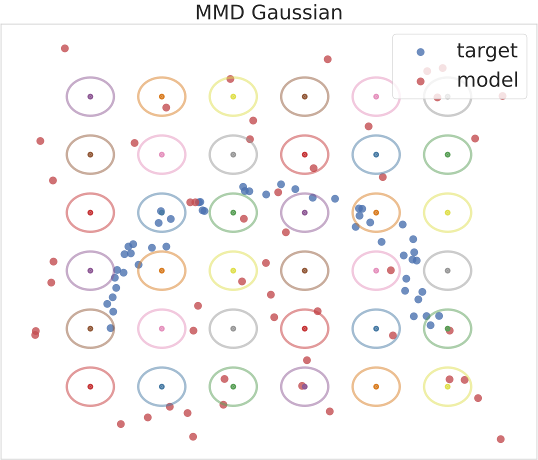
A simple 2-D example

Witness gradient, MMD with exp. quad. kernel $k(x, y)$



A simple 2-D example

What the kernels $k(x, y)$ look like



A data-adaptive gradient penalty: NeurIPS 2018

- New gradient regulariser Arbel, Sutherland, Binkowski, G. [NeurIPS 2018]
- Also related to Sobolev GAN Mroueh et al. [ICLR 2018]

On gradient regularizers for MMD GANs

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Modified witness constraint:

$$\widetilde{MMD} := \sup_{\boxed{\|f\|_S \leq 1}} [\mathbb{E}_P f(X) - \mathbb{E}_Q f(Y)]$$

where

$$\|f\|_S^2 = \underbrace{\|f\|^2_{L_2(P)}}_{\text{L_2 norm control}} + \underbrace{\|\nabla f\|^2_{L_2(P)}}_{\text{Gradient control}} + \underbrace{\lambda \|f\|_k^2}_{\text{RKHS smoothness}}$$

Maximise $\widetilde{MMD}$ wrt critic features

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Modified witness constraint:

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Problem: not computationally feasible: $O(n^3)$ per iteration.

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Maximise scaled MMD over critic features:

$$SMMD(P, \lambda) = \sigma_{P,\lambda} \; MMD$$

where

$$\sigma^2_{P,\lambda} = \lambda + \int k(h_\psi(x), h_\psi(x))\, dP(x) + \sum_{i=1}^{d} \int \partial_i \partial_{i+d} k(h_\psi(x), h_\psi(x)) \; dP(x)$$

Replace expensive constraint with cheap upper bound:

$$\|f\|_S^2 \leq \sigma^{-1}_{P,\lambda} \; \|f\|_k^2$$

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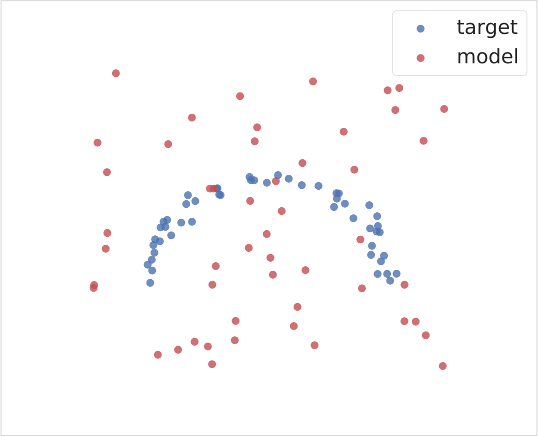
Idea: rather than regularise the critic or witness function, regularise features directly

Simple 2-D example revisited

Samples from target P and model Q



Simple 2-D example revisited

Use kernels $k(h_\psi(x), h_\psi(y))$ with features

$$h_\psi(x) = L_3\left(\begin{bmatrix} x \\ L_2(L_1(x)) \end{bmatrix}\right)$$

where L_1, L_2, L_3 are fully connected with quadratic nonlinearity.

Simple 2-D example revisited

Witness gradient, maximise $SMMD(P, \lambda)$
to learn $h_\psi(x)$ for $k(h_\psi(x), h_\psi(y))$

vector field movie, use Acrobat Reader to play

Simple 2-D example revisited

What the kenels $k(h_\psi(x), h_\psi(y))$ look like

isolines movie, use Acrobat Reader to play

Our empirical observations

Data-adaptive critic loss:

- Witness function class for $SMMD(P, \lambda)$ depends on P.
  - Without data-dependent regularisation, maximising MMD over features h_ψ of kernel $k(h_\psi(x), h_\psi(y))$ can be unhelpful.
  - WGAN-GP is a pretty good data-dependent regularisation strategy
- Similar regularisation strategies apply to variational form in f-GANs

  Roth et al [NeurIPS 2017, eq. 19 and 20]

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Roth et al [NeurIPS 2017, eq. 19 and 20]

Alternate critic and generator training:

- Weaker critics can give better signals to poor (early stage) generators.
- Incomplete training of the critic is also a regularisation strategy

Linear vs nonlinear kenels

- Critic features from DCGAN: an f-filter critic has f, $2f$, $4f$ and $8f$ convolutional filters in layers 1-4. LSUN 64×64.

$k(h_\psi(x), h_\psi(y))$, $f = 64$, KID=3

${h_\psi}^\top(x) h_\psi(y)$, $f = 64$, KID=4

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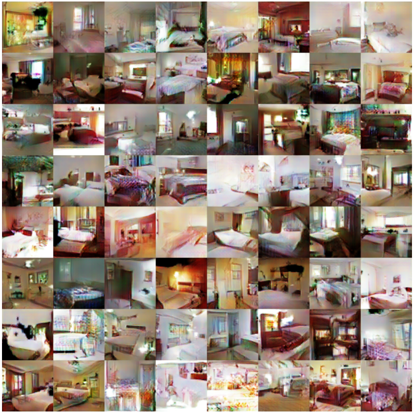
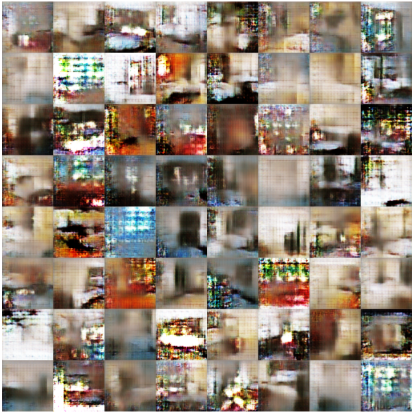


$k(h_\psi(x), h_\psi(y))$, $f = 16$, KID=9



${h_\psi}^\top(x) h_\psi(y)$, $f = 16$, KID=37

The theory

Scaled MMD vs Wasserstein-1 (NeurIPS 18)

Let $k_\psi = k \circ h_\psi$.

Wasserstein-1 bounds SMMD,

$$SMMD(P, Q) \leq \frac{Q_k \kappa^L}{d_L \alpha^L} \mathcal{W}(P, Q)$$

- Conditions on the neural network layers:
  - $h_\psi : \mathcal{X} \to \Re^s$ fully-connected L-layer network, Leaky-ReLU$_\alpha$ activations whose layers do not increase in width
  - Width of ℓth layer is d_ℓ.
  - κ is the bound on condition number of the weight matrices W^ℓ
- Conditions on the kernel and gradient regulariser:
  - k satisfying mild smoothness conditions, summarised in $Q_k < \infty$.
  - μ is a probabilty measure with support over $\mathcal{X}$,

$$\int k(x, x) d\mu(x) + \sum_{i=1}^{d} \int \partial_i \partial_{i+d} k(x, x) \; d\mu(x)$$

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Unbiased gradients of MMD, WGAN-GP (ICLR 18)

Subject to mild conditions on

- Critic mappings h_ψ (conditions hold for almost all feedforward networks: convolutions, max pooling, ReLU,....)
- kernel k (a growth assumption)
- Target distribution P, generator network $Y \sim G_\theta(Z)$ (densities not needed, second moments must exist),

Then for μ-almost all ψ, θ where μ is Lebesgue,

$$\mathbf{E}_{\substack{X\sim P\\ Z\sim R}}\left[\partial_{\psi,\theta} k(h_\psi(X), h_\psi(G_\theta(Z)))\right] = \partial_{\psi,\theta}\mathbf{E}_{\substack{X\sim P\\ Z\sim R}}\left[k(h_\psi(X), h_\psi(G_\theta(Z)))\right].$$

and thus MMD gradients unbiased.

Also true for WGAN-GP.

Bias of MMD GAN critic (ICLR 18)

Gradient bias when critic trained on a separate dataset?

Recall definition of MMD for P vs Q

$$MMD(P, Q; F) := \sup_{\|f\| \leq 1} \left[\mathbf{E}_P f(X) - \mathbf{E}_Q f(Y) \right]$$

$$(F = \text{unit ball in RKHS } \mathcal{F})$$

Define f_{tr} as discriminator witness trained on $\{x_i^{\mathrm{tr}}\}_{i=1}^m \overset{\text{i.i.d.}}{\sim} P$, $\{y_i^{\mathrm{tr}}\}_{i=1}^n \overset{\text{i.i.d.}}{\sim} Q$.

Then

$$\left[\mathbf{E}_P f_{tr}(X) - \mathbf{E}_Q f_{tr}(Y) \right] \leq MMD(P, Q; F)$$

Downwards bias. Unless bias is in f_{tr} constant, biased gradients too.

Same true for WGAN-GP.

Bias of MMD GAN critic (ICLR 18)

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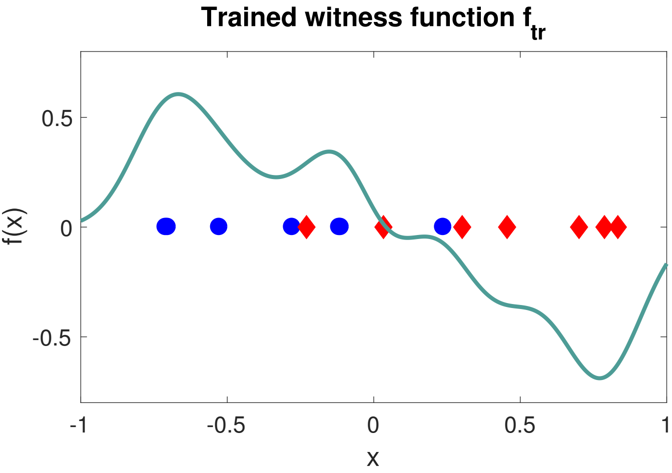
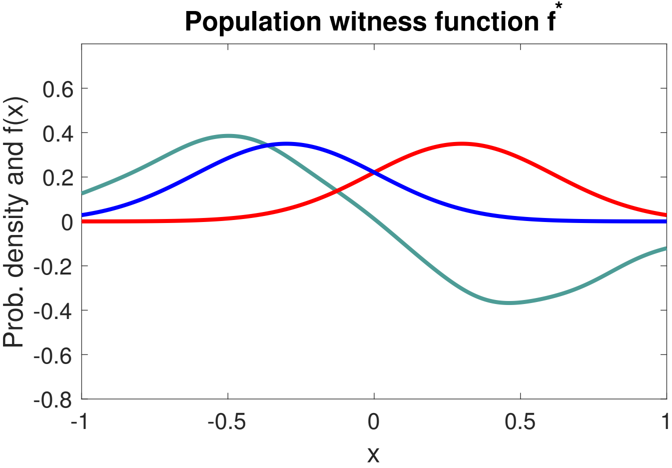
Training minibatch critic function f_{tr}



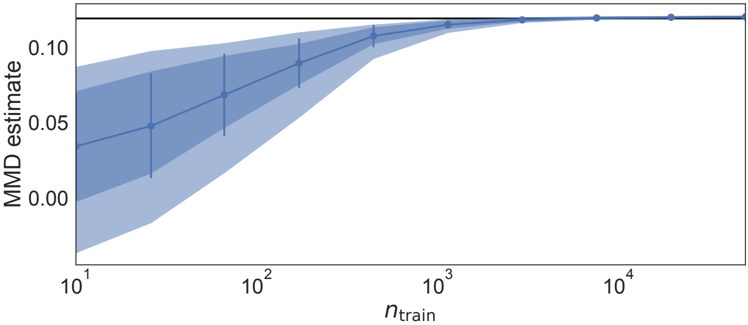
Bias of MMD GAN critic (ICLR 18)

Population critic function f^*



Bias of MMD GAN critic (ICLR 18)

Bias in MMD vs training minibatch size:



Evaluation and experiments

Evaluation of GANs

The inception score? Salimans et al. [NeurIPS 2016]

Based on the classification output $p(y|x)$ of the inception model Szegedy et al. [ICLR 2014],

$$E_X \exp KL(P(y|X) \| P(y)).$$

High when:

- predictive label distribution $P(y|x)$ has low entropy (good quality images)
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Problem: relies on a trained classifier! Can't be used on new categories (celeb, bedroom...)

Evaluation of GANs

The Frechet inception distance? Heusel et al. [NeurIPS 2017]

Fits Gaussians to features in the inception architecture (pool3 layer):

$$FID(P, Q) = \|\mu_P - \mu_Q\|^2 + \mathrm{tr}(\Sigma_P) + \mathrm{tr}(\Sigma_Q) - 2\mathrm{tr}\left((\Sigma_P \Sigma_Q)^{\frac{1}{2}}\right)$$

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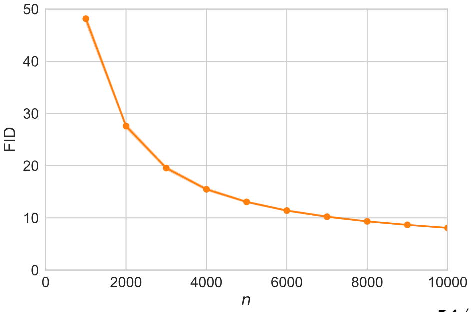
Problem: **bias**. For finite samples can consistently give incorrect answer.

- Bias demo, CIFAR-10 train vs test



Evaluation of GANs

The FID can give the wrong answer in theory.

Assume m samples from P and $n \to \infty$ samples from Q.

Given two alternatives:

$$P_1 \sim \mathcal{N}(0, (1 - m^{-1})^2) \qquad P_2 \sim \mathcal{N}(0, 1) \qquad Q \sim \mathcal{N}(0, 1).$$

Clearly,

$$FID(P_1, Q) = \frac{1}{m^2} > FID(P_2, Q) = 0$$

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Evaluation of GANs

The FID can give the wrong answer in practice.

Let $d = 2048$, and define

$$P_1 = \text{relu}(\mathcal{N}(\mathbf{0}, I_d)) \quad P_2 = \text{relu}(\mathcal{N}(\mathbf{1}, .8\Sigma + .2I_d)) \qquad Q = \text{relu}(\mathcal{N}(\mathbf{1}, I_d))$$

where $\Sigma = \frac{4}{d} CC^T$, with C a $d \times d$ matrix with iid standard normal entries.

For a random draw of C:

$$FID(P_1, Q) \approx 1123.0 > 1114.8 \approx FID(P_2, Q)$$

With $m = 50\,000$ samples,

$$FID(\widehat{P_1}, Q) \approx 1133.7 < 1136.2 \approx FID(\widehat{P_2}, Q)$$

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The kernel inception distance (KID)

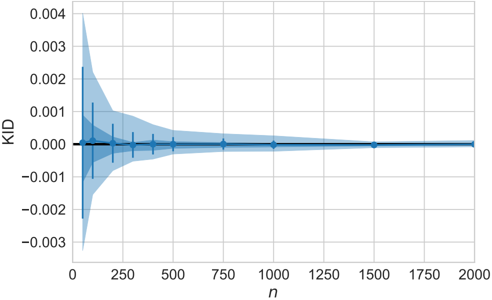
The Kernel inception distance Binkowski, Sutherland, Arbel, G. [ICLR 2018]

Measures similarity of the samples' representations in the inception architecture (pool3 layer)

MMD with kernel

$$k(x, y) = \left(\frac{1}{d}x^\top y + 1\right)^3.$$

- Checks match for feature means, variances, skewness
- Unbiased : eg CIFAR-10 train/test



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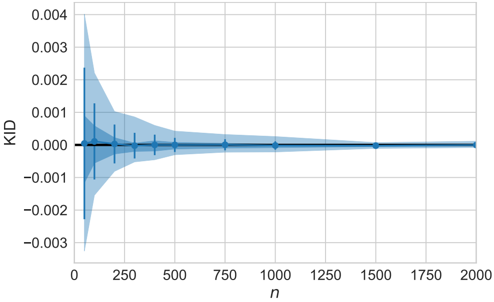

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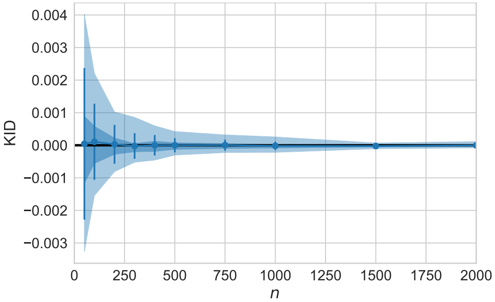

..."but isn't KID is computationally costly?"

"Block" KID implementation is cheaper than FID: see paper (or use Tensorflow implementation)!

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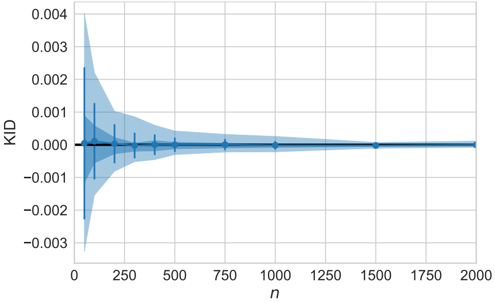

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Also used for automatic learning rate adjustment: if $KID(\widehat{P}_{t+1}, Q)$ not significantly better than $KID(\widehat{P}_t, Q)$ then reduce learning rate.

[Bounliphone et al. ICLR 2016]

Related: "An empirical study on evaluation metrics of generative adversarial networks", Xu et al. [arxiv, June 2018]

Benchmarks for comparison (all from ICLR 2018)

Spectral Normalization for Generative Adversarial Networks

Takeru Miyato[1], Toshiki Kataoka[1], Masanori Koyama[2], Yuichi Yoshida[3]
{miyato, kataoka}@preferred.jp
oyama asanori@gmail.com
yos i.ac.jp
works, Inc. [2]Ritsumeikan University [3]National Institute of Informatics

We combine with scaled MMD

Sobolev GAN

Youssef Mroueh[†], Chun-Liang Li[∘,⋆], Tom Sercu[†,⋆], Anant Raj[◊,⋆] & Yu Cheng[†]
† IBM Research AI
∘ Carnegie Mellon University
◊ Max Planck Institute for Intelligent Systems
⋆ denotes Equal Contribution
{mroueh,chengyu}@us.ibm.com, chunlial@cs.cmu.edu,
tom.sercu1@ibm.com, anant.raj@tuebingen.mpg.de

Demystifying MMD GANs

Mikołaj Bińkowski*
Department of Mathematics
Imperial College London
mikbinkowski@gmail.com

Dougal J. Sutherland*, Michael Arbel & Arthur Gretton
Gatsby Computational Neuroscience Unit
Uni College London
,michael.n.arbel,arthur.gretton}@gmail.com

Our ICLR 2018 paper

Boundary-Seeking Generative Adversarial Networks

R Devon Hjelm*
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Yoshua Bengio
MILA, University of Montréal, CIFAR, IVADO
yoshua.bengio@umontreal.ca

Results: unconditional imagenet 64×64

KID scores:

- BGAN: 47
- SN-GAN: 44
- SMMD GAN: 35

ILSVRC2012 (ImageNet) dataset, 1 281 167 images, resized to 64 × 64. 1000 classes.

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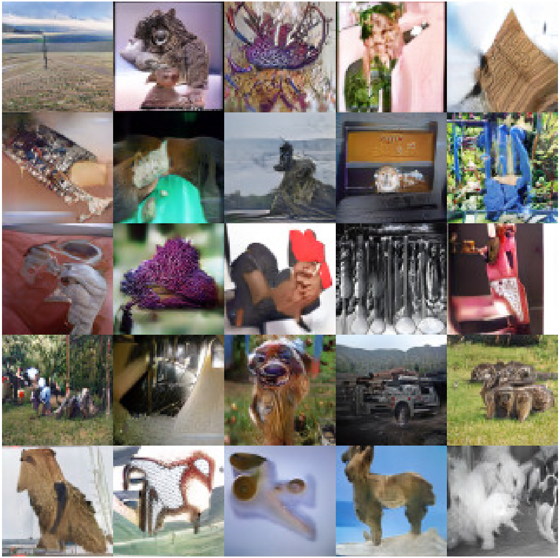

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Summary

- GAN critics rely on two sources of regularisation
  - Regularisation by incomplete training
  - Data-dependent gradient regulariser
- Some advantages of hybrid kernel/neural features:
  - MMD loss still a valid critic when features not optimal (unlike WGAN-GP)
  - Kernel features do some of the "work", so simpler h_ψ features possible.

"Demystifying MMD GANs," including KID score, ICLR 2018:
https://github.com/mbinkowski/MMD-GAN

Gradient regularised MMD, NeurIPS 2018:
https://github.com/MichaelArbel/Scaled-MMD-GAN

Post-credit scene: MMD flow

From NeurIPS 2019:

Maximum Mean Discrepancy Gradient Flow

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Questions?

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